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A MODEL OF DEEP INELASTIC POLARIZED ELECTROPRODUCTION

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A realistic phenomenological model combining parton/QCD ideas with lower energy SU(6) constraints is proposed for the shape and evolution of the leading spin-dependent structure function $G^{ep}(x, Q^2)$ in polarized electroproduction. Close's broken-SU(6) ansatz is used to relate appropriately defined polarized quark-parton distribution densities to unpolarized ones at the matching momentum scale $Q^2 = Q_0^2$. The differences between spin and helicity distribution densities as well as the complications due to perturbative QCD and parton k_T (with related target-mass) effects are taken into account. Evolution to higher ($>10\,\mathrm{GeV}^2$) values of Q^2 (where target-mass effects can be neglected) yields experimentally testable numerical predictions that are presented through various plots. The value of Q_0 is self-consistently determined to be about 0.5 GeV.

1. Introduction

Deep inelastic spin-dependent ep (or μp) scattering has received a fair amount of theoretical attention [1] in the past. But a realistic phenomenological prediction of the form (i.e. the dependence on both x and Q^2, the two standard deep inelastic variables) of the leading structure function $G^{ep}(x, Q^2)$ has been lacking so far. It has, however, become a timely task to make such a prediction in view of polarized target experiments currently in progress at SLAC and CERN [2]. The aim of the present paper is try to perform this task on the basis of ideas reported by us [3] in a previous communication.

We propose a parton/QCD type of a model to give the leading description of spin effects in this process. Our basic approach is to introduce requisite polarized quark-parton distribution densities which can be related to corresponding unpolarized ones at the matching momentum scale (explained below) $Q^2 = Q_0^2$ by Close's broken-SU(6) ansatz [4]. We further employ a generalized covariant parton formulation [5] which can successfully incorporate QCD plus parton transverse momentum (k_T) and the related target-mass (M) effects. A mechanism is thereby

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automatically generated which modifies the naive relations of Close [4] in a calculable way. Evolution to much higher values of $Q^2(\gg M^2)$ is then carried out by standard renormalization-group methods [6]. Using a popular and successful parameterization for the unpolarized quark distribution densities, we thus obtain the values of $G^{ep}(x, Q^2)$ in the interval $0<x\leqslant 1$ and for Q^2 ranging from $10\,\mathrm{GeV}^2$ upwards.

The description of the nucleon is central to a theoretical treatment of any type of ℓN scattering. Over the years, two different descriptions have emerged. One, motivated by the creditable performance of low-energy SU(6) phenomenology, views a nucleon as a composite of just three constituent quarks. The other, inspired by the highly successful deep inelastic parton picture, regards it (in the infinite momentum frame) as made of three valence quarks plus a sea of quarks and antiquarks and gluons. These two descriptions are supposėdly related by Melosh-like transformations [7]. Nevertheless, one knows the following from experience. The first description works in the low Q^2 region and begins to fail as Q^2 increases. The second, on the other hand, is known to be more appropriate for somewhat higher momentum transfers squared and cannot be extrapolated down to the very low Q^2 domain. This difference notwithstanding, it has been proposed [8] that the two descriptions can be matched at a process-independent scale [9] $Q^2=Q_0^2$. The latter is presumably related to the dimensional parameter Λ in quantum chromodynamics, but the explicit relation is expected to be controlled by non-perturbative effects and is unknown. We take up this idea in combining Close's broken-SU(6) ansatz with the parton description at $Q^2=Q_0^2$.

The employment of SU(6) symmetry to relate the polarized and unpolarized structure functions in ep scattering has been an old and much used idea from the starting days of deep inelastic physics (a review and references may be found in Close's book [1]). However, many have been the pitfalls in this approach. For instance, the attribution of an exact SU(6)-symmetric **56**-plet wave function to the nucleon leads to the experimentally unacceptable result $F_2^{en}(x)/F_2^{ep}(x)\geqslant\frac{2}{3}$, independent of x, for the neutron to proton ratio of the standard unpolarized scale function F_2. Close [4] tackled this problem by resorting to broken-SU(6). Motivated by dynamical considerations such as the Melosh transformation [7], he allowed arbitrary strengths of the isovector and isoscalar diquark configurations (spectators relative to the participator quark) instead of taking them symmetrically, as done in the **56**-plet. This procedure avoids the above unacceptable result and yet yields non-trivial relations between polarized and unpolarized quark-parton distributions. Nevertheless, there arises an additional problem as explained below.

In the parton picture of polarized deep inelastic eN scattering, the spin function $G^{eN}(x)$ is linearly related to the helicity distribution densities of quarks in the nucleon. If one uses the naive version of Close's [4] relations to link these to unpolarized quark distribution densities and (legitimately) ignores the polarization of the $q\bar{q}$ sea, a simple form for $G^{eN}(x)$ obtains. This form must, however, be incorrect; in combination with the Bjorken sum-rule $\int_0^1 dx\, G^{e(p-n)}(x) = \frac{1}{6} g_A/g_V$, it implies the bad SU(6) value, $\frac{5}{3}$, for the ratio of the axial and vector weak couplings of the nucleon. Sea polarization cannot cure this, since it leaves $G^{e(p-n)}$ unaffected. There have been *ad hoc* attempts [10] to overcome this difficulty by imposing a universal "dilution factor" on Close's relations and also phenomenology [11] along this line. However, we shall not pursue that approach since it is quite arbitrary.

Our resolution [3] of the above difficulty has been through a proper recognition of the importance of the primordial transverse momenta of the quark partons. Their role in breaking the SU(6) symmetry (and hence in diluting the value of g_A/g_V from $\frac{5}{3}$ to about $\frac{5}{4}$) is well-known in non-relativistic quark models [12]. The point that we make is that in the parton model framework also, their presence naturally generates the dilution of g_A/g_V if Close's ansatz is used (at the matching momentum scale $Q^2 = Q_0^2$) on the spin rather than helicity distribution densities. These two densities differ via parton transverse momenta. One can show [3] in the covariant parton model formulation [5] how these differences explicitly modify the naive version of Close's relation at $Q^2 = Q_0^2$. However, since Q_0^2 is expected to be not very different from the average transverse momentum squared k_T^2 of quarks, the effects of the latter have to be completely taken into account without expanding in powers of $\langle k_T^2 \rangle / Q_0^2$ as was done earlier [3].

We develop a formalism which not only accomplishes the above goal but also takes into account the non-leading QCD effects. Parton k_T effects can be easily handled in the covariant parton model [5]. To tackle these effects we introduce modified quark parton distribution densities which are simply related to QCD moments and impose the Close ansatz [4] on them. In terms of these modified distributions, the Bjorken sum rule can be rewritten as a clean relation that is valid for all Q^2 and is unaffected by QCD. The use of this relation at Q_0^2 leads to a self-consistent determination of $Q_0 \approx 0.5$ GeV, given g_A/g_V and parametrizations of unpolarized quark distributions as inputs. Once our theory is developed at $Q^2 = Q_0^2$, the evolution to higher Q^2 ($\gg M^2$) is done using the standard renormalization group procedure [6] and leads to testable predictions on the shape of $G^{ep}(x, Q^2)$ at sufficiently high values of Q^2 and for $0 < x \leqslant 1$.

The plan of the rest of the paper is as follows. Sect. 2 is devoted to a short recapitulation of relations between $G^{ep}(x, Q^2)$ and the usual polarized quark distribution densities. In sect. 3 the modified distribution densities are introduced to incorporate quantum chromodynamic effects properly and the Close ansatz is implemented on them at $Q^2 = Q_0^2$. Sect. 4 contains a treatment of parton k_T with related target mass effects at $Q^2 = Q_0^2$ in terms of a covariant ξ-scaling formalism. In sect. 5 we evolve via the renormalization group to higher values of $Q^2 (\gg M^2)$. Sect. 6 is addressed to the numerical aspects of this work relevant to current experiments and presents our conclusions.

2. Usual polarized quark densities

Consider deep inelastic ep scattering with the proton polarized parallel ($\uparrow\uparrow$) or antiparallel ($\uparrow\downarrow$) to the helicity of the incoming electron. Let $-Q^2$ be the square of the four-momentum transfer between projectile and target and let x and y be the standard Bjorken and inelasticity variables, respectively. The leading spin-dependence of the process, in the Bjorken limit ($Q^2 \to \infty$, x fixed) is now described by the spin function $G^{\mathrm{ep}}(x, Q^2)$ through the formula

$$\frac{\mathrm{d}^2\sigma^{\uparrow\uparrow}}{\mathrm{d}x\,\mathrm{d}y} - \frac{\mathrm{d}^2\sigma^{\uparrow\downarrow}}{\mathrm{d}x\,\mathrm{d}y} = \frac{8\pi\alpha^2}{Q^2}(2-y)G^{\mathrm{ep}}(x, Q^2)\,. \tag{2.1}$$

Here $G^{\mathrm{ep}}(x, Q^2) = (2x)^{-1}MQ^2G_1(x, Q^2)$ in terms of the structure function G_1, defined in ref. [1] and terms involving positive powers of M^2/Q^2 have been dropped.

In the parton picture, ignoring terms with m_{q}^2/Q^2(m_{q} = current mass of quark partons), one is led to the following expression for the spin function:

$$G^{\mathrm{ep}}(x, Q^2) = \tfrac{1}{2}\sum_{\mathrm{q}} e_{\mathrm{q}}^2 \int \mathrm{d}^2k_{\mathrm{T}}\Delta q^{\mathrm{h}}(x, Q^2, k_{\mathrm{T}}^2)\,, \tag{2.2a}$$

$$\Delta q^{\mathrm{h}}(x, Q^2, k_{\mathrm{T}}^2) \equiv q_+(x, Q^2, k_{\mathrm{T}}^2) - q_-(x, Q^2, k_{\mathrm{T}}^2)\,. \tag{2.2b}$$

In eqs. (2.2) e_{q} is the charge (in units of e) of the quark of flavour q, k_{T} is the transverse momentum of the participator quark with a longitudinal fraction x of the momentum of the proton – envisaged as moving fast along the direction z(say) – and $q_{+,-}(x, Q^2, k_{\mathrm{T}}^2)$ are the corresponding parton distribution densities of positive, negative helicity quarks. We can call $\Delta q^{\mathrm{h}}(x, Q^2, k_{\mathrm{T}}^2)$ the differential helicity distribution density. A theory of this function is essentially a theory of G^{ep}.

It is also possible to introduce spin distribution densities $q_{\uparrow\downarrow}(x, Q^2, k_{\mathrm{T}}^2)$ of quarks with up, down spin along the z-direction. One may further define a differential spin-distribution density

$$\Delta q^{\mathrm{s}}(x, Q^2, k_{\mathrm{T}}^2) \equiv q_\uparrow(x, Q^2, k_{\mathrm{T}}^2) - q_\downarrow(x, Q^2, k_{\mathrm{T}}^2)\,. \tag{32.3}$$

The two differential densities Δq^{h} and Δq^{s} can be related by use of rotational invariance. If k_{L} and $|\boldsymbol{k}|$ are respectively the longitudinal and the total momentum of the quark parton, it follows that

$$\Delta q^{\mathrm{h}}(x, Q^2, k_{\mathrm{T}}^2) = \frac{k_{\mathrm{L}}}{|\boldsymbol{k}|}\Delta q^{\mathrm{s}}(x, Q^2, k_{\mathrm{T}}^2)\,. \tag{2.4}$$

A derivation of eq. (2.4) is given in the appendix.

3. Modified quark distribution densities and the Close ansatz

The polarized quark distribution densities of sect. 2 are simply related to the spin function. However, they are problematic in discussing QCD effects and in combining those with the usual flavour symmetry constraints on proton. QCD complications are best discussed in terms of moments of scale functions. We shall broadly follow the notation of Kodaira et al. [13]. The nth moment of the valence part (v) of $G^{ep}(x, Q^2)$,

$$\Delta M_n^{\mathrm{v}}(Q^2) \equiv \int_0^1 \mathrm{d}x\, x^{n-1} G_{\mathrm{v}}^{ep}(x, Q^2)\,, \tag{3.1}$$

has the form

$$\Delta M_n^{\mathrm{v}}(Q^2) = \tfrac{1}{2} \sum_{q_{\mathrm{v}}} e_{\mathrm{q}}^2 \Delta M_n^{\mathrm{q,v}}(Q^2)\,, \tag{3.2a}$$

$$e_{\mathrm{q}}^2 \Delta M_n^{\mathrm{q,v}}(Q^2) = a_n^{\mathrm{q,v}}(\mu^2) E_{1\mathrm{q,v}}^n(Q^2/\mu^2, g)\,, \tag{3.2b}$$

with

$$E_{1\mathrm{q,v}}^n(Q^2/\mu^2, g) = E_{1\mathrm{q,v}}^n(1, \bar{g}(Q^2)) \exp\left[-\int_{\bar{g}(\mu^2)}^{\bar{g}(Q^2)} \mathrm{d}\bar{g}\, \frac{\gamma_n(\bar{g})}{\beta(\bar{g})}\right], \tag{3.3}$$

where $a_n^{\mathrm{q,v}}(\mu^2)$ comes [13] from the polarized proton matrix element of a completely symmetric spin-n twist-2 non-singlet operator* $R_{1\mathrm{q,v}}^{\mu_1 \cdots \mu_n}$ renormalized at $Q^2 = \mu^2$. $E_{1\mathrm{q,v}}^n(Q^2, g)$ is related to the coefficient function accompanying $R_{1\mathrm{q,v}}^{\mu_1 \cdots \mu_n}$ in the operator product expansion relevant for $G_{\mathrm{v}}^{ep}(x, Q^2)$. γ_n is the anomalous scale dimension of $R_{1\mathrm{q,v}}^{\mu_1 \cdots \mu_n}$ and it governs [14] the Q^2 evolution of the valence part of $G^{ep}(x, Q^2)$. Finally $\bar{g}^2(Q^2) = g^2[1 + (7g^2/16\pi^2) \ln(Q^2/\mu^2)]^{-1}$ for six quark flavours and $\beta(\bar{g})$ is the usual renormalization group function.

A comparison of eqs. (2.2), (3.1) and (3.2) leads to the relations

$$\Delta M_n^{\mathrm{q,v}}(Q^2) = \int_0^1 \mathrm{d}x\, x^{n-1} \int \mathrm{d}^2 k_{\mathrm{T}} \Delta q_{\mathrm{v}}^{\mathrm{h}}(x, Q^2, k_{\mathrm{T}}^2)\,, \tag{3.4a}$$

$$e_{\mathrm{q}}^2 \int_0^1 \mathrm{d}x\, x^{n-1} \int \mathrm{d}^2 k_{\mathrm{T}}\, \Delta q_{\mathrm{v}}^{\mathrm{h}}(x, Q^2, k_{\mathrm{T}}^2)$$
$$= a_n^{\mathrm{q,v}}(\mu^2) \exp\left[-\int_{\bar{g}(\mu^2)}^{\bar{g}(Q^2)} \mathrm{d}\bar{g}\, \frac{\gamma_n(\bar{g})}{\beta(\bar{g})}\right] E_{1\mathrm{q,v}}^n(1, \bar{g}(Q^2))\,. \tag{3.4b}$$

Eqs. (3.4) makes clear what is problematic with the usual quark-parton distribution density of sect. 2. In the lowest order $\bar{g} = 0$ case, $E_{1\mathrm{q,v}}^n = 1$ and $a_n^{\mathrm{q,v}}$ is a number so that everything is fine. With chromodynamic interactions present, however, $\Delta q_{\mathrm{v}}^{\mathrm{h}}$ depends on $E_{1\mathrm{q,v}}^n(1, \bar{g}(Q^2))$ via eqs. (3.4). But we seek a parton distribution density which, although Q^2 dependent, is controlled by the wave function (and hence operator matrix element) of the proton and not by a coefficient function such as $E_{1\mathrm{q,v}}^n$; the latter appears in the operator product expansion of two electromagnetic

* We use the index q_{v} instead of i as in ref. [13]. The parameter μ is given in terms of the standard [6] QCD scale parameter Λ by $\mu = \Lambda \exp(8\pi^2/7g^2)$ for six quark flavours.

currents and has nothing to do with the proton. Only such a density would be a natural quantity on which to impose a symmetry of the proton wave function, such as broken SU(6), at the matching momentum scale Q_0. To this end, we introduce modified quark-parton distribution densities $\tilde{q}_+(x, Q^2, k_T^2)$ and $\Delta\tilde{q}^h = \tilde{q}_+ - \tilde{q}_-$ defined by

$$e_q^2 \int_0^1 dx\, x^{n-1} \int d^2k_T\, \Delta\tilde{q}_v^h(x, Q^2, k_T^2) = a_n^{q,v}(\mu^2) \exp\left[-\int_{\bar{g}(\mu^2)}^{\bar{g}(Q^2)} d\bar{g}\, \frac{\gamma_n(\bar{g})}{\beta(\bar{g})}\right]. \quad (3.5)$$

It is clear from a comparison with eqs. (3.4) that these densities are independent of $E_{1q,v}^n(1, \bar{g}(Q^2))$.

Apart from the lack of dependence on $E_{1q,v}^n(1, \bar{g}(Q^2))$, these modified parton densities possess an additional advantage. The statement of the Bjorken sum rule in terms of them is rather simple to all orders in the QCD coupling. It is now well-known [13] that this sum rule, written in terms of $G^{e(p-n)}(x)$, is modified to $O(\bar{g}^2)$. In particular,

$$\int_0^1 dx\, G^{e(p-n)}(x, Q^2) = \frac{1}{6}\frac{g_A}{g_V}\left(1 - \frac{1}{4\pi}\bar{g}^2(Q^2)\right) + O(\bar{g}^4).$$

However, the sum rule ultimately originates from the physical fact that, for $n = 1$, $R_{1q,v}^{\mu}$ is e_q^2 times the q-contribution to the ordinary weak axial current so that [13]

$$\gamma_1(\bar{g}) = 0, \quad (3.6a)$$

$$\sum_q [(a_1^q)^p - (a_1^q)^n] = \tfrac{1}{3} g_A/g_V. \quad (3.6b)$$

Eqs. (3.6) are independent of $\bar{g}$; the dependence on the latter creeps into the modified Bjorken sum rule through $E_{1q,V}^n(1, \bar{g})$. On the other hand, our $\Delta\tilde{q}_v^h$ are free of the same. Thus the corresponding statement in terms of the $\Delta\tilde{q}_v^h$ would be $\bar{g}$-independent.

More specifically, defining

$$\Delta\tilde{M}_n^{q,v}(Q^2) \equiv \int_0^1 dx\, x^{n-1} \int d^2k_T\, \Delta\tilde{q}_v(x, Q^2, k_T^2)$$
$$= \frac{1}{e_q^2} a_n^{q,v}(\mu^2) \exp\left[-\int_{\bar{g}(\mu^2)}^{\bar{g}(Q^2)} d\bar{g}\, \frac{\gamma_n(\bar{g})}{\beta(\bar{g})}\right], \quad (3.7)$$

we have

$$\Delta\tilde{M}_1^{u,v}(Q^2) - \Delta\tilde{M}_1^{d,v}(Q^2) = g_A/g_V \quad (3.8)$$

for all Q^2 and to all orders in $\bar{g}$. Eq. (3.8) will be of considerable importance to us in sect. 6 in the self-consistent determination of the matching momentum scale Q_0. We merely note here that eq. (3.7) may be rewritten with Q_0 as a reference value.

Thus

$$\Delta \check{M}_n^{\mathrm{q,v}}(Q^2) = \Delta \check{M}_n^{\mathrm{q,v}}(Q_0^2) \exp\left[-\int_{\bar{g}(Q_0^2)}^{\bar{g}(Q^2)} \mathrm{d}\bar{g}\, \frac{\gamma_n(\bar{g})}{\beta(\bar{g})}\right], \tag{3.9a}$$

where

$$\Delta \check{M}_n^{\mathrm{q,v}}(Q_0^2) = \frac{1}{e_{\mathrm{q}}^2} a_n(\mu^2) \exp\left[-\int_{\bar{g}(\mu^2)}^{\bar{g}(Q_0^2)} \mathrm{d}\bar{g}\, \frac{\gamma_n(\bar{g})}{\beta(\bar{g})}\right]. \tag{3.9b}$$

The modified quark distribution densities can be easily related to the usual ones introduced in sect. 2. By definition,

$$\Delta M_n^{\mathrm{q,v}}(Q^2) = \Delta \check{M}_n^{\mathrm{q,v}}(Q^2) E_{1\mathrm{q,v}}^n(1, \bar{g}(Q^2)). \tag{3.10}$$

Introduce a function $H^{\mathrm{q,v}}(x, Q^2)$ such that

$$\int_0^1 \mathrm{d}x\, x^{n-1} H^{\mathrm{q,v}}(x, Q^2) \equiv E_{1\mathrm{q,v}}^n(1, \bar{g}(Q^2)). \tag{3.11}$$

From eqs. (3.4a), (3.7) and (3.10) and the convolution property of Mellin transforms it follows that

$$\Delta q_{\mathrm{v}}^{\mathrm{h}}(x, Q^2, k_{\mathrm{T}}^2) = \int_x^1 \frac{\mathrm{d}y}{y} H^{\mathrm{q,v}}\left(\frac{x}{y}, Q^2\right) \Delta \tilde{q}_{\mathrm{v}}^{\mathrm{h}}(y, Q^2, k_{\mathrm{T}}^2). \tag{3.12}$$

Analogously, for the quark spin distribution densities we have

$$\Delta q_{\mathrm{v}}^{\mathrm{s}}(x, Q^2, k_{\mathrm{T}}^2) = \int_x^1 \frac{\mathrm{d}y}{y} H^{\mathrm{q,v}}\left(\frac{x}{y}, Q^2\right) \Delta \tilde{q}_{\mathrm{v}}^{\mathrm{s}}(y, Q^2, k_{\mathrm{T}}^2). \tag{3.13}$$

The $\Delta \tilde{q}_{\mathrm{v}}^{\mathrm{s}}$ of eq. (3.13) are the appropriate modified densities on which we shall use the Close ansatz [4]. However, we also have to introduce modified unpolarized densities $\tilde{q}_{\mathrm{v}}(x, Q^2, k_{\mathrm{T}}^2)$ to which these will be related. Thus we have

$$\check{M}_n^{\mathrm{q,v}}(Q^2) \equiv \int_0^1 \mathrm{d}x\, x^{n-1} \int \mathrm{d}^2 k_{\mathrm{T}}\, \tilde{q}_{\mathrm{v}}(x, Q^2, k_{\mathrm{T}}^2) = C_n^{\mathrm{q,v}}(Q_0^2) \exp\left[-\int_{\bar{g}(Q_0^2)}^{\bar{g}(Q^2)} \mathrm{d}\bar{g}\, \frac{\gamma_n(\bar{g})}{\beta(\bar{g})}\right]. \tag{3.14}$$

In eq. (3.14) $C_n^{\mathrm{q,v}}$ are the unpolarized operator matrix-elements corresponding to $a_n^{\mathrm{q,v}}$ and γ_n are the same anomalous dimensions as in the polarized case [13].

There is one more point to be made before implenting Close's ansatz. We shall work in the approximation where the polarization of the quark–antiquark pairs and the gluonic sea can be neglected in comparison with that of valence quarks. This approximation can be justified. Contributions from antiquarks (i.e. $\int \mathrm{d}^2 k_{\mathrm{T}} (\bar{q}_+ + \bar{q}_-)$) and gluons to the structure of the nucleon can be regarded as approximately zero at the matching scale Q_0^2 since at this scale, a nucleon is known to behave as an effective system with three valence quarks. Positivity of the polarized antiquark ($\bar{q}_{\pm}$) and of the corresponding gluonic distributions then allows us to justify the above approximation at $Q^2 = Q_0^2$. The evolution of this initially small sea polarization is governed by the singlet sector of the Altarelli–Parisi equations [14] namely,

$$\frac{\mathrm{d}\Delta S^{\mathrm{h}}(x,t)}{\mathrm{d}t} = \frac{\alpha(t)}{2\pi}\int_x^1 \frac{\mathrm{d}y}{y}\left[\Delta S^{\mathrm{h}}(y,t)\,\Delta P_{\mathrm{qq}}\left(\frac{x}{y}\right) + 2f\,\Delta G(y,t)\,\Delta P_{\mathrm{qG}}\left(\frac{x}{y}\right)\right], \tag{3.15a}$$

$$\frac{\mathrm{d}\Delta G(x,t)}{\mathrm{d}t} = \frac{\alpha(t)}{2\pi}\int_x^1 \frac{\mathrm{d}y}{y}\left[\Delta S^{\mathrm{h}}(y,t)\,\Delta P_{\mathrm{Gq}}\left(\frac{x}{y}\right) + \Delta G(y,t)\,\Delta P_{\mathrm{GG}}\left(\frac{x}{y}\right)\right], \tag{3.15b}$$

where $\Delta S^{\mathrm{h}}(x,t) \equiv \sum_{\mathrm{q}} \int \mathrm{d}^2k_{\mathrm{T}}\, \Delta q^{\mathrm{h}}(x, Q^2, k_{\mathrm{T}}^2)$ and summation is over all the quark flavours; other quantities are as defined in ref. [14]. The sea polarization at deep inelastic values of Q^2 can be obtained numerically from eqs. (3.15) if the Close ansatz and the assumption of the negligible sea polarization are used as boundary conditions at $Q^2 = Q_0^2$. A sea polarization, thus generated, is, however, expected [11] to be small at all Q^2 in view of the fact that the relevant polarized Altarelli–Parisi functions $\Delta P_{\mathrm{G,q}}(z)$ and $\Delta P_{\mathrm{G,G}}(z)$ – unlike their unpolarized counterparts – do not go to ∞ as $z \to 0$. This expectation is confirmed numerically in ref. [11]. It was found that $|\int \mathrm{d}^2k_{\mathrm{T}}\, \Delta\bar{q}^{\mathrm{h}}| \ll |\int \mathrm{d}^2k_{\mathrm{T}}\, \Delta q_{\mathrm{v}}^{\mathrm{h}}|$ over any range of x and for Q^2 values that are of interest to us. Note that this does not contradict the claim in ref. [15] that the q̄q sea is substantially polarized, i.e. that $|\int \mathrm{d}^2k_{\mathrm{T}}\, \Delta\bar{q}^{\mathrm{h}}|/|\int \mathrm{d}^2k_{\mathrm{T}}\, \bar{q}|$ is sizeable for large x. The point is that the qq̄ sea itself is negligible except at x near zero where polarization effects are small since the diffractive contribution conserves helicity. Thus we can ignore the net qq̄ sea contribution to the r.h.s. of eq. (2.2a) without any further ado.

The spin function can now be rewritten as

$$G_{\mathrm{v}}^{\mathrm{ep}}(x, Q^2) = \tfrac{4}{9}G_{\mathrm{u,v}}^{\mathrm{ep}}(x, Q^2) + \tfrac{1}{9}G_{\mathrm{d,v}}^{\mathrm{ep}}(x, Q^2), \tag{3.16a}$$

$$G_{\mathrm{q,v}}^{\mathrm{ep}}(x, Q^2) = \tfrac{1}{2}\int \mathrm{d}^2k_{\mathrm{T}}\, \Delta q_{\mathrm{v}}^{\mathrm{h}}(x, Q^2, k_{\mathrm{T}}^2). \tag{3.16b}$$

In view of eq. (3.12), we can now write

$$G_{\mathrm{q,v}}^{\mathrm{ep}}(x, Q^2) = \int_x^1 \frac{\mathrm{d}y}{y} H^{\mathrm{q,v}}\left(\frac{x}{y}, Q^2\right)\tilde{G}_{\mathrm{q,v}}^{\mathrm{ep}}(y, Q^2), \tag{3.17a}$$

$$\tilde{G}_{\mathrm{q,v}}^{\mathrm{ep}}(x, Q^2) = \tfrac{1}{2}\int \mathrm{d}^2k_{\mathrm{T}}\, \Delta\tilde{q}_{\mathrm{v}}^{\mathrm{h}}(x, Q^2, k_{\mathrm{T}}^2). \tag{3.17b}$$

In our notation the correct relations among the valence quark distribution densities following from Close's broken-SU(6) ansatz [4] at $Q^2 = Q_0^2$ are

$$\Delta\tilde{u}_{\mathrm{v}}^{\mathrm{s}}(x, Q_0^2, k_{\mathrm{T}}^2) = \tilde{u}_{\mathrm{v}}(x, Q_0^2, k_{\mathrm{T}}^2) - \tfrac{2}{3}\tilde{d}_{\mathrm{v}}(x, Q_0^2, k_{\mathrm{T}}^2), \tag{3.18a}$$

$$\Delta\tilde{d}_{\mathrm{v}}^{\mathrm{s}}(x, Q_0^2, k_{\mathrm{T}}^2) = -\tfrac{1}{3}\tilde{d}_{\mathrm{v}}(x, Q_0^2, k_{\mathrm{T}}^2). \tag{3.18b}$$

Thus eqs. (2.4), (3.17b) and (3.18) imply the relations

$$\tilde{G}^{\mathrm{ep}}_{\mathrm{u,v}}(x, Q_0^2) = \tfrac{1}{2}\int \mathrm{d}^2k_{\mathrm{T}}\,\frac{k_{\mathrm{L}}}{|\boldsymbol{k}|}[\tilde{u}_{\mathrm{v}}(x, Q_0^2, k_{\mathrm{T}}^2) - \tfrac{2}{3}\tilde{d}_{\mathrm{v}}(x, Q_0^2, k_{\mathrm{T}}^2)]\,, \tag{3.19a}$$

$$\tilde{G}^{\mathrm{ep}}_{\mathrm{q,v}}(x, Q_0^2) = \tfrac{1}{2}\int \mathrm{d}^2k_{\mathrm{T}}\,\frac{k_{\mathrm{L}}}{|\boldsymbol{k}|}[-\tfrac{1}{3}\tilde{d}_{\mathrm{v}}(x, Q_0^2, k_{\mathrm{T}}^2)]\,. \tag{3.19b}$$

We see how the $k_{\mathrm{L}}/|\boldsymbol{k}|$ factor in the k_{T} integration changes the naive version of Close's relations.

Eq. (3.8), which is the equivalent of the Bjorken sum rule, may also be rewritten at $Q^2 = Q_0^2$. Eqs. (2.4), (3.7), (3.8), (3.18) and the vanishing contribution of the sea-quarks to the p–n combination imply

$$\int_0^1 \mathrm{d}x \int \mathrm{d}^2k_{\mathrm{T}}\,\frac{k_{\mathrm{L}}}{|\boldsymbol{k}|}[\tilde{u}_{\mathrm{v}}(x, Q_0^2, k_{\mathrm{T}}^2) - \tfrac{1}{3}\tilde{d}_{\mathrm{v}}(x, Q_0^2, k_{\mathrm{T}}^2)] = \frac{g_{\mathrm{A}}}{g_{\mathrm{V}}}\,. \tag{3.20}$$

In the naive version the factor $k_{\mathrm{L}}/|\boldsymbol{k}|$ is missing and instead of $\tilde{u}_{\mathrm{v}}$, $\tilde{d}_{\mathrm{v}}$ one has the usual u_{v}, d_{v} so that the l.h.s. of eq. (3.20) reduces in that case to the bad SU(6) value $\frac{5}{3}$ by flavour normalization.

4. Parton k_{T} and related target-mass effects at $Q^2 = Q_0^2$

Our eventual goal is to predict $G^{\mathrm{ep}}(x, Q^2)$ in a region of fairly high Q^2 (>10 GeV^2 say). Effects due to primordial parton transverse momenta and the target mass (the two are, in fact, related) might be neglected there since $\mathrm{O}(\langle k_{\mathrm{T}}^2\rangle/Q^2)$ terms would be quite small. However, at $Q^2 = Q_0^2$, $G^{\mathrm{ep}}(x, Q_0^2)$ is related to $\mathrm{O}(\langle k_{\mathrm{T}}^2\rangle/Q_0^2)$ terms arising due to the presence of $k_{\mathrm{L}}/|\boldsymbol{k}|$ in eq. (3.19). Since Q_0 is expected [9] to be less than 1 GeV, such terms must be considered. This we do by following the covariant formulation of the parton picture due to Barbieri et al. [5].

In this formalism, the quark interacting with the photon is put on the mass-shell. This leads to the emergence of the right [16] subasymptotic scaling variable $\xi = 2x[1 + (1 + 4M^2x^2/Q^2)^{1/2}]^{-1}$. Moreover, $\langle k_{\mathrm{T}}^2\rangle/Q_0^2$ is related [17] to the only available parameter M^2/Q_0^2. Consequently, the transverse momentum effects on Close's ansatz manifest themselves as the target mass effects. The main feature of this formulation, which is of interest to us, is the following: for all parton distribution densities the separate dependence on x and k_{T}^2 is to be combined into a single dependence on the covariantly constructed variable $\rho = 2k \cdot pM^{-2}$, where p is the four-momentum of the proton. Thus, in particular, $\tilde{u}_{\mathrm{v}}(x, Q^2, k_{\mathrm{T}}^2) = \tilde{u}_{\mathrm{v}}(\rho, Q^2)$ and $\tilde{d}_{\mathrm{v}}(x, Q^2, k_{\mathrm{T}}^2) = \tilde{d}_{\mathrm{v}}(\rho, Q^2)$. Moreover, ρ is bounded by unity (since the spectator system is timelike with $(p-k)^2 > 0$) and varies [5] in the range $\xi \leqslant \rho < 1$. consequently, the k_{T} integration in the usual infinite-momentum-frame calculations is related to a ρ-integration by

$$\int \mathrm{d}^2 k_\mathrm{T} \frac{k_\mathrm{L}}{|\boldsymbol{k}|} f(x, k_\mathrm{T}^2) = x(2x-\xi)^{-1}\pi M^2 \int_\xi^1 \mathrm{d}\rho\, f(\rho) \tag{4.1}$$

for any suitable function f.

From the above discussion it follows that eqs. (3.19) can be rewritten as

$$\hat{G}^{\mathrm{ep}}_{\mathrm{u,v}}(x, Q_0^2) = \frac{x}{2(2x-\xi_0)}[\xi_0 \hat{U}_\mathrm{v}(\xi_0, Q_0^2) - \tfrac{2}{3}\xi_0 \hat{D}_\mathrm{v}(\xi_0, Q_0^2)]\,, \tag{4.2a}$$

$$\hat{G}^{\mathrm{ep}}_{\mathrm{d,v}}(x, Q_0^2) = \frac{x}{2(2x-\xi_0)}[-\tfrac{1}{3}\xi_0 \hat{D}_\mathrm{v}(\xi_0, Q_0^2)]\,, \tag{4.2b}$$

where

$$\xi_0 = 2x\left[1+\left(1+\frac{4M^2x^2}{Q_0^2}\right)^{1/2}\right]^{-1}$$

$$\hat{U}_\mathrm{v}(\xi_0, Q_0^2) = \pi M^2 \int_{\xi_0}^1 \mathrm{d}\rho\, \tilde{u}_\mathrm{v}(\rho, Q_0^2)\,, \tag{4.3a}$$

$$\hat{D}_\mathrm{v}(\xi_0, Q_0^2) = \pi M^2 \int_{\xi_0}^1 \mathrm{d}\rho\, \tilde{d}_\mathrm{v}(\rho, Q_0^2)\,. \tag{4.3b}$$

Eqs. (4.2) and (4.3) are our final statements on the Close ansatz [4] with partons k_T and related target-mass effects taken fully into account. We further note that the statement corresponding to the Bjorken sum-rule eq. (3.20) at fixed $Q^2 = Q_0^2$ can now be rewritten as

$$\int_0^1 \mathrm{d}x \frac{x}{(1+4M^2x^2/Q_0^2)^{1/2}}[\hat{U}_\mathrm{v}(\xi_0, Q_0^2) - \tfrac{1}{3}\hat{D}_\mathrm{v}(\xi_0, Q_0^1)] = \frac{g_\mathrm{A}}{g_\mathrm{V}}\,. \tag{4.4}$$

5. Evolution to higher Q^2

Eq. (3.9a) relates moments of $\tilde{G}^{\mathrm{ep}}_{\mathrm{q,v}}(x, Q^2)$ at two different scales Q^2 and Q_0^2, respectively. This equation can be inverted in a standard manner [6]. Convolution properties of the Mellin transforms, when combined with eqs. (3.7), (3.9) and (3.17b), lead to the following relation:

$$\hat{G}^{\mathrm{ep}}_{\mathrm{q,v}}(x, Q^2) = \int_x^1 \frac{\mathrm{d}y}{y} T\left(\frac{x}{y}, \frac{Q^2}{Q_0^2}\right) \hat{G}^{\mathrm{ep}}_{\mathrm{q,v}}(y, Q_0^2)\,, \tag{5.1}$$

where we introduced the function $T(x, Q^2/Q_0^2)$ defined as follows:

$$\int_0^1 \mathrm{d}x\, x^{n-1} T(x, Q^2/Q_0^2) \equiv \exp\left[-\int_{\bar{g}(Q_0^2)}^{\bar{g}(Q^2)} \mathrm{d}\bar{g}\, \frac{\gamma_n(\bar{g})}{\beta(\bar{g})}\right]. \tag{5.2}$$

Eq. (5.1) describes the evolution of the function $\tilde{G}^{\rm ep}_{\rm q,v}$ from Q_0^2 to Q^2. The use of eq. (3.17) with the function H as defined by eq. (3.11) now leads to the relation

$$G^{\rm ep}_{\rm q,v}(x, Q^2) = \int_x^1 \frac{\mathrm{d}y}{y} \int_y^1 \frac{\mathrm{d}z}{z} H^{\rm q,v}\left(\frac{x}{y}, Q^2\right) T\left(\frac{y}{z}, \frac{Q^2}{Q_0^2}\right) \tilde{G}^{\rm ep}_{\rm q,v}(z, Q_0^2)\,. \tag{5.3}$$

Eqs. (5.3), (4.2) and (3.16a) together constitute our central result. The H and T functions can be extracted via eqs. (3.11) and (5.2) from the knowledge [6, 13] of $E_{\rm 1q,v}(1, \bar{g}(Q^2))$, $\gamma_n(\bar{g})$ and $\beta(\bar{g})$. Thus, given the parametrizations of $\hat{U}_{\rm v}$ and $\hat{D}_{\rm v}$, one could obtain $G^{\rm ep}(x, Q^2)$ from eq. (5.3). We do this in the next section.

By virtue of eqs. (3.8) and (3.17b), it follows that the function $\tilde{G}^{\rm ep}(x, Q^2) = \frac{4}{9}\tilde{G}^{\rm ep}_{\rm u,v}(x, Q^2) + \frac{1}{9}\tilde{G}^{\rm ep}_{\rm d,v}(x, Q^2)$ and the corresponding $\tilde{G}^{\rm en}(x, Q^2)$ obey the Q^2 independent unmodified Bjorken sum rule

$$\int_0^1 \mathrm{d}x\, \tilde{G}^{\rm ep-en}(x, Q^2) = \tfrac{1}{6} g_{\rm A}/g_{\rm V}\,. \tag{5.4}$$

Next, we have from eq. (5.3) that

$$\begin{aligned}\int_0^1 \mathrm{d}x\, G^{\rm ep}_{\rm q,v}(x, Q^2) &= \int_0^1 \mathrm{d}t\, H^{\rm q,v}(t, Q^2) \int_0^1 \mathrm{d}s\, T(s, Q^2/Q_0^2) \int_0^1 \mathrm{d}x\, \tilde{G}^{\rm ep}_{\rm q,v}(x, Q_0^2) \\ &= E^1_{\rm 1q,v}(1, \bar{g}(Q^2)) \int_0^1 \mathrm{d}x\, \tilde{G}^{\rm ep}_{\rm q,v}(x, Q_0^2)\,,\end{aligned} \tag{5.5}$$

where in the last step eqs. (3.11) and (5.2) [with $n = 1$, and $\gamma_1 = 0$] have been used. However, it is known [13] that, to second order in $\bar{g}$,

$$E^1_{\rm 1q,v}(1, \bar{g}(Q^2)) = 1 - \frac{\bar{g}^2(Q^2)}{4\pi^2} + \mathrm{O}(\bar{g}^4) \tag{5.6}$$

is flavour independent. Eqs. (5.4), (5.5) and (3.16a) lead to the result

$$\int_0^1 \mathrm{d}x\, G^{\rm ep-en}(x, Q^2) = \frac{1}{6}\frac{g_{\rm A}}{g_{\rm V}}\left[1 - \frac{\bar{g}^2(Q^2)}{4\pi^2} + \mathrm{O}(\bar{g}^4)\right]. \tag{5.7}$$

Thus our expression for $G(x, Q^2)$ obeys the correct modified [13] Bjorken sum rule to $\mathrm{O}(\bar{g}^2)$.

6. Numerology and discussion

We shall use the theoretical model developed in earlier sections to make numerical predictions on the spin structure function $G^{\rm ep}(x, Q^2)$. Our numerical work will take only the lowest order QCD into account, although the formalism developed above is quite general. The mass-dependent quark distributions $\hat{D}_{\rm v}(\xi_0, Q_0^2)$ and $\hat{U}_{\rm v}(\xi_0, Q_0^2)$ which enter our final eq. (5.3) through $G^{\rm ep}_{\rm q,v}$ of eqs. (4.2) are obtained from the parametrizations of Buras et al. [18] which make use of the experimental data around $Q^2 \sim 1.8$ (GeV)^2 and fix the QCD scale $\Lambda^2 = 0.09$ (GeV)^2. The use of such parametrizations, determined from lowest order QCD, may not be strictly justified around $Q_0 \sim 0.5$ GeV. However, we have chosen not to use some dynamical models such as the bag model or the harmonic oscillator model [8, 9] and introduce more parameters. Instead, in the spirit that this is the first detailed parton/QCD type of a model for $G^{ep}(x, Q^2)$, we have preferred to extrapolate lowest order QCD to its extreme. All our results should thus be taken modulo (rather significant) higher order corrections in the effective QCD coupling $\bar{g}(Q^2)$.

The value of Q_0^2 which is obtained from a numerical evaluation of the Bjorken sum rule [eq. (4.4)] is 0.26 (GeV)^2. The major dilution of g_A/g_V (compared to the SU(6) value $\frac{5}{3}$) comes from the factor $(1+4M^2x^2/Q^2)^{1/2}$ in the denominator of eq. (4.4). The value of g_A/g_V is not particularly sensitive to the dependence of the distributions $\hat{U}_v(\xi_0, Q_0^2)$, $\hat{D}_v(\xi_0, Q_0^2)$ on the nucleon mass M. The use of the corresponding x-distribution leads to a slightly lower value (namely 0.25 GeV^2) for Q_0^2. In what follows we adopt this value of Q_0 and the parametrizations of valence quark distributions in terms of the x-variable. This value of Q_0^2 implies (for all Q^2)

$$\int_0^1 dx \int d^2k_T \, \Delta\tilde{u}_v^h(x, Q^2, k_T^2) = 1.01 , \tag{6.1}$$

$$\int_0^1 dx \int d^2k_T \, \Delta\tilde{d}_v^h(x, Q^2, k_T^2) = -0.25 . \tag{6.2}$$

These are significantly diluted in comparison with the corresponding integrals for the spin densities which coincide with the broken SU(6) values (respectively $\frac{4}{3}$ and $-\frac{1}{3}$) following from Close's original ansatz [4].

The solid line in fig. 1 is our prediction for the shape of $xG^{ep}(x, Q^2)$ at $Q^2 = 4$ (GeV)^2. Also shown (dashed line) is the prediction that one obtains if parton k_T is completely neglected. Evidently, the dilution due to transverse momentum effects

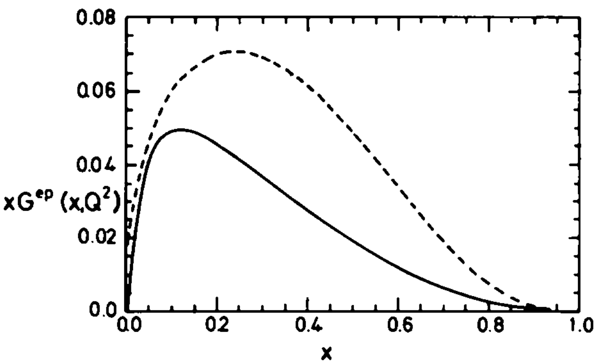
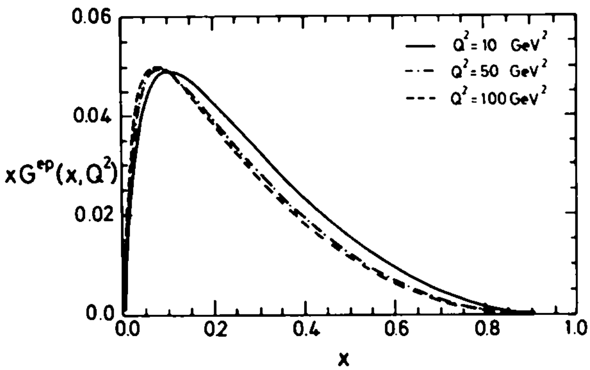


Fig. 1. $xG^{ep}(x, Q^2)$ as function of x for $Q^2 = 4$ GeV^2. The dashed curve is for the corresponding quantity with k_T^2 ignored.

(a)



(b)

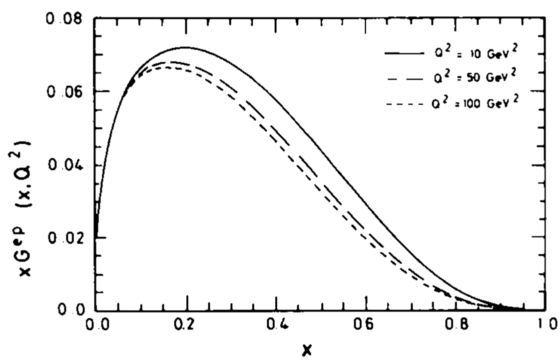


Fig. 2. (a) $xG^{ep}(x, Q^2)$ as function of x for $Q^2 = 10\,\mathrm{GeV}^2$ (full curve), $Q^2 = 50\,\mathrm{GeV}^2$ (dashed curve) and $Q^2 = 100\,\mathrm{GeV}^2$ (dotted curve), respectively. (b) As (a) but with k_T^2 ignored.

is quite significant. This figure has been given only to demonstrate this effect rather than for comparison with experiment since our theory is not so reliable at such a low value of Q^2. However, the order of magnitude between the solid and the dashed lines does not change much with Q^2 even up to $100\,\mathrm{GeV}^2$. The point is that the effects due to parton transverse momenta are largely controlled by k_T^2/Q_0^2 (rather than by k_T^2/Q^2) in a Q^2 insensitive way. Fig. 2a displays our predictions for $xG^{ep}(x, Q^2)$ at three different values of $Q^2 = 10$, 50 and $100\,\mathrm{GeV}^2$, respectively. (Fig. 2b shows the same plots but with parton-k_T effects neglected). One sees the well-known scaling violation pattern, i.e. the decrease (increase) of the structure function for large (small) x as Q^2 goes up, familiar from the study of the unpolarized non-singlet structure functions. This qualitative similarity in the two cases arises from the fact that the scale breaking in both cases is governed by the same anomalous dimensions. The same pattern is also apparent in fig. 3 which displays the variation (with Q^2) of the integral $\int_{x_1}^{x_2} \mathrm{d}x\, G^{ep}(x, Q^2)$, over small bins $x_2 - x_1$ in x. These quantities are expected to be measured [2] in forthcoming CERN experiments on polarized μp scattering and would provide a testing ground for the model proposed

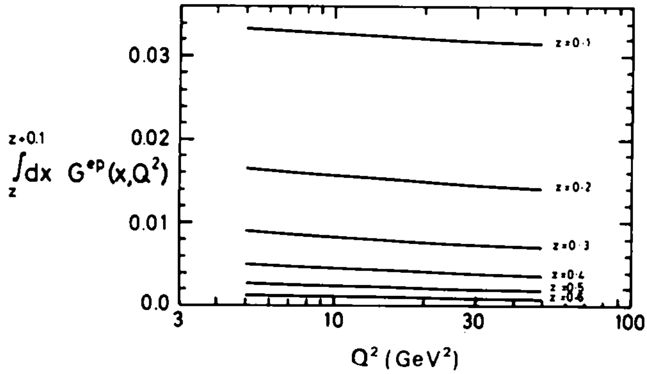


Fig. 3. The integrals $\int_z^{z+0.1} dx\, G^{ep}(x, Q^2)$ as functions of Q^2 for $z = 0.1$ to 0.6 in intervals of 0.1.

here. One important feature in all these figures is the rather weak Q^2 dependence of the plotted quantities. In view of this, the experimental extraction of the x-dependence or shape of G^{ep} would be easier compared to the study of its Q^2 evolution.

In conclusion, we have shown that the low-energy description of the nucleon suggested by Close [4] yields a theoretically consistent picture of polarized deep inelastic scattering provided the transverse momenta of the quarks are properly taken into account. Although the average transverse momenta of the quarks are not large compared to the usual deep inelastic values of $\sqrt{Q^2}$ [17], they produce significant effects at the matching momentum scale (Q_0) where the low-energy description of the nucleon is supposed to match with its parton picture necessary for the deep inelastic domain. This is a consequence of the fact that $\langle k_T^2 \rangle$ is not very small compared to Q_0^2. The dynamical higher twist effects (i.e. those arising from the presence of operators with twists >2 in the operator product expansion of currents) are completely neglected in this analysis, although our retention of the target mass does account for the kinematical higher twist effects at Q_0^2. The former effects could significantly affect our ansatz (at Q_0^2) in the $x \sim 1$ region even if they are unimportant [19] at higher Q^2. Moreover, as $x \to 1$, there will be additional higher twist effects arising from the kinematically implied [20] off-shell nature of the quarks. We have neglected these effects in the spirit of the covariant parton model of Barbieri et al. [5]. Similarly, for x very close to zero, higher order QCD effects may change our results. Nevertheless, we feel that our model has a good chance of being valid over a substantial range of x in between the end values. At least, it can provide a starting point for the comparison of the forthcoming muon data with theory.

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Appendix

RELATION BETWEEN SPIN AND HELICITY DISTRIBUTION DENSITY

Describe the helicity states of a quark-parton by $|\pm\rangle$. On the other hand, let its spin-states – spinning along the longitudinal (relative to the proton) z-direction – be designated $|\uparrow, \downarrow\rangle$. The two types of states are related by the polar rotation (azimuthal complications being inessential):

$$\begin{aligned} |+\rangle &= \cos \tfrac{1}{2}\theta|\uparrow\rangle + \sin \tfrac{1}{2}\theta|\downarrow\rangle , \\ |-\rangle &= -\sin \tfrac{1}{2}\theta|\uparrow\rangle + \cos \tfrac{1}{2}\theta|\downarrow\rangle , \end{aligned} \tag{A.1}$$

where $\cos\theta = k_{\mathrm{L}}/|\boldsymbol{k}|$. Eq. (A.1) tells us that the probability of finding a positive helicity quark within the proton is $\cos^2 \frac{1}{2}\theta$ times that of finding a spin-up quark plus $\sin^2 \frac{1}{2}\theta$ times the same for finding a spin-down quark, and similarly for a quark of negative helicity. Thus we have

$$\begin{aligned} q_+(x, Q^2, k_{\mathrm{T}}^2) &= \cos^2 \tfrac{1}{2}\theta q_\uparrow(x, Q^2, k_{\mathrm{T}}^2) + \sin^2 \tfrac{1}{2}\theta q_\downarrow(x, Q^2, k_{\mathrm{T}}^2) , \\ q_-(x, Q^2, k_{\mathrm{T}}^2) &= \sin^2 \tfrac{1}{2}\theta q_\uparrow(x, Q^2, k_{\mathrm{T}}^2) + \cos^2 \tfrac{1}{2}\theta q_\downarrow(x, Q^2, k_{\mathrm{T}}^2) . \end{aligned} \tag{A.2}$$

From eqs. (A.2) follows the relation

$$\Delta q^{\mathrm{h}}(x, Q^2, k_{\mathrm{T}}^2) = \Delta q^{\mathrm{s}}(x, Q^2, k_{\mathrm{T}}^2) \cos\theta , \tag{A.3}$$

which leads to eq. (2.4). We note further from eq. (A.2) that

$$q(x, Q^2, k_{\mathrm{T}}^2) = q_+(x, Q^2, k_{\mathrm{T}}^2) + q_-(x, Q^2, k_{\mathrm{T}}^2) = q_\uparrow(x, Q^2, k_{\mathrm{T}}^2) + q_\downarrow(x, Q^2, k_{\mathrm{T}}^2) ,$$

as expected. An integrated version of eq. (A.3) also follows from the covariant parton model approach [3].

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