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LUMINOSITY FUNCTION FOR TWO-PHOTON PROCESSES IN THE SINGLE TAGGING MODE

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An approximate formula is given for the luminosity function relating the processes ee → eeX and $\gamma^*\gamma^* \rightarrow$ X in the case where only one of the outgoing electrons is tagged whereas the other one is restricted to small angles (anti-tagging). The range of validity of this approach is discussed and a numerical comparison with the exact formula is given. It is also shown that multiple use of this formula can give the single tagging efficiency with good precision in cases where the anti-tagging condition is not imposed on the second electron.

1. Introduction

The general problem of calculating the QED factors (or 'luminosity functions') relating the processes ee → eeX and $\gamma^*\gamma^* \rightarrow$ X has been discussed, in a more general framework than that of the equivalent photon approximation, in a recent paper [1]. Expressions were given for the double tagged luminosity (both scattered electrons detected at angles $\gg m_e/E$ (E = beam energy, m_e = electron mass) and single tagged luminosity (only one electron detected). For the double tagging case no kinematical approximations were made, the luminosity functions being found by numerical integration of the exact transverse-transverse luminosity function [2, 3]. In the single tagging case, where it is necessary to consider angles $\lesssim m_e/E$ for the undetected electron, simplifying kinematical assumptions were made so that the integration over the variable Q_1^2 (see ref. [1] for all notation and definitions) could be done analytically. The present paper presents an exactly equivalent but algebraically simpler form of this approximate single tagging function and investigates its accuracy with different constraints on the 'unobserved' electron. The case of 'single tagging' (electron 1 unobserved over the whole angular range) is compared with 'single anti-tagging' or 'tagging by absence' [4] where it is required that the electron does *not* appear in one or more angular ranges. For the single tagging case multiple use of the approximate formula, in a way described in sect. 5, is found to

considerably increase the precision of the calculation.

A recent paper by Carimalo et al. [5] has also considered single tagging experiments where one virtual photon is quasi-real; equivalent to our assumption, in deriving the approximate formula, that $Q_1^2 \ll z^2$. Ref. [5] is more general in the sense that effects of longitudinal photons from the tagged electron are included and not all variables of the produced $\gamma\gamma$ system are integrated out. The aim of the present work is to give accurate and simple expressions for the purely transverse-transverse luminosity function useful for extracting total photon-photon cross sections.

2. Single tagged luminosity formulae

In ref. [1] a threefold differential luminosity function for tagging one electron (electron 2) at angle θ_2 ($\theta_2 \gg m_e/E$) where the other electron (electron 1) is at an arbitrary angle corresponding to the fully allowed range of Q_1^2: $Q^2_{\min} < Q^2 < Q^2_{\max}$, where

$$Q^2_{\min} = \frac{m_e^2}{4E^2}\frac{x_1^2}{1-x_1},$$

$$Q^2_{\max} = 1 - x_1,$$

was given (ref. [1], eq. (11)). A similar formula can also be written for the case where electron 1 is in the limited angular range

$$0 < \theta_1 < \theta_{\max}.$$

In this case $Q^2_{\max}$ is given by

$$Q^2_{\max} = (1-x_1)\sin^2\left(\tfrac{1}{2}\theta_{\max}\right).$$

The corresponding threefold differential luminosity function is then

$$\frac{d^3\mathcal{L}}{d\theta_2 dx_1 dx_2} = \frac{\alpha^2}{8\pi^2} K' \cot\tfrac{1}{2}\theta_2 \left[\frac{(K'-2x_1)^2}{K'^2} + 1\right] \times \left\{\left[\ln\frac{2E}{m_e}\frac{(1-x_1)}{x_1}\sin\tfrac{1}{2}\theta_{\max}\right]\left[\frac{\{K'-2(x_2+Q_2^2)\}^2}{K'^2}+1\right] - 2\frac{(1-x_1)}{x_1^2}\right\}, \qquad K' \equiv z^2 + Q_2^2. \tag{1}$$

Formula (1) is apparently 'non-factorising', i.e. variables of both scattered electrons appear together in the factors of the formula. However, it is easy to show when the

approximate relation for z^2 (eq. (10) of ref. [1]) is substituted in eq. (1) the following factorisable form is found:

$$\frac{d^3\mathcal{L}}{d\theta_2 dx_1 dx_2} = \frac{\alpha^2}{8\pi^2} f_1(x_1, \theta_{max}) f_2(x_2, \theta_2),$$

$$f_1(x_1, \theta_{max}) = \left[\ln \frac{2E}{m_e} \frac{(1-x_1)}{x_1} \sin \tfrac{1}{2}\theta_{max}\right]\left[\frac{(x_1-2)^2}{x_1} + x_1\right] - 2\frac{(1-x_1)}{x_1},$$

$$f_2(x_2, \theta_2) = \left[\frac{(x_2 + Q_2^2 - 2)^2}{x_2 + Q_2^2} + x_2 + Q_2^2\right] \cot \tfrac{1}{2}\theta_2,$$

$$Q_2^2 = (1-x_2)\sin^2 \tfrac{1}{2}\theta_2. \tag{2}$$

The factorisability is clearly an advantage for use in Monte Carlo generators. Another advantage is that it is possible to perform the integration over the angular variable of electron 2 analytically, so that the differential luminosity of experimental interest, $d\mathcal{L}/dz$, requires numerical integration over only one variable, x_1. Changing variables in eq. (2) from θ_2, x_2, x_1 to Q_2^2, z, x_1, gives

$$\frac{d^3\mathcal{L}}{dQ_2^2 dz dx_1} = \frac{\alpha^2}{2\pi^2} z \frac{f_1}{x_1^2} \frac{1}{Q_2^2}\left[z^2 + Q_2^2 + \frac{2x_1^2}{z^2 + Q_2^2} - 2x_1\right]. \tag{3}$$

Integrating over Q_2^2,

$$\frac{d^2\mathcal{L}}{dz dx_1} = \frac{\alpha^2}{2\pi^2} z \frac{f_1}{x_1^2}\left[\left(\frac{2x_1^2}{z^2} + z^2 - 2x_1\right)\ln Q_2^2 - \frac{2x_1^2}{z^2}\ln\left(z^2 + Q_2^2\right) + Q_2^2\right]_{Q_L^2}^{Q_U^2} \tag{4}$$

$Q_{U,L}^2$ are the upper and lower limits of the integration. For an angular range $\theta_L < \theta_2 < \theta_U$, $Q_{U,L}^2$ are given by

$$Q_{U,L}^2 = \frac{(1 - z^2/x_1)\sin^2 \tfrac{1}{2}\theta_{U,L}}{\left[1 + (1/x_1 - 1)\sin^2 \tfrac{1}{2}\theta_{U,L}\right]} \tag{5}$$

Eq. (5) follows from the definition of Q^2 and eq. (10) of ref. [1]. $d\mathcal{L}/dz$ can now be found from eqs. (4), (5) by numerical integration over x_1.

3. Experimental tagging constraints

The use of eq. (4) and, in particular the accuracy of the calculation, depend crucially on the configuration of the experimental tagging system and whether or not

information is available on the presence or absence of electrons *outside* the acceptance of the tagging system itself. Two extreme cases may be considered:

(i) Single tagging. In this case, one electron is seen in the tagging detector, but the experimenter is (intentionally or unintentionally) 'blind' to the presence of a second electron, even if it occurs in a second, symmetrically placed tagging detector. This is the definition of 'single tagging' used in ref. [1]. To calculate the tagging efficiency the 'unobserved' electron is integrated over its full kinematic range from $Q^2_{\min}$ to $Q^2_{\max} = 1 - x_1$.

(ii) Single anti-tagging. This type of experiment, considered first in ref. [4], assumes that one electron is detected in a certain range of polar angle $\theta_L < \theta < \theta_U$ say, and no other electron is seen in the range $\theta_L < \theta < \pi - \theta_L$. It is then inferred that the unobserved electron must lie in the angular cone of half angle θ_L about the beam opposite in direction to that of the detected electron. The tagging efficiency is given by integrating the Q^2 of the unobserved electron from $Q^2_{\min}$ to $Q^2 = (1 - x_1)\sin^2 \frac{1}{2}\theta_L$.

In practice the single anti-tagging possibility has only been realised on the compact PLUTO detector at PETRA. Other detectors at PETRA and PEP all have a dead region between the tagging system and the rest of the detector. The approximate angular acceptances of the tagging systems, and the size of the dead region of some of the PETRA detectors are as follows:

Detector	Tagging acceptance (mrad)	Dead region (mrad)
PLUTO	23–70	none
CELLO	25–50	50–153
TASSO	18–60	60–207
JADE	34–70	70–240

To use the 'anti-tag' definition on CELLO, TASSO and JADE allowance must be made for the possibility that an electron enters the dead region. This can clearly be done by suitably modifying the range of integration. However, careful attention must be paid to the accuracy of the formula in this case, as the angles of the scattered electrons are now large. This problem is considered below.

4. Accuracy of the approximate formula

'Accuracy' as used here refers only to errors resulting from kinematical approximations. It should not be forgotten, that for any specific process, the observed cross section may be much more affected by the neglect, here and in ref. [ ], of longitudinal photons, and by momentum transfer dependances of the virtual photon-photon processes. We are here concerned only with well identifiable errors in the purely QED factors.

If θ_{max} in eq. (2) is sufficiently small it is expected from general arguments that eqs. (2)–(4) will be accurate. The essential condition for deriving eq. (11) of ref. [1] from the general expression eq. (9) is that

$$Q_1^2 \ll z^2.$$

This will be a good approximation if θ_{max} satisfies

$$\sin^2 \tfrac{1}{2}\theta_{max} \sim \tfrac{1}{4}\theta_{max}^2 \ll z^2.$$

In practice, at PETRA energies, only values of $z \gtrsim 0.03$ are of interest, implying that

$$\theta_{max} \ll 60 \text{ mrad}.$$

Note that the condition for good accuracy of eqs. (2)–(4) relates the tagging angle (usually determined by purely technical considerations) to the energy scaled variable z. So if the formula has a given accuracy at a mass of 900 MeV or $z = 0.03$ for the $\gamma\gamma$ system at PETRA energies ($E = 15$ GeV), it will have similar accuracy only for masses $\gtrsim 6$ GeV at LEP energies ($E = 100$ GeV). For smaller masses the exact transverse-transverse function (eq. (9) of ref. [1]) is needed.

To study the accuracy of eq. (4), an angle θ_0 is chosen satisfying the condition $\frac{1}{4}\theta_0^2 \ll z^2$, but sufficiently large that for $\theta > \theta_0$ the m_e/E^2Q^2 terms in eq. (9) of ref. [1] may be dropped. A suitable choice is $\theta_0 = 100 m_e/E \sim 3{\cdot}4$ mrad ($E = 15$ GeV). An accurate estimation of the luminosity function in the region from θ_0 to θ_{max} can then be found by 4-dimensional numerical integration as described in ref. [1]. Notice that since the Q_2^2 integration has now been done analytically eq. (4) above is completely equivalent to eq. (11) of ref. [1] where a twofold numerical integration over Q_2^2 and x_1 was required. So the accuracy of eq. (4) is found by comparing two expressions for $\mathrm{d}\mathcal{L}/\mathrm{d}z$:

'exact' calculation:

$$\frac{\mathrm{d}\mathcal{L}^{ex}}{\mathrm{d}z} = \int \frac{\mathrm{d}^2\mathcal{L}}{\mathrm{d}z\,\mathrm{d}x_1}(\theta_U, \theta_L, \theta_0, z)\,\mathrm{d}x_1$$

$$+ \int_{\theta_L}^{\theta_U} \mathrm{d}\theta_2 \int_{\theta_0}^{\theta_{max}} \mathrm{d}\theta_1 \int \mathrm{d}\phi \int \mathrm{d}x_1 \frac{\mathrm{d}^5\mathcal{L}}{\mathrm{d}z\,\mathrm{d}\theta_2\,\mathrm{d}\theta_1\,\mathrm{d}\phi\,\mathrm{d}x_1};$$

'approximate' calculation:

$$\frac{\mathrm{d}\mathcal{L}^{ap}}{\mathrm{d}z} = \int \frac{\mathrm{d}^2\mathcal{L}}{\mathrm{d}z\,\mathrm{d}x_1}(\theta_U, \theta_L, \theta_{max}, z)\,\mathrm{d}x_1.$$

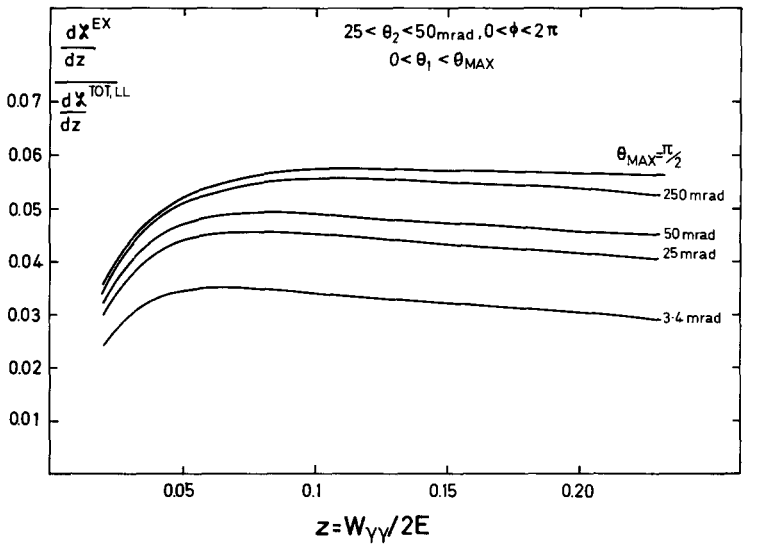

Fig. 1. Differential luminosity function for various angular tagging intervals.

$d^2\mathcal{L}/dz\,dx_1$ is given by eq. (4) and $d^5\mathcal{L}/dz\,d\theta_2\,d\theta_1\,d\phi\,dx_1$ by eq. (9) of ref. [1], with exchange of x_2 for z as variable and neglect of the m_e^2/E^2Q^2 terms. Fig. 1 shows curves of the ratio $(d\mathcal{L}^{ex}/dz)/(d\mathcal{L}^{tot,LL}/dz)$ where $d\mathcal{L}^{tot,LL}/dz$ is the leading log approximation to the total luminosity (eq. (6) of ref. [1]). $\theta_L, \theta_U = 25$, 50 mrad are chosen and a range of θ_{max} values are taken. More than 50% of the luminosity can be seen to lie at angles less than 3.4 mrad for the unobserved electron. It is also evident that the simple $\ln(\theta_{max}/\theta_{min})$ law for the angular dependence of the luminosity that follows from the equivalent photon approximation breaks down badly for $\theta_{max} > 250$ mrad. Table 1 compares $(d\mathcal{L}^{ex}/dz)/(d\mathcal{L}^{tot,LL}/dz)$ with $(d\mathcal{L}^{ap}/dz)/(d\mathcal{L}^{tot,LL}/dz)$ for $\theta_{max} = 0.025, 0.050, 0.25, \frac{1}{2}\pi$ rad and various values of z. The accuracy of the approximate calculation is $\lesssim 1\%$ for $\theta_{max} = 25, 50$ mrad. It is a few percent at $\theta_{max} = 250$ mrad, and $\sim 20\%$ for $\theta_{max} = \frac{1}{2}\pi$. Eq. (4) used in the straightforward way shown in table 1 therefore gives an accurate estimate of 'single anti-tag' luminosity with $\theta_{max} \sim 20$ mrad as used by PLUTO, but only a rather poor estimate of the single tagging efficiency where the unobserved electron is integrated over the full angular range.

The error in the numerical integration for the entries in table 1 is estimated to be of the order of $\lesssim 1\%$ in the worst case ($z = 0.02$).

It turns out however that by using eq. (4) in a slightly more complicated way it can also give much more precise estimates of the single tagging efficiency. How this may be done is described in the next section.

TABLE 1

$R^{ex} = \frac{d\mathcal{L}^{ex}}{dz} \Big/ \frac{d\mathcal{L}^{tot,LL}}{dz}$, $R^{ap} = \frac{d\mathcal{L}^{ap}}{dz} \Big/ \frac{d\mathcal{L}^{tot,LL}}{dz}$
$25 < \theta_2 < 50$ mrad $\Delta = (R^{ap} - R^{ex})/R^{ex}$
$0 < \phi < 2\pi$ $E = 15$ GeV

			$\theta_{max} = 25$ mrad		
z	0.02	0.06	0.11	0.16	0.21
R^{ap}	0.03049	0.04543	0.04476	0.04296	0.04130
R^{ex}	0.02987	0.04502	0.04458	0.04286	0.04120
Δ(%)	2.0	0.9	0.4	0.2	0.2
			$\theta_{max} = 50$ mrad		
R^{ap}	0.03265	0.04906	0.04862	0.04689	0.04525
R^{ex}	0.03228	0.04832	0.04830	0.04671	0.04509
Δ(%)	1.1	1.5	0.7	0.4	0.4
			$\theta_{max} = 250$ mrad		
R^{ap}	0.03765	0.05745	0.05759	0.05600	0.05441
R^{ex}	0.03410	0.05292	0.05596	0.05440	0.05332
Δ(%)	10.4	8.6	2.9	2.9	2.0
			$\theta_{max} = \frac{1}{2}\pi$		
R^{ap}	0.04305	0.06653	0.06727	0.06583	0.06430
R^{ex}	0.03414	0.05337	0.05729	0.05672	0.05659
Δ(%)	26.0	24.7	17.4	16.1	13.6

5. Use of the approximate formula to give single tagging efficiency with good precision

The poor accuracy of eq. (4) for $\theta_{max} \gtrsim 250$ mrad is due to the condition $Q_1^2 \ll z^2$, used in the derivation, being badly violated. However, the typical range of tagging angles for the *observed* electron is still such that the condition $Q_2^2 \ll z^2$ is not badly violated (the forward tagging systems of all the PETRA detectors have maximum angles less than 80 mrad). Suppose the single tagging efficiency is needed for the angular range $25 < \theta_2 < 50$ mrad. As shown above the contribution C_1 to the luminosity for

$$0 < \theta_1 < 3.4 \text{ mrad}, \qquad 25 < \theta_2 < 50 \text{ mrad},$$

is given to good accuracy for $z \gtrsim 0.01$ by straightforward use of eq. (4). It remains to calculate the remaining contribution (to compare with the values given in table 1, θ_1

is integrated up to $\frac{1}{2}\pi$ rather than π: this changes the luminosity function by only a fraction of a percent)

$$C_2\text{: } 3.4 \text{ mrad} < \theta_1 < \tfrac{1}{2}\pi, \qquad 25 < \theta_2 < 50 \text{ mrad}.$$

As eq. (4) makes no kinematical approximations for electron 2 this can be done by reversing the labels of electrons 1 and 2 and using the equation twice:

$$\text{for } 0 < \theta_1 < 50 \text{ mrad}, \qquad 3.4 \text{ mrad} < \theta_2 < \tfrac{1}{2}\pi,$$

$$\text{and } 0 < \theta_1 < 25 \text{ mrad}, \qquad 3.4 \text{ mrad} < \theta_2 < \tfrac{1}{2}\pi.$$

Subtracting these gives the luminosity function for

$$25 < \theta_1 < 50 \text{ mrad}, \qquad 3.4 \text{ mrad} < \theta_2 < \tfrac{1}{2}\pi.$$

Exchanging the labels this is identical to C_2. Adding now C_1 and C_2 gives the desired single tagged luminosity function for the range:

$$0 < \theta_1 < \tfrac{1}{2}\pi, \qquad 25 < \theta_2 < 50 \text{ mrad}.$$

Numerical results of the calculation are given in table 2. Agreement is found with $(\mathrm{d}\mathfrak{L}^{\mathrm{ex}}/\mathrm{d}z)/(\mathrm{d}\mathfrak{L}^{\mathrm{tot,LL}}/\mathrm{d}z)$ at a level of better than one percent, except for the lowest 2 values of $z = 0.02$, 0.06 where errors of 12%, 3.2%, respectively, are found.

TABLE 2
Single tagging luminosity function calculation by multiple use of eq. (4) (definitions as in table 1)

$L1$:	$0 < \theta_1 < 3.4$ mrad;		$25 < \theta_2 < 50$ mrad		
	$z = 0.02$	0.06	0.11	0.16	0.21
R^{ap}	0.02429	0.03501	0.03363	0.03166	0.02992
$L2$:	$0 < \theta_1 < 50$ mrad;		$3.4 < \theta_2 < \frac{1}{2}\pi$		
R^{ap}	0.2112	0.27187	0.29497	0.30490	0.30960
$L3$:	$0 < \theta_1 < 25$ mrad;		$3.4 < \theta_2 < \frac{1}{2}\pi$		
R^{ap}	0.1973	0.25178	0.27149	0.27935	0.28258
$L2 - L3$:	$25 < \theta_1 < 50$ mrad;		$3.4 < \theta_2 < \frac{1}{2}\pi$		
R^{ap}	0.01390	0.02009	0.02348	0.02555	0.02702
$L1 + (L2 - L3)$:	$25 < \theta_1 < 50$ mrad;		$0 < \theta_2 < \frac{1}{2}\pi$		
R^{ap}	0.03820	0.05510	0.05711	0.05721	0.05694
R^{ex}	0.03414	0.05337	0.05729	0.05672	0.05659
$\Delta(\%)$	12.0	3.2	−0.3	0.9	0.6

The extension of this calculation to allow for dead areas is straightforward. Notice that the subtracted form of eq. (4) used as an intermediate step in this calculation does in fact give a *double tagging* luminosity function valid for the case where electron 1 is at small angles ($Q_1^2 \ll z^2$) and electron 2 is at arbitrary angles. For many double tagging applications it may be sufficiently accurate to use eq. (4) (1-dimensional numerical integration) rather than the formulae of ref. [1] where, in the general case of no kinematical restriction, a 4-dimensional numerical integration is needed.

6. Summary and conclusions

The approximate single tagging luminosity function, given in eq. (11) of ref. [1], has been shown to reduce to a factorisable form which can be integrated analytically over the variable Q_2^2, thus yielding a twofold differential luminosity function in the variables x_1, z – eq. (4) above. The accuracy of this formula for different angular ranges of tagging has been studied by comparison with the general expression for the transverse-transverse luminosity function – eq. (9) of ref. [1]. The results of this comparison are shown in table 1. The precision is of the order of a percent or better down to values $W_{\gamma\gamma}/2E = z$ as low as 0.02 for angles of the unobserved electron below 50 mrad. However integrating over the full angular range the approximate formula overestimates the tagging efficiency by $\sim 20\%$ for z values $\lesssim 0.1$. The single tagging luminosity curves given in ref. [1] will have errors of a similar magnitude. However, by multiple use of eq. (4) a much more accurate estimate of the single tagging efficiency can be made. Such a calculation is presented in table 2. Use of the formula to give an anti-tagging efficiency where the unobserved electron is required to be at angles below 20 mrad, as done by the PLUTO collaboration in their analysis of multihadronic 2γ events, is expected to have errors below the 1% level, even for z values as low as 0.02.

Finally, it should be stressed again, that 'accuracy' is used above only in the sense of well identifiable errors in the QED factors in the cross section, other effects such as the Q^2 dependance of the virtual $\gamma\gamma$ cross section, and the effect of longitudinal photons have been completely neglected.

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