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Measuring the Charges of QCD Jets

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Abstract. We investigate the effects of perturbative branching upon the accuracy with which one can determine the charge of the underlying QCD quantum from the charge structure of a given hadronic jet. We show explicitly how at asymptotic Q^2 we lose all such charge information. We investigate these effects at current PETRA energies using the Monte Carlo program of Fox and Wolfram; and find that a reasonably accurate charge determination is still possible at these energies. We suggest the variation of the jet charge structure with multiplicity, at a given energy, as a sensitive probe of the onset of perturbative branching inside jets.

1. Introduction

The problem of identifying the charge of the QCD quantum that gives rise to a given jet of hadrons is clearly important. In two previous papers [1, 2] we have shown how to construct charge measures that are insensitive to charge fluctuations arising during the soft hadronisation. In this paper we investigate how the accuracy of such charge measures is affected by perturbative bremsstrahlung.

We shall work throughout in the leading logarithm approximation (LLA) [3], or close variants there-of, both in our analytic calculations and in our numerical calculations where we use the jet Monte Carlo program of Fox and Wolfram [4, 5].

First we review the derivation of the charge measure of [2] for soft jets, and point out the qualitative impact of perturbative branching. We follow this with a quantitative picture of quark jets at very high Q^2. We illustrate all this with a quark jet possessing an energy of about 10^{15} GeV—the grand unification (GUTS) scale. We then explicitly demonstrate how at truly asymptotic Q^2 *all* charge information (event-by-event) is lost.

Having clarified what happens at very high Q^2, we return to the practical question of what happens at PETRA energies. At such intermediate energies analytic calculations are unlikely to be reliable, so we perform our calculations with the Fox and Wolfram Monte Carlo [4] which contains both (modified) LLA perturbative branching and subsequent hadronisation. We shall find that the perturbative aspect is indeed important at PETRA energies; nonetheless the reliability of our charge measure is in fact roughly as good as was estimated in our previous work [2], so that we can expect to be able to infer the quark charge reliably in most events.

2. A Confinement-Safe Charge Measure

In this section we briefly describe the charge measure that was introduced in [1, 2]. The reader should consult those references for further details [6].

Consider a jet of hadrons produced from some initial quark, q. We suppose this jet to be dominated by the soft hadronisation process so that its properties are similar to those of the jets observed in ordinary soft hadronic collisions. In particular assume (i) the jet possesses short range charge correlations (SRCC); (ii) a uniform and finite density of hadrons in rapidity (away from the fragmentation region); (iii) a leading particle effect; and (iv) small transverse momenta.

We label the hadrons in the jet by their rapidities. In units of rapidity let the charge correlation length be l_c (which we expect to be about 1 or 2 units). Break up the rapidity interval occupied by the jet into adjacent sections of length l_c as in Fig. 1. Label the faces of these sections $1, 2, \ldots, M$ as shown, and their rapidities by $y(1), \ldots, y(M)$. Now, consider the net charge of the hadrons with $y > y(i)$; call it $Q(q; i)$. This net charge will equal the charge of q (since by the leading particle effect the quark q should lie in one of the fastest hadrons, with $y > y(i)$) plus the charge flowing from the part of the event

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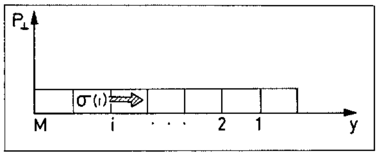

Fig. 1. The jet of hadrons, plotted against rapidity and transverse momentum, is partitioned into boxes whose linear dimension is the charge correlation length l_c. The jet is effectively one-dimensional so that its transverse dimension is less than l_c. The initial quark typically will be in the first box, and the charge flowing across the i'th face will be some antiquark charge plus some random charge fluctuation $\sigma(i)$ as shown

with $y < y(i)$ into the part of the jet with $y > y(i)$. Since the hadrons with $y > y(i)$ have net mesonic quantum numbers (disregarding the unlikely case that $y(i)$ separates a baryon antibaryon pair) this charge flow across $y = y(i)$ will consist of an antiquark charge plus some integer valued charge fluctuation centred about zero. So we have

$$Q(q;i) = q + \langle \bar{q} \rangle \pm \sigma \tag{1}$$

where σ is the standard deviation of the zero-centred charge fluctuation. Now by SRCC the σ fluctuations across different boundaries i and j are independent, so if we average $Q(q;i)$ from $i = 1$ to M we obtain

$$Q_M(q) = \frac{1}{M} \sum_{i=1}^{M} Q(q;i) = q + \langle \bar{q} \rangle \pm \frac{\sigma}{\sqrt{M}} \tag{2}$$

This is the simplest version of the charge measure introduced in [1, 2]. Note that it is designed to be applied to a single jet, rather than averaged over many events, and note also how the precision of the measure increases rapidly with M. $\bar{q}$ is of course calculable; in the case when we only include $\bar{u}$ and $\bar{d}$ we have $\langle \bar{q} \rangle = -\frac{1}{6}$.

Now in the above we assumed that the incident quark always ends up in a hadron with $y > y(i)$ for any i. In reality there will be some probability for this quark to be slowed down below $y = y(i)$. Call this probability $P_q(W_i)$ where $\frac{1}{2} W_i$ is the energy of the portion of the jet with $y > y(i)$ in a frame where $y(i) = 0$. By the leading particle hypothesis $P_q(W_i)$ should fall rapidly with increasing W_i; and indeed an analysis [7] of ν and $\bar{\nu}$ data [8] suggests that at moderate Q^2,

$$P_q(W_i) \approx \frac{1.4}{W_i} \tag{3}$$

for W_i not too small. It is also possible that σ should vary with i, in which case we must use $\sigma(i)$ in our calculations. We then find that the appropriate generalisation of (2) is

$$Q_M(q) = \sum_{i=1}^{M} Q(q;i) \frac{[1 - P_q(W_i)]}{\sigma^2(i)} \cdot \left\{ \sum_{i=1}^{M} \frac{[1 - P_q(W_i)]^2}{\sigma^2(i)} \right\}^{-1}$$

$$= q + \langle \bar{q} \rangle \pm \left\{ \sum_{i=1}^{M} \frac{[1 - P_q(W_i)]^2}{\sigma^2(i)} \right\}^{-1/2} \tag{4}$$

Note the way the terms in the above sum are weighted. A given term $Q(q,i)$ will contribute the less to the charge measure the smaller the probability that it contains the initial quark, i.e. the larger is $P_q(W_i)$; and also the larger is the random fluctuation $\sigma(i)$. This is just common-sense; in addition the charge measure of (4) is designed to be the "best possible" measure given our assumptions.

We now turn to a preliminary discussion of the expected effects of perturbative branching upon our initial assumptions and the consequent accuracy of the charge measure.

(*i*) We expect that the SRCC property of the soft hadronisation should be unaffected by any prior perturbative branching. However, the branching process itself violates SRCC: a quark antiquark pair produced early on in the branching will, at the point prior to hadronisation, be far apart in phase space and hence will end up in hadrons far apart in phase space. If one hadron is at $y(I)$ and the other at $y(J)$ then we see that the extra error induced in the charge measure, (2), is

$$\delta Q_M(q) \approx \frac{I - J}{M} x \text{ quark charge} \tag{5}$$

So if $I - J$ is comparable to M the measure loses all precision.

(*ii*) Perturbative branching increases the number of quarks and gluons that hadronise in a given jet and hence will increase the density of hadrons per l_c interval; call it $n(i)$. We expect $\sigma(i) \propto \sqrt{n(i)}$, and we see from (4) that the error on the charge measure increases as

$$\delta Q_M(q) \propto n^{1/2} \tag{6}$$

(*iii*) Perturbative bremsstrahlung implies that the incident quark loses momentum before hadronisation. This weakens the leading particle effect in the quark jet, reducing the coefficients $[1 - P_q(W_i)]$ in (4), with an effect on the error that one can see quantitatively in (4).

(*iv*) The transverse momenta in a jet increase with increasing perturbative branching. This does not directly affect our charge measure, but it will in general falsify our claim that this is the "best possible" charge measure.

When a jet grows in the transverse as well as in the longitudinal directions, SRCC implies that it can be decomposed three dimensionally into boxes of length l_c such that charge fluctuations across the faces of different boxes are independent whatever their orientation. We can still apply our one dimensional measure to such a jet but it is clear that we are then only using a little of the information provided by SRCC and should expect that better measures will be possible.

3. Properties of QCD jets

In developing a semi-quantitative understanding of the way QCD jets develop we shall work within the leading log approximation [3] (LLA). While this should give us a good insight into how things change with Q^2, the reader should beware of taking the results too literally at the quantitative level. For example the rapid multiplicity increase with Q^2 shall be one of our primary concerns; but the detailed LLA prediction [9] is not expected to be accurate [10]. This arises because the average multiplicity has the schematic form [9]

$$\langle n \rangle \sim \exp(a\sqrt{\ln Q^2}) \tag{7}$$

so that the average x is

$$\langle x \rangle \sim \langle n \rangle^{-1} \sim \exp(-a\sqrt{\ln Q^2}) \tag{8}$$

and hence

$$\ln^2 \langle x \rangle \sim \ln Q^2 \tag{9}$$

Thus factors of $\ln Q^2$ can be outweighed by factors of $(\ln x)^2$ and lower order terms are important. This will change the parameter a from the LLA result, but the form of (7) will probably remain unchanged [10]. This illustrates our earlier caution.

We shall use the LLA to tell us how a quark of virtuality $\sim Q^2$ turns into a "jet" of quarks and gluons of virtuality $Q_0^2 \sim 1\,\text{GeV}^2$, which is the lowest Q_0^2 at which we can pretend to have any confidence in the LLA. The LLA development of a QCD jet is very slow, being primarily governed by the variable ($N_{c,f} = n\bar{o}$ colours, flavours)

$$Y = \frac{6}{11N_c - 2N_f} \log\left[\frac{\alpha_s(\mu^2)}{\alpha_s(Q^2)}\right] = \frac{6}{11N_c - 2N_f} \log\left[\frac{1}{\alpha_s(Q^2)}\right] \tag{10}$$

where we have taken (as we shall continue to do from now on) $\mu^2 = Q_0^2$ and $\alpha_s(Q_0^2) = 1$, which corresponds to $\Lambda \approx 500$ MeV. (We shall for simplicity be careless about the fact that N_f increases with Q^2). Of course, we are primarily interested in the structure of the quark jet in terms of its final observable hadrons, rather than in terms of some intermediate partons. We shall make the conventional assumption that the number of hadrons is roughly proportional to the number of partons of virtuality $Q_0^2 \sim 1\,\text{GeV}^2$, and also that the phase space distribution of these hadrons closely follows the distribution of the partons which is not an unreasonable assumption in any fragmentation/recombination picture of hadronisation.

We will now go through the list of assumptions (*i*)–(*iii*) that we made in Sect. 2 and see what happens in each case as Q^2 becomes large.

(*i*) Let $x_v(Q_0^2)$ be the fraction of its original momentum that the initial "valence" quark possesses when its virtuality has been degraded to Q_0^2. If Q^2 is small then $x_v(Q_0^2) \sim 1$, there is little perturbative branching, and the situation is essentially as in the naive parton model (to which our charge measure best applies) where any eventual slowing down of the quark occurs during the hadronisation stage. As Q^2 increases the LLA predicts that

$$\langle x_v \rangle = e^{-(16/9)Y} \approx [\alpha_s(Q^2)]^{0.46}. \tag{11}$$

This is a slow decrease (in going from PETRA to GUTS energies $\langle x_v \rangle$ decreases by about a factor of 4). Moreover the absolute decrease is in itself not particularly relevant to us; we are more interested in whether the number of hadrons which are faster than the hadron containing the valence quark increases with Q^2 or not. We translate this question into the corresponding question in terms of the quarks and gluons of virtuality $Q_0^2 \sim 1\,\text{GeV}^2$. In [3] an expression for the number of such non-valence quanta possessing $x > x_0$ has been derived for small x_0 in the LLA. In our region of interest it may be approximated as

$$\langle n(x > x_0) \rangle = \frac{4e^{z-5.6Y}}{9\sqrt{2\pi z}}\left\{1 + \frac{1}{8z} + \frac{20}{3}\frac{Y}{z}\left(1 - \frac{3}{8z}\right)\right\} \tag{12}$$

where

$$z = 2\sqrt{6Y \log 1/x_0}$$

We plot $\langle n(x > \langle x_v \rangle)\rangle$ as a function of Y in Fig. 2. The main points are:

(*a*) up to GUTS energies $\langle n(x > \langle x_v \rangle)\rangle$ varies only slowly, and the valence quark is usually the fastest parton. (The precise numbers shown for PETRA/LEP energies are overestimates because the values of $\langle x_v \rangle$ at these energies are too large to allow the reliable use of (12)).

(*b*) $n(x > \langle x_v \rangle)$ *does* increase with Q^2 and eventually becomes very large. Here "eventually" $\gg$ GUTS! Of course, the separate treatment of QCD only makes sense below GUTS energies; so any discussion of higher Q^2 is not of any direct physical interest.

While the above tells us that for most events the valence quark is the leading parton, it does not tell us the width of the x_v distribution. In particular what is the probability that the valence quark finds itself slowed down to $x_v \sim 0$ just before hadronisation? We can find the leading power behaviour of the number density dn_v/dx_v near $x_v = 0$ by looking at the moments of the valence quark momentum distribution, $G_v(n, Y)$, which have the usual LLA expression [3]

$$G_v(n, Y) = e^{-A_n Y} \tag{13}$$

where the A_n are the appropriate anomalous dimensions, and noting that the n at which $G_v(n, Y)$ first

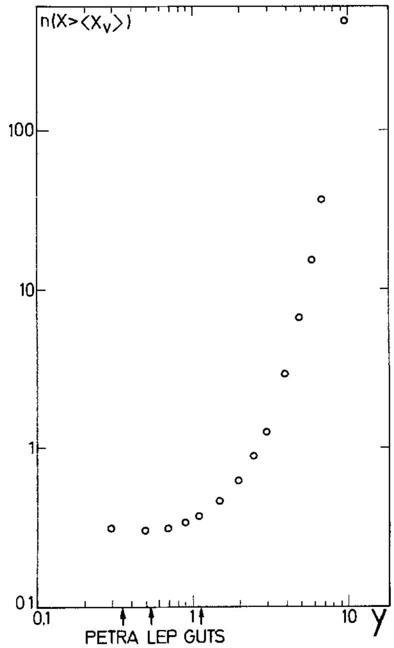

Fig. 2. The number of secondary partons with virtuality $Q_0^2 \sim 1$ GeV^2, and with momentum greater than the average final momentum of the initial quark, is plotted versus the variable Y (see text). Energies corresponding to PETRA, LEP and GUTS are indicated

diverges as we decrease n reflects the leading power behaviour of dn_v/dx_v near $x_v = 0$. We find (leading power behaviour only)

$$\frac{dn_v}{dx_v} \underset{x_v \sim 0}{\sim} \text{const.} \tag{14}$$

or, in terms of longitudinal rapidity, y,

$$\frac{dn_v}{dy} \sim e^{-|y-y_{\max}|} \tag{15}$$

Since we expect [2, 7] that at the hadronisation stage the valence quark will develop a tail at low x_v that is at least as large as that given by (15) it is apparent that at all energies of any possible physical interest the initial quark's relative momentum distribution is such as not to seriously reduce the accuracy of our charge measure in (2) and (4).

(*ii*) Short range charge compensation—the idea that if there is a positively charged particle produced at some point in phase space in the jet, then the corresponding negative particle will be close in phase space, the correlation length being typically 0(1) in units of rapidity (or its transverse generalisation),—is a well established and understood property of the jets produced in normal soft hadron–hadron collisions [11], and preliminary analyses [12] of quark jets confirm this expectation. So, as remarked earlier, a breakdown of SRCC at the hadronic level can only occur if $q\bar{q}$ pairs have managed to become distant in phase space *prior* to the hadronisation: then by SRCC the hadrons containing the q and $\bar{q}$ will be about as distant in phase space as were the $q, \bar{q}$ just before hadronisation. In a perturbative QCD jet this happens with the production of $q\bar{q}$ pairs early in the branching process (i.e. pairs of a large virtuality). The corresponding $q, \bar{q}$ with a virtuality $Q_0^2 \sim 1$ GeV^2 will have a large relative invariant mass $\sim 0(Q/Q_0)$ and will be correspondingly distant in phase space. Of course $q\bar{q}$ production at a vertex characterized by virtuality $\sim \bar{Q}^2$, will be suppressed relative to gluon production by a factor of $\alpha(\bar{Q}^2)$. However, the large number of vertices more than compensates for this. A crude calculation tells us that the first $q\bar{q}$ pair will indeed be formed with a virtuality Q_L^2 that is $0(Q^2)$, so that the q and $\bar{q}$ will find themselves in hadrons that are very far apart in phase space.

Despite the serious breakdown of SRCC implied by all this, we now show that our charge measure is not much affected by the production of such $q\bar{q}$ pairs, because it turns out that the fractional momenta of $q, \bar{q}$ at $Q_0^2 \sim 1$ GeV^2 are similar, and this is what is important in our charge measure.

So suppose that the $q\bar{q}$ pair is produced by a gluon of momentum fraction $\bar{x}$ and virtuality $\bar{Q}^2$. The $g \to q\bar{q}$ decay probability is not especially peaked, so the q and $\bar{q}$ at this stage carry comparable fractions of $\bar{x}$. By the time the q and $\bar{q}$ have degraded their virtuality to $Q_0^2 \sim 1$ GeV^2, they will have average momentum fractions

$$\langle x_q \rangle \sim \langle x_{\bar{q}} \rangle \sim \frac{\bar{x}}{2} e^{-(16/9)\bar{Y}} \tag{16}$$

where we use (11) with $\bar{Y} = Y(\bar{Q}^2)$. Thus although $M_{q\bar{q}} \sim 0(Q)$, longitudinally $x_q/x_{\bar{q}} \sim$ finite. This is not counterintuitive: consider a pair of massless quarks q, q' with opposite transverse momenta $p_\perp$, and with longitudinal momenta $xP, x'P \gg p_\perp$. Then if their invariant mass is $M_{q\bar{q}}$,

$$\begin{aligned} M_{q\bar{q}}^2 &= 2((x^2P^2 + p_\perp^2)^{1/2}, xP, p_\perp)((x'^2P^2 + p_\perp^2)^{1/2}, x'P, -p_\perp) \\ &\approx \left(\frac{x'}{x} + \frac{x}{x'} + 2\right) p_\perp^2 \\ &= \frac{(x+x')^2}{xx'} p_\perp^2 > 4p_\perp^2. \end{aligned} \tag{17}$$

So as long as the $q, \bar{q}$ longitudinal momenta are much bigger than their invariant mass (which is always true, at least by logarithms if not by powers of Q) we expect

$$\frac{x_q}{x_{\bar{q}}} \sim \text{finite.}$$

A better estimate for x_q/x_q, obtained by evaluating $(\langle x_q^2 \rangle - \langle x_q \rangle^2)^{1/2}$ and hence the width of the x_a distribution, turns out to be roughly

$$\frac{x_q}{x_{\bar{q}}} \sim [\alpha_s(\bar{Q}^2)]^{-1/3} \tag{18}$$

(assuming $x_q > x_{\bar q}$). So the longitudinal rapidity interval Δy between the q and $\bar q$ is typically

$$\Delta y \approx \left|\ln \frac{x_q}{x_{\bar q}}\right| \approx Y(\bar Q^2) \tag{19}$$

which even at the highest energies is comparable to the correlation length of the usual short range correlations.

(*iii*) In the standard one-dimensional soft jet the density of particles per unit rapidity is more or less constant with rapidity and with energy: increasing multiplicity comes from the increasing range of rapidity available with increasing energy. So the charge fluctuations are also constant in rapidity and the error in our charge measure improves as $\frac{1}{\sqrt{N}}$ where N is the total multiplicity (see (2)). What happens in an asymptotic QCD jet? As we have seen, $\langle N \rangle \propto e^{a\sqrt{\ln Q^2}}$. Now the total rapidity length $\propto \ln Q^2$. Hence for at least some of the rapidity length $\frac{dN}{dy} \to \infty$ as $Q^2 \to \infty$ and as argued in Sect. 2 the corresponding fluctuations also become large: $\sigma(y) \propto \sqrt{\frac{dN}{dy}} \to \infty$.

Now, what is useful for picking out the incident quark is the portion of the jet slower than the incident quark, i.e. possessing $x < \langle x_v \rangle$. What is the particle density there? Asymptotically using (11) and (12) we see that $x \sim \langle x_v \rangle$ the density of particles is approximately

$$\frac{dn(x = \langle x_v \rangle)}{dy} \propto e^{Y} \tag{20}$$

and increases with decreasing x. So the error in (4) is

$$\left[\sum_{i=1}^{\sim \ln Q^2} \frac{1}{\sigma^2(y_i)}\right]^{-1/2} \propto (\ln Q^2)^{\delta > 0}. \tag{21}$$

Thus we see explicitly that as Q^2 increases, the increasing density of particles eventually renders our charge measure useless. Of course, "eventually" may not mean much: surely the highest sensible energy for a QCD jet is the GUTS mass scale $\sim 10^{15}$ GeV. So we now exemplify all the above by calculating our charge measure for a typical quark jet of 10^{15} GeV, and we will estimate its error.

A GUTS Jet

Consider a quark jet of energy $\sim 10^{15}$ GeV, originating from an off-shell quark of a similar virtuality. As we have seen, even at such an energy the initial quark is probably the leading parton just before hadronisation and the production of $q\bar q$ pairs has no degrading effect on such a longitudinal charge measure. So the main factor affecting the accuracy of our charge measure is the density of hadrons.

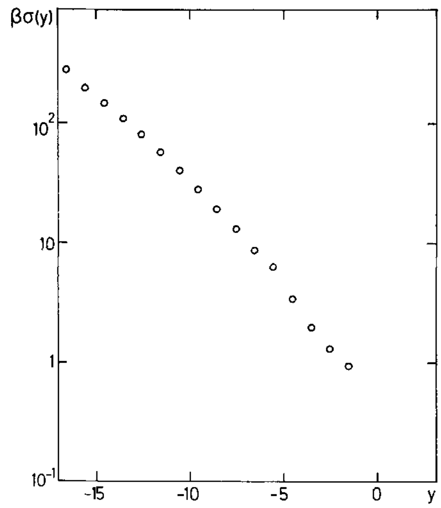

Fig. 3. The square root of the number of partons with virtuality $Q_0^2 \sim 1$ GeV^2 per unit rapidity interval is plotted versus the rapidity ($y = 0$ corresponds to the rapidity of the incident quark) for a quark jet where the incident quark has a virtuality $\sim 10^{15}$ GeV This also equals the quantity $\beta\sigma$ – see text

Using (12) we calculate the density of partons versus rapidity, $\frac{dn}{dy}$. As we remarked in Sect. 2 the charge fluctuation increases as

$$\sigma(y) = \frac{1}{\beta}\sqrt{\frac{dn}{dy}} \tag{22}$$

where β^{-2} is the conversion factor in going partons of virtuality $\sim Q_0^2 = 1$ GeV^2, to the final hadrons. We plot the resulting $\beta\sigma(y)$ in Fig. 3. Now, in such a situation the best one can do (see Sect. 2) is to pick one charged hadron per unit rapidity, and then the value of the charge measure becomes

$$Q(q) = \left(q - \frac{1}{6}\right) \pm \frac{\sigma}{\sqrt{\left\{\sum_{i=1}^{\sim 35} \beta^2 \Big/ \frac{dn}{dy_i}\right\}}} \tag{23}$$

were previous estimates [2] give $\sigma \sim 1$. Reading the values off Fig. 3 gives us

$$Q(q) = q - \tfrac{1}{6} \pm \frac{\sigma}{\beta} \cdot \frac{1}{\sqrt{2.14}}. \tag{24}$$

We expect $\sigma, \beta \sim 0(1)$, so very roughly we find that applying our charge measure to a quark jet gives us the quark "charge" with an error of

$$\delta Q \approx \pm \frac{1}{\sqrt{2.14}} \approx 0.7. \tag{25}$$

Thus the accuracy of the charge measure is poor—no better than one would obtain at the higher DORIS energies.

Note that if the jet had had a density of particles which was constant in rapidity and which was comparable to the density observed at low energies (as would be the case for a jet of energy 10^{15} GeV but virtuality 1 GeV) then the error would have been

$$\delta Q \sim \pm \frac{1}{\sqrt{35}} \sim \pm 0.17 \tag{26}$$

since the length of the GUTS jet is about 35 rapidity units. The difference between (25) and (26) is a direct measure of the extent to which perturbative effects have degraded the charge information at the highest energy at which one can reasonably discuss QCD jets.

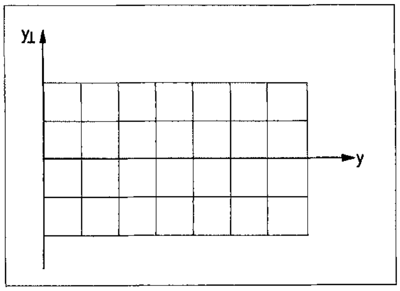

Fig. 4. A jet of hadrons at large Q^2 where transverse momenta have become large, although still small compared to the longitudinal momenta. The jet is plotted against longitudinal rapidity y, and transverse rapidity, $y_\perp \sim \frac{1}{2}\ln(m^2+p_\perp^2)$, and is partitioned into boxes of dimension l_c. In contrast to the soft jet in Fig. 1, the jet now possesses non-trivial transverse structure

4. Loss of all Charge Information as $Q^2 \to \infty$

We wish to demonstrate in this section that, quite independently of the charge measure employed, the charge information on an event by event basis is completely lost as $Q^2 \to \infty$.

To begin we wish to encapsulate diagrammatically the information SRCC provides us about the charge structure of a "three" dimensional QCD jet. To do this we simply extend to 3 dimensions what we did in Fig. 1 for one dimensional soft jets. The QCD jet is partitioned into boxes of size l_c: a two dimensional section is shown in Fig. 4. The longitudinal variable is rapidity, while the transverse variable is its transverse analogue: $\ln p_\perp$ for $p_\perp \gg$ masses. SRCC tells us that the charge fluctuations across all the faces of this structure are independent. (We might be tempted to construct a multidimensional generalisation and improvement of our one-dimensional measure. However, such an effort would be unrewarding because we already know that transverse SRCC is broken by the perturbative production of $q\bar{q}$ pairs).

So for the first part of the argument consider the jet decomposed as in Fig. 4. Assume each box is populated with m charged hadrons. We wish to know: what is the probability that such a jet with a net charge q located somewhere near the front of the jet, would look like a neutral jet? Or, in other words, what is the probability that a neutral jet possesses a hadronic charge structure typical of a charged jet?

To see how to answer this question let us first pose it in the context of the simpler 1 dimensional model in Fig. 1b. We characterize the charge structure of the jet by the charge fluctuation across the sides of the boxes; let σ(i) be the fluctuation across the i'th side from the right. Let $\sigma_{\text{nat}}(i)$ be a "natural" random value of $\sigma(i)$. Suppose then we have a jet of charge q (residing in the fastest box) then for it to look like a typical neutral jet, whose charge fluctuations will be some typical $\sigma_{\text{nat}}(i)$, the actual charge fluctuations of this charged jet must all be shifted by q, away from their "natural" values:

$$\sigma(i) = \sigma_{\text{nat}}(i) - q \tag{27}$$

In the particularly simple case where

$$q^2 \gg \langle \sigma^2_{\text{nat}}(i) \rangle \tag{28}$$

we clearly will have a suppression factor, S, like

$$S = [P(\sigma = q)]^N \tag{29}$$

where N is the length of the jet (using the charge correlation length as a unit) and $p(\sigma = q)$ is the probability that a charge fluctuation takes the value q numerically. More generally, if we relax the condition (28), we expect S to have the form

$$S = \alpha^N \tag{30}$$

where α is some number less than one, simply because all the charge fluctuations across different sides are more or less independent. An interesting special case is for

$$q^2 \ll \langle \sigma^2_{\text{nat}}(i) \rangle \tag{31}$$

when, assuming a Gaussian form,

$$\frac{dN}{d\sigma} \propto e^{-\sigma^2/2\langle\sigma^2\rangle} \tag{32}$$

we see that

$$\alpha \sim e^{-q/\sqrt{\langle\sigma^2\rangle}} \tag{33}$$

So as $\langle \sigma^2 \rangle$ increases, which typically will happen if the particle density increases, the probability for a charged jet to look like a neutral jet increases rapidly.

We will now extend this argument to the case of a fat jet as in Fig. 4. The central observation is that the neutral jet will look like a charge q jet if we can find a string of boxes from the front of the jet to the rear of the jet which looks like a (one-dimensional) q jet. Let the suppression factor across each of such a one-dimensional string be α, and let L be the length of the string, then the total suppression will be

$$S = \Sigma \alpha^L \tag{34}$$

where Σ represents the sum over all possible paths.

When $S \geq 1$ this means that the *number* of paths is so great as to overwhelm the individual suppression factor of α^L.
Now the total number of paths will be of the order

$$\text{number of paths} \sim 5^L \tag{35}$$

because at each box we can move out in 5 possible directions. So very crudely we can replace (34) by

$$S = (5\alpha)^L \tag{36}$$

Thus if $5\alpha > 1$, the suppression for any individual string is overwhelmed by the huge number of possible strings, and all charge information is lost. Our argument has been very crude, but doing better (e.g. using a honeycombed jet with more faces) merely serves to strengthen the result.

For a quark charge, q, we expect $\alpha > 1/5$ even for a soft jet. In a QCD jet since the number of boxes $\sim (\ln Q)^3$ while the total multiplicity $\sim \exp(a\sqrt{\ln Q^2})$, it is evident that the portion of the jet where nearly all of the multiplicity resides will have $\alpha \sim 1$ and will carry no charge information.

So we have seen that that portion of a QCD jet which carries most of the multiplicity carries no charge information: this certainly includes the portion of the jet with

$$x_0 \lesssim 0(e^{-6Y}) \tag{37}$$

which we can see by asking when the average multiplicity per unit box necessarily starts growing. So the longitudinal rapidity range available for any charge measure is

$$\delta y \sim \ln x_0 \sim \text{const.}\, x\, Y. \tag{38}$$

We have seen that there are $0(e^Y)$ partons faster than the initial quark. To maximise our chance of picking out the net jet charge we minimise the various charge fluctuations by assuming that the initial quark and the very fast partons all form separate subjets. So we have $0(e^Y)$ subjets each of a usable length in longitudinal rapidity that is $0(Y)$ units. Can we pick out a charge q that resides in one of these jets? Since each of these subjets is essentially one-dimensional we apply our charge measure to this collection of subjets, and find that the error on the measure is typically

$$\text{error} \sim 0\left(\frac{e^Y}{\sqrt{Y}}\right) \underset{Y \to \infty}{\longrightarrow} \infty \gg q. \tag{39}$$

So we claim on the basis of these arguments (which could be strengthened by including the effects of $q\bar{q}$ production) that in a given QCD jet all charge information is lost as $Q^2 \to \infty$.

One should note that this statement only applies to a *given* jet. If for example we were to be provided with unlimited numbers of, say, u jets, then however, high the virtuality of the initial u quarks, we could always use enough jets for our average that the average net charge would give us the u quark charge (with the usual modifications of course) to arbitrarily good accuracy. It is amusing that the presence of the same perturbative showering makes it difficult to conceive of a source of quark jets, all of the same type, as $Q^2 \to \infty$.

5. Monte Carlo Studies

In this section we shall resort to the Fox and Wolfram (FW) [4] computer Monte Carlo simulation of QCD jets in order to see quantitatively how drastically perturbative jet evolution degrades the accuracy of our optimal (for "soft" jets) charge measure $Q^N(J)$ at current PETRA energies [12]. We shall also argue that a clear-cut signature of such an underlying perturbative cascade mechanism is the strong expected correlation between jet multiplicity and leading charge structure; this should be visible at present energies in contrast to other signature, such as subjet structure [3], which only become manifest around LEP energies.

To study these issues we have run the FW program at $\sqrt{s} = 30$ GeV to produce one hundred $e^+e^- \to$ two jets events; i.e. two-hundred single jet events. We do not include c, b flavours because of theoretical uncertainties concerning their decays. In general we would expect some degradation of all charge measures; most noticeably those involving only the fastest hadrons. "Data" from some runs at lower energies will also be used.

General Partonic Structure of FW Quark Jets

In the FW program a highly virtual parton, initially off-shell by $Q^2 \lesssim O(s)$ initiates a branching process. The branching is allowed to continue until any given parton falls below Q_0^2 off-shell, at which point it decays no further. Finally a conventional hadronisation scheme is applied [4]. In all our work we use $Q_0 = 1$ GeV.

The FW model improves upon the naive collinear kinematics of the LLA, and this turns out to lead to a *reduction* in the amount of branching as compared to the LLA. In Fig. 5 we display the number density histogram, dN/dx_v, obtained with the FW program at $\sqrt{s} = 30$ GeV, $Q_0 = 1$ GeV. To obtain a similar density in the *LLA* we must employ a higher cut-off of $Q_0 \approx 1.7$ GeV. We have also run the program at $\sqrt{s} = 200$ GeV, leading to dN/dx_v as in Fig. 6. To reproduce this with a LLA spectrum requires a cut-off of $Q_0 = 1.3$ GeV. So, as we would anticipate, the FW program and the LLA approach each other at higher energies.

The results of perturbative showering at $\sqrt{s} = 30$ GeV may be summarised as follows. (i) As we see in Fig. 5 the initial quark is not slowed down very much. (ii) The number density of secondary partons, dN/dx, is plotted in Fig. 7, and is not large. Its main compo-

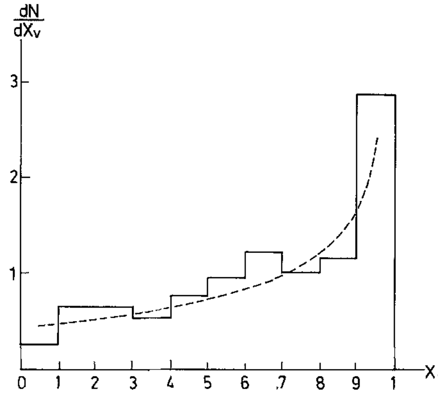

Fig. 5. Valence quark distribution after branching down to $Q_0^2=1$ GeV at $\sqrt{s}=30$ GeV, extracted from 200 events of FW Monte Carlo. The dashed curve shows the LLA expectation at the Y value which fits the $\langle x_v \rangle$ of the distribution

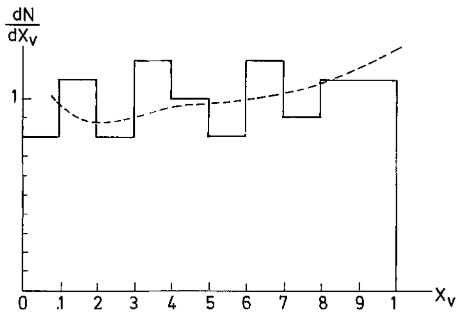

Fig. 6. As Fig. 5 but $\sqrt{s}=200$ GeV

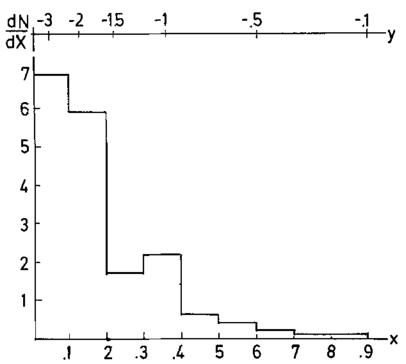

Fig. 7. Number density of secondary partons at $\sqrt{s}=30$ GeV, $Q_0^2=1$ GeV^2. Extracted from FW results

nent is glue: on average ~ 2 per jet. (iii) Secondary $q\bar{q}$ production is rare, about 0.1 secondary quarks per jet, and such $q\bar{q}$ end up on average 2 units of rapidity apart in longitudinal phase space.

So we may expect that at PETRA energies perturbative showering will not have degraded the charge information in jets too severely.

Charge Measure and the FW Results

We shall now consider how reliably the charge measure $Q_N(J)$ of (2) serves to distinguish a '+' QCD jet, i.e. $u, \bar{d}, \bar{s} \ldots$ from a '−' QCD jet, $\bar{u}, d, s$. As we discussed at length in (2) distinguishing between $u, \bar{d}, \bar{s}$ is unlikely to be possible using only the charge measure, although some indirect means were proposed.

We shall examine the confidence level, $C(Q_N(J))$, with which the criterion $Q_N(J) \geqq O \Rightarrow$ '+' jet, $Q_N(J) < O \Rightarrow$ '−' jet, allows one to identify $u, \bar{d}, \bar{s}$ versus $\bar{u}, d, s$ FW jets. We shall see later that such a confidence level, for a criterion which applies to *all* events, can be extracted from 2-jet $e^+ s^-$ data, where one has no prior knowledge of which jet is which.

We shall be concerned with $N=1$ and $N=n_c$. The former is just equivalent to using the charge of the fastest charged hadron to identify the jet and has in its favour directness and lack of possible experimental ambiguity.

The FW results for our $\sqrt{s}=30$ GeV, $Q_0=1$ GeV PETRA sample of 200 events are $C(Q_1(J) = 0.7 \pm 0.03$, $C(Q_{n_c}(J)) = 0.75 \pm 0.03$. From our naive model for 'soft' jets in (2) we would have estimated $C(Q_1(J)) = .67$, $C(Q_{n_c}(J)) = 0.8$ taking $n_c = \langle n_c \rangle = 6$. The agreement is reasonable. In both cases there is seen to be a significant improvement in confidence if one uses the full charged multiplicity of the jet as opposed to relying on the fastest charge alone.

Use of the combined 2-jet measure of (2): $\Delta Q(J_1, J_2) = \frac{1}{N}[N_1 Q_{N_1}(J_1) - N_2 Q_{N_2}(J_2)]$ with the criterion $J_1=$'+', $J_2=$'−' if $\Delta Q > 0$, or vice versa for $\Delta Q < 0$, gives $C(\Delta Q) = 0.8 \pm 0.04$ for FW jets. This is to be compared with a confidence of 0.7 if one identifies each jet of a 2-jet event using the fastest charge when these are opposite, and determines at random when the fastest charges are identical.

Having seen that our charge measure will correctly identify jets about 8 times out of 10 at upper PETRA energy we now turn to the signature of perturbative branching contained in the charge structure.

6. Correlation Between Multiplicity and Charge Structure

Even at $\sqrt{s}=30$ GeV, the internal perturbative showering has already had a marked effect on the final hadron multiplicity. This can be seen in Fig. 8: at $\sqrt{s}=30$ GeV the average multiplicity is *twice* what one obtains from those jets which suffer no perturbative branching (because they are produced with $Q^2 < Q_0^2$). Now increased perturbative showering $\Leftrightarrow$ the valence quark is slower $\Leftrightarrow$ the charge measure works less well. Hence we would expect a marked decrease of the reliability of the leading particle charge measure with increasing multiplicity at a given energy; and, conversely, the observation of such an effect at PETRA energies we would claim to be a strong signature of internal perturbative showering.

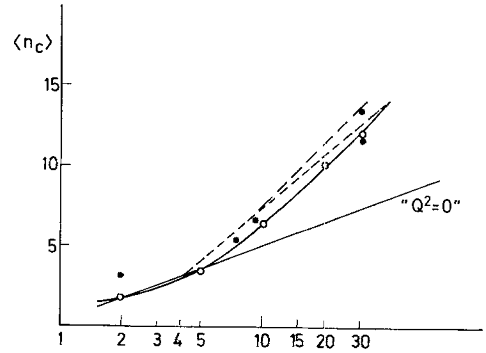

Fig. 8. Average hadronic charged multiplicity vs $\sqrt{s}$. The black points are selected e^+e^- data (see [14]). The open points and solid curve are FW results. The "$Q^2=0$" line is the FW expectation with no branching. The dashed line is a Feynman–Field $q\bar{q}$ Monte Carlo expectation, and the dashed-dotted is the $q\bar{q}g$ expectation

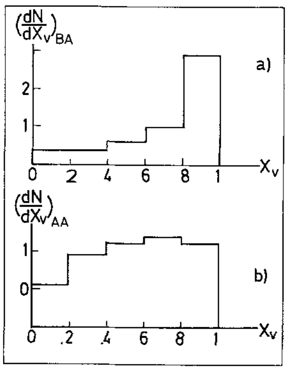

Fig. 9 a, b x_v distribution corresponding to $n_c < 6$ and $n_c \geqq 6$ respectively

In Fig. 9 we plot the number densities for below and above average multiplicities: $(dN/dx_v)_{BA}$ and $(dN/dx_v)_{AA}$ respectively. Note how sharply peaked the former is, while the latter is very much like the overall number density at *LEP* energies (see Fig. 6): the correlation between hadron multiplicity and x_v is very strong. We now evaluate our charge measures for these two sets of events. We find

$C(Q_1(J))_{BA} = 0.76 \pm 0.04$, $C(Q_1(J))_{AA} = 0.64 \pm 0.04$ and $C(Q_{n_c}(J))_{BA} = 0.79 \pm 0.04$, $C(Q_{n_c}(J))_{AA} = 0.69 \pm 0.04$.

The observation at PETRA of such a strong charge structure-multiplicity correlation (one would expect only a weak effect with soft jets) would be indicative of underlying perturbative branching.

7. Extraction of $C(Q_N(J))$ and the Optimum Measure

$C(Q_N(J))$ is easily extracted from a sample of 2-jet events by measuring the average

$$P(Q_N(J)) = -\langle \bar{Q}_{N_1}(J_1)\bar{Q}_{N_2}(J_2)\rangle$$

where the product is taken for each two-jet event and

$$\bar{Q}_N(J) = \begin{cases} 1 & Q_N(J) \geqq 0. \\ -1 & Q_N(J) < 0. \end{cases}$$

It is then trivial to show that

$$C(Q_N(J)) = \frac{1}{2} + \frac{\sqrt{P(Q_N(J))}}{2}$$

By measuring the analogue of P for different charge measures, for example the Feynman-Field Z-weighted charge sum [6], and our own $Q_N(J)$, one can then obtain from 2-jet e^+e^- data an objective assessment of how well these event-by-event measures work relative to each other. This is preferable to measuring $-\langle Q(J_1)Q(J_2)\rangle$ for the measure itself, which has already been done in (12) for the Feynman-Field measure, because this average depends on the shape of the distribution in the charge measure, as opposed to how well it differentiates between jets of different underlying charge, which is what we are interested in.

Using $P(Q_N(J))$ on $\sqrt{s} = 30$ GeV PETRA data the FW results would predict:

$$\begin{aligned} P(Q_1(J)) &= 0.16 \pm 0.05 \\ P(Q_{n_c}(J)) &= 0.25 \pm 0.06 \\ P(Q_1(J))_{AA} &= 0.08 \pm 0.05 \\ P(Q_1(J))_{BA} &= 0.27 \pm 0.09 \\ P(Q_{n_c}(J))_{AA} &= 0.14 \pm 0.07 \\ P(Q_{n_c}(J))_{BA} &= 0.34 \pm 0.09 \end{aligned}$$

Here P_{BA}, P_{AA} denote the average over two-jet events where *both* jets have respectively below average or above average charged multiplicity. The experimental observation of the predicted big difference between P_{AA} and P_{BA} would, as explained earlier, be a strong indication of the presence of perturbative branching in the PETRA jets.

Of course, for neutrino or antineutrino production, where the flavour of the final current quark is fixed one can measure the confidence levels directly.

We finally make some remarks concerning the optimum charge measure. It is clear that we can do rather better than using $Q_N(J)$ to decide the underlying charge. $Q_N(J)$ only depends on the sequence of charges ordered in rapidity. For N charged particles there are 2^N such permutations. For instance $N=2$ has $++$, $--$, $+-$, $-+$ as the 4 permutations. By studying a large sample of Monte Carlo events one could, in principle, determine the probability $p(P)$ that the permutation P results from a '+' QCD jet, $1-p(P)$ that it results from a '−' QCD jet. If $p(P) > \frac{1}{2}$ then any jet having charge permutation P should be identified as '+' or vice-versa for $p(P) < \frac{1}{2}$.

Such a procedure is guaranteed, since it utilizes all available information, to work with a better confidence level than $Q_N(J)$ which can assign the same value of measure to two different permutations.

8. Conclusions

In this paper we have conducted a detailed investigation of the effects of perturbative branching upon our ability to decide from the charge structure of the hadrons in a given QCD jet the charge of the initial QCD quantum: in particular within the context of a charge measure which we had shown earlier [2] to be insensitive to the charge fluctuations incurred during the final soft hadronisation.

Our numerical calculations at the highest currently available energies of about 30 GeV were performed using the Monte Carlo program of Fox and Wolfram. We found that in a typical 2 jet event at such an energy our charge measure would correctly identify the $u\bar{u}$ (for example) orientation some 80% of the time. This is close to our previous, naive estimates in [2] and is a significant improvement on the 70% obtained by using only the leading charges in each jet.

We made some asymptotic analytic estimates. In particular we explicitly evaluated our charge measure for a jet of virtuality $\sim 10^{15}$ GeV, and the charge identification was found to have become poor by this energy (which is the highest of any possible physical interest in pure QCD).

We showed that as $Q^2 \to \infty$ all charge information is lost on an event by event basis; not just with our charge measure, but with any charge measure.

Finally we pointed out that the variation of the multiplicity at a given energy is strongly correlated to the amount of the perturbative showering in a given jet, and that this is sensitively reflected in the charge structure of the jets.

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