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Radiative Quark Mass Generation and a Fourth Quark Family

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Abstract. We study the possibility to calculate masses and mixing angles of the known three quark families from radiative corrections connecting them to a heavy fourth family. In a left-right symmetric electroweak model with chiral horizontal gauge interactions, we obtain for the heavy quark masses values in the TeV range.

The spontaneously broken gauge theories of electroweak interactions have a large variety of mass scales. On one hand there are the gauge boson masses in the 100 GeV range linked with a large vacuum expectation value of the same order of magnitude. On the other hand the masses of known quarks and leptons ($u, d, e, \nu_e; c, s, \mu, \nu_\mu; t?, b, \tau, \nu_\tau$) are rather small compared to the vector boson mass scale. This leads one to suspect that the observed fermions are in fact massless in first approximation [1] and the actual masses result from higher order radiative corrections. Mechanisms for radiative mass generation were proposed in different frameworks: chiral horizontal gauge symmetries [2, 3], extended technicolour symmetry [4, 5] and grand unified theories [6, 7]. In all these cases the light masses of present day quarks come about by feading down some small portions of heavy fermion masses through mixing effects due to gauge boson loops or sometimes also due to Higgs-scalar loops.

The simplest situation one can imagine would be that there is just one "heavy" family with a mass of the order of the vacuum expectation value which would provide masses to the known "light" families listed above. This leads us to consider, for a given electric charge, fermion mass matrices of the form

$$m = \begin{pmatrix} \alpha_1(u_4+v_4) & 0 & 0 & v_1 \\ 0 & \alpha_2(u_4+v_4) & 0 & v_2 \\ 0 & 0 & \alpha_3(u_4+v_4) & v_3 \\ u_1 & u_2 & u_3 & u_4+v_4 \end{pmatrix} \tag{1}$$

The $u_j, v_j (j=1,2,3,4)$ are the zeroth-order elements and $\alpha_1, \alpha_2, \alpha_3 \neq 0$ originate from radiative corrections, so that $\alpha_1, \alpha_2, \alpha_3 \gg 1$. Before we explain the mechanism which can produce such radiative higher order terms we study what can be expected from (1) for the masses of the three "light" families. For this we shall assume that because of a (possibly spontaneous) breaking of the left-right symmetry $|u_j| \ll |v_k| (j, k = 1, 2, 3, 4)$. After showing that (1) leads to a satisfactory phenomenology we demonstrate that it can be obtained, in a left-right symmetric electroweak theory with chiral vertical and horizontal gauge interactions. We shall restrict ourselves to considering quarks only because of the presumably special status of neutrinos in the lepton sector. In the following, indices which refer to the quark electric charge ($Q=\frac{2}{3}$ for u, c, t and h quarks and $Q=-\frac{1}{3}$ for d, s, b and g quarks) will usually be omitted. We shall concentrate on considering the $Q=\frac{2}{3}$ states explicitly. The treatment of the $Q=-\frac{1}{3}$ states is exactly analogous.

The mass matrix (1) is defined on a basis with definite transformation properties with respect to the gauge symmetries. In order to find the physical states with a definite mass we diagonalize (1) by $\mathcal{U}_R m \mathcal{U}_L^{-1}$, which means that

$$\mathrm{diag}\,(m^2) = \mathcal{U}_L m^+ m \mathcal{U}_L^{-1} = \mathcal{U}_R m m^+ \mathcal{U}_R^{-1}, \tag{2}$$

where $\mathcal{U}_L$ and $\mathcal{U}_R$ are two, in general, different unitary matrices acting on the left- and right-handed states, respectively. In fact, the form (1) implies $\mathcal{U}_L \neq \mathcal{U}_R$, since it is not Hermitian.

With $u_j = 0 (j=1, \ldots, 4)$ and introducing $v_j =$

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$\rho_j e^{i\varphi_j}$ $(j = 1, \ldots, 4)$ we get from (1):

$$m^+ m = \begin{pmatrix} \alpha_1^2 \rho_4^2 & 0 & 0 & \alpha_1 \rho_1 \rho_4 e^{i(\varphi_1 - \varphi_4)} \\ 0 & \alpha_2^2 \rho_4^2 & 0 & \alpha_2 \rho_2 \rho_4 e^{i(\varphi_2 - \varphi_4)} \\ 0 & 0 & \alpha_3^2 \rho_4^2 & \alpha_3 \rho_3 \rho_4 e^{i(\varphi_3 - \varphi_4)} \\ \alpha_1 \rho_1 \rho_4 e^{i(\varphi_4 - \varphi_1)} & \alpha_2 \rho_2 \rho_4 e^{i(\varphi_4 - \varphi_2)} & \alpha_3 \rho_3 \rho_4 e^{i(\varphi_4 - \varphi_3)} & \rho^2 \end{pmatrix} \tag{3}$$

where $\rho^2 = \rho_1^2 + \rho_2^2 + \rho_3^2 + \rho_4^2$. The phases φ_j can be absorbed by redefining the phases of the original basis fields:

$$\psi_u \to e^{i\varphi_1}\psi_u \qquad \psi_t \to e^{i\varphi_3}\psi_t$$
$$\psi_c \to e^{i\varphi_2}\psi_c \qquad \psi_h \to e^{i\varphi_4}\psi_h \tag{4}$$

Of course, in general, these redefinitions introduce CP-violation in the gauge interactions. The diagonalizing unitary matrices now have the form

$$\mathscr{U}_{L(R)} = \mathscr{O}_{L(R)} \operatorname{diag}(e^{-i\varphi_k}), \tag{5}$$

where $\mathscr{O}_{L(R)} \in SO(4)$ are real (orthogonal) matrices. We parametrize them according to

$$\mathscr{O}(\theta, \beta, \gamma; \delta_1, \delta_2, \delta_3) = \mathscr{A}(\theta, \beta, \gamma)\mathscr{B}(\delta_1, \delta_2, \delta_3), \tag{6}$$

where

$$\mathscr{A}(\theta, \beta, \gamma) = \begin{pmatrix} \cos\beta\cos\theta & -\cos\gamma\sin\theta - \sin\gamma\sin\beta\cos\theta & \sin\gamma\sin\theta - \cos\gamma\sin\beta\cos\theta & 0 \\ \cos\beta\sin\theta & \cos\gamma\cos\theta - \sin\gamma\sin\beta\sin\theta & -\sin\gamma\cos\theta - \cos\gamma\sin\beta\sin\theta & 0 \\ \sin\beta & \sin\gamma\cos\beta & \cos\gamma\cos\beta & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \tag{7}$$

$$\mathscr{B}(\delta_1, \delta_2, \delta_3) = \begin{pmatrix} \cos\delta_1 & -\sin\delta_1\sin\delta_2 & -\sin\delta_1\cos\delta_2\sin\delta_3 & -\sin\delta_1\cos\delta_2\cos\delta_3 \\ 0 & \cos\delta_2 & -\sin\delta_2\sin\delta_3 & -\sin\delta_2\cos\delta_3 \\ 0 & 0 & \cos\delta_3 & -\sin\delta_3 \\ \sin\delta_1 & \sin\delta_2\cos\delta_1 & \sin\delta_3\cos\delta_1\cos\delta_2 & \cos\delta_1\cos\delta_2\cos\delta_3 \end{pmatrix} \tag{8}$$

The δ_j is the rotation angle in the $(j4)$ plane (between the j'th and the 4'th family) whereas θ, β, γ are the rotation angles among the three light families as introduced by Maiani [8] as a parametrization of the Kobayashi-Maskawa mixing matrix [9].

The hierarchy of masses of the various families (the first family being much lighter than the second, the second much lighter than the third etc.) is reproduced if $\alpha_1 \ll \alpha_2 \ll \alpha_3 \ll 1$. Under these conditions we can diagonalize the matrix (3) approximately in analytic form. It is easy to verify that the mixing angles of the left-handed states between the heavy and light families are

$$\delta_{jL} = \alpha_j \frac{\rho_j \rho_4}{\rho^2} \quad (j = 1, 2, 3). \tag{9}$$

Applying the rotation with these angles on (3) we obtain the mass squared matrix of the light-family left-handed states:

$$m_L^2 = \alpha^2 \rho^2 y_4^2 \begin{pmatrix} x_1^2(1 - y_1^2) & -x_1 x_2 y_1 y_2 & -x_1 x_3 y_1 y_3 \\ -x_1 x_2 y_1 y_2 & x_2^2(1 - y_2^2) & -x_2 x_3 y_2 y_3 \\ -x_1 x_3 y_1 y_3 & -x_2 x_3 y_2 y_3 & x_3^2(1 - y_3^2) \end{pmatrix} \tag{10}$$

where

$$\alpha^2 = \alpha_1^2 + \alpha_2^2 + \alpha_3^2,$$
$$x_j = \frac{\alpha_j}{\alpha} \quad (j = 1, 2, 3), \; y_j = \frac{\rho_j}{\rho} \quad (j = 1, \ldots, 4). \tag{11}$$

In terms of these parameters the mass m_h of the fourth family quark is given by

$$m_h^2 = \rho^2 \left(1 + \alpha^2 y_4^2 \sum_{j=1}^{3} x_j^2 y_j^2\right). \tag{12}$$

The diagonalization of m_L^2 gives us the mixing angles of the left-handed states in the light family sector. In the same approximation as before we obtain:

$$\theta_L = -\frac{x_1}{x_2}\frac{y_1 y_2}{y_1^2 + y_4^2}, \quad \beta_L = -\frac{x_1}{x_3}\frac{y_1 y_3}{1 - y_3^2},$$
$$\gamma_L = -\frac{x_2}{x_3}\frac{y_2 y_3}{1 - y_3^2}, \tag{13}$$

whereas the masses are given by

$$m_u^2 = \rho^2 \frac{\alpha_1^2 y_4^4}{y_1^2 + y_4^2}, \quad m_c^2 = \rho^2 \alpha_2^2 \frac{y_4^2(y_1^2 + y_4^2)}{1 - y_3^2},$$
$$m_t^2 = \rho^2 \alpha_3^2 y_4^2 (1 - y_3^2). \tag{14}$$

In deriving these relations we used the experimental fact $m_u \ll m_c \ll m_t$, too.

In order to determine the mixing of the right-handed states we must diagonalize the hermitean matrix mm^+. This can be done with ordinary degenerate perturbation theory by considering in mm^+ the terms proportional to α^2 as a perturbation. The right-handed mixing angles, however, appear only in right-handed weak interactions which are suppressed by the large masses of the corresponding gauge bosons. In the moment we have no reason to pursue the details of this calculation.

The mixing angles show a clear pattern. They are all relatively small due to the hierarchy of masses. The mixing angles between the heavy and the light families (see (9)) are the smallest ones because they are suppressed by $y_4 = \frac{\rho_4}{\rho}$ in addition to the suppression factor α_j. In general, the mixing between neighbouring families is the largest, for example $\gamma_L \gg \beta_L$ because of the assumed $x_2 \gg x_1$. We shall see later on that the $\alpha_j (j=1,2,3)$ are the same for both charges. Therefore, for $y_1 = y_2 = y_3 = 0$ the mass ratios among the families are independent of the charge and all mixing angles vanish a situation not very far from reality.

However, we shall see below that the actual mass ratios require y_1, y_2, y_3 to be larger than y_4. Furthermore the charge dependent off-diagonal elements in the mass matrix (1) make the mass ratios different for the two charges and also produce nonvanishing mixing angles. It can be shown that for the mixing of the right-handed states the situation is somewhat different. There exists considerable mixing among the heavy and light families and, in general, all the mixing angles have the tendency to be larger than for the left-handed states.

Before we can say something about the mass ratios and the various mixing angles we must have some idea about the various parameters: $y_j (j=1,\dots,4)$ and $\alpha_j (j=1,2,3)$, which appear in (13) and (14) and similarly $y'_j (j=1,\dots,4)$ for the $Q=-\frac{1}{3}$ states.

The mass matrix (1) can arise in models with spontaneously broken horizontal gauge symmetry and chiral interactions similar to the model of Barr and Zee [2] for the electron-muon mass problem (for previous work on horizontal gauge interactions see also [10–14]). We choose the gauge group $SU(2)_{LV} \otimes SU(2)_{RV} \otimes U(1)_{B-L} \otimes SU(4)_{LH} \otimes SU(4)_{RH}$ containing the vertical left-right symmetric extension of the Glashow-Weinberg-Salam model: $SU(2)_{LV} \otimes SU(2)_{RV} \otimes U(1)_{B-L}$ [15, 16] and a horizontal chiral left-right symmetric group $SU(4)_{LH} \otimes SU(4)_{RH}$ to describe the four family symmetry. These four quark families are represented by the $B-L=\frac{1}{3}$ fields $\psi_L^{\alpha a}(x)$ and $\psi_R^{\rho r}(x)$ which are doublets under $SU(2)_{LV}$ resp. $SU(2)_{RV}$ (indices α resp. ρ) and quartets under $SU(4)_{LH}$ resp. $SU(4)_{RH}$ (indices a resp. r). In order to have an anomaly free set with respect to the horizontal group it is also necessary to introduce four horizontal antiquartet "mirror" families which we assume to be very heavy.

For the breaking of the gauge symmetries a possible set of Higgs-scalar fields involves the $B-L=1$ vertical doublets $\chi_L^\alpha(x)$ and $\chi_R^\rho(x)$ and three $B-L=0$ horizontal quartet fields $\varphi_{LI}^a(x)$ and $\varphi_{RI}^r(x) (I=1,2,3)$. Here and in the following we use the same index convention as for the quark fields above. In addition, there are the $B-L=0$ fields with two vertical respectively two horizontal indices: $\sigma_{V\rho}^\alpha(x)$ and $\sigma_{Hr}^a(x)$. In fact, the symmetry breaking can be achieved also by other Higgs-field combinations. But we found these the easiest set. It is clear that these Higgs-fields cannot produce Yukawa coupling for the fermions. Therefore, to generate quark masses, we must introduce $B-L=0$ scalar fields $\phi_{\rho r}^{\alpha a}(x)$ which are doublets with respect to the vertical groups and quartets (resp. antiquartets) with respect to the horizontal groups. Then the Yukawa couplings with these fields are

$$\mathscr{L}_y = -G\tilde{\psi}_{R\rho r}(x)[\phi_{\rho r}^{\alpha a}(x)]^+ \psi_L^{\alpha a}(x) + \text{h.c.} \tag{15}$$

The vacuum expectation values, necessary to produce the various boson or fermion masses, are the following:

$$\begin{aligned}
&\langle 0|\chi_L^\alpha|0\rangle = \delta_{\alpha 2} v_{\chi L}, \quad \langle 0|\chi_R^\rho|0\rangle = \delta_{\rho 2} v_{\chi R},\\
&\langle 0|\varphi_{LI}^a|0\rangle = \delta_{aI} v_{\varphi L}^a, \quad \langle 0|\varphi_{RI}^r|0\rangle = \delta_{rI} v_{\varphi R}^r,\\
&\langle 0|\sigma_{V\rho}^\alpha|0\rangle = \delta_{\alpha\rho} v_{\sigma V}^{(\alpha)}, \quad \langle 0|\sigma_{Hr}^a|0\rangle = \delta_{ar} v_{\sigma H}^{(a)},\\
&\langle 0|\phi_{\rho r}^{\alpha a}|0\rangle = \delta_{\alpha\rho}(\delta_{a4} v_{\phi(\alpha r)} + \delta_{r4} u_{\phi(a\rho)}).
\end{aligned} \tag{16}$$

It can be shown that there is a finite region in the Higgs-coupling parameter space, where the minimum of the Higgs potential has this form. The vacuum expectation values of the χ_L, χ_R and σ_V fields break the vertical $SU(2)_{LV} \otimes SU(2)_{RV}$ symmetry in the usual way. The three quartets φ_{LI} and φ_{RI} are needed in order to break down the horizontal $SU(4)_{LH} \otimes SU(4)_{RH}$ completely. The vacuum expectation value of σ_H is responsible for the mixing of the $SU(4)_{LH}$ gauge fields with the $SU(4)_{RH}$ gauge fields which is important for the radiative quark mass generation.

The zeroth order quark masses are given by the terms $u_{\phi 1}^a$ and $v_{\phi r}^1$ for the $Q=\frac{2}{3}$ quarks and by $u_{\phi 2}^a$ and $v_{\phi r}^2$ respectively for the $Q=-\frac{1}{3}$ quarks. These produce the quark mass matrix (1) with $\alpha_1=\alpha_2=\alpha_3=0$. At low energies the right-handed vertical and both, the left- and right-handed horizontal couplings, are suppressed by large mass values of the corresponding gauge bosons. This requires for the vacuum expectation values $|v_{\chi L}|, |v_{\sigma V}|, |v_\phi| < |v_{\chi R}|$, where a factor of about 10 is already consistent with present data on right-handed vertical currents [16]. The suppression of horizontal currents requires $|v_{\varphi L}|$, $|v_{\varphi R}|, |v_{\sigma H}| \gg |v_{\chi L}|, |v_{\sigma V}|$, where at least a factor $10^{-2}-10^{-3}$ is needed [13, 14]. We assume also that

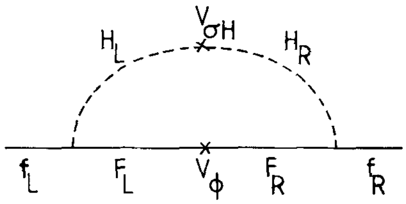

Fig. 1. One-loop radiative correction to the fermion mass matrix giving the terms with $\alpha_{1,2,3}$ in (1). $F_{L,R}$ denotes the quark in the massive family, $f_{L,R}$ is the corresponding one (with the same electric charge) in one of the light families. $H_{L,R}$ denotes flavour changing horizontal gauge bosons

the left-right horizontal symmetry is strongly broken, at least as much as the left-right vertical symmetry. This leads to $|u^a_{\phi 1}| \ll |v^1_{\phi r}|$ and $|u^a_{\phi 2}| \ll |v^2_{\phi r}|$. In the following we shall neglect the $u_a = u^a_{\phi 1,2}$. Non-vanishing u_a's would make a contribution to the t and b mass respectively. But the masses of the u and c quark (and d and s quark) would still be zero.

A consequence of this symmetry breaking scheme is the mixing of the left-handed and right-handed gauge fields belonging to the left-handed and right-handed gauge groups, respectively. This mixing produces the terms proportional to α_1, α_2 and α_3 in the mass matrix (1) due to the horizontal gauge-boson loop graph shown in Fig. 1. This graph yields the following expression for the α_j's:

$$\alpha_j = \frac{3g_H^2}{32\pi^2} \sin 2\lambda_j \left(\ln \frac{M_{2j}^2}{M_{1j}^2} + \frac{m_F^2}{M_{2j}^2 - m_F^2} \cdot \ln \frac{M_{2j}^2}{m_F^2} - \frac{m_F^2}{M_{1j}^2 - m_F^2} \ln \frac{M_{1j}^2}{m_F^2} \right). \tag{17}$$

In (17) g_H is the coupling constant of the horizontal gauge interactions to the fermions, λ_j is the left-right mixing angle and M_{1j} and M_{2j} are the masses of the mixed left-handed and right-handed gauge bosons providing the necessary flavour change between the light and the massive family. m_F is the mass of the fermion from the heavy family. One should note that there are also vertical contributions to the α_j's similar to Fig. 1. But they are presumably negligible due to the smallness of the vertical left-right gauge boson mixing. Furthermore, because of the off-diagonal vacuum expectation values of the ϕ-field (see (16)), the mass matrix of the right-handed horizontal gauge bosons differs somewhat from that of the left-handed ones. This produces small flavour-nondiagonal pieces for the graph in Fig.1. They are of the order $\alpha_j |v_\phi|/|v_\sigma|$ or $\alpha_j |v_\phi|/|v_\varphi|$ and, therefore, negligibly small. Without knowledge of the horizontal vector boson masses M_{1j} and M_{2j} and the horizontal boson coupling constant g_H it is impossible to give accurate values for the α_j's which are needed to calculate the various quark masses and mixing angles. For an estimate we assume $g_H \cong 1.6\, g_V \cong 1$ and

$$\sin 2\lambda_j \left(\ln \frac{M_{2j}^2}{M_{1j}^2} + \frac{m_F^2}{M_{2j}^2 - m_F^2} \ln \frac{M_{2j}^2}{m_F^2} - \frac{m_F^2}{M_{1j}^2 - m_F^2} \ln \frac{M_{1j}^2}{m_F^2} \right) \cong 1 \tag{18}$$

Then we obtain $\alpha_j \cong 10^{-2}$. Of course, due to the splitting of the horizontal gauge boson masses and in particular due to different mixing angles λ_j we expect some hierarchy in the α_j values. In the following we shall assume $\frac{\alpha_1}{\alpha_2} = 0.1$, $\frac{\alpha_2}{\alpha_3} = 0.05$ and $\alpha_3 = 0.05$. This splitting pattern of the α_j's could come about, for example, if $\sin 2\lambda_3$ is near its maximal value but $\sin 2\lambda_2$ and $\sin 2\lambda_1$ are correspondingly smaller. Given these α_j values it is still not possible to predict the light quark masses and their mixing angles uniquely. Instead we fix the best known masses: $m_c = 1.2$ GeV and $m_b = 4.5$ GeV and look for such parameter values y_1, y_2 and y_3 (with $\sum_i y_i^2 = 1$) for $Q = \frac{2}{3}$ and y'_1, y'_2 and y'_3 (with $\sum_i y_i'^2 = 1$) for $Q = -\frac{1}{3}$ which yield appropriately small mixing angles. Such parameters are:

$$y_1 = 0.81,\ y_2 = 0.48,\ y_3 = 0.32,\ y_4 = 0.030 \tag{19}$$
$$y'_1 = 0.21,\ y'_2 = 0.53,\ y'_3 = 0.82,\ y'_4 = 0.038$$

This gives for the quark masses which were left undetermined and for the mixing angles:

$$\begin{matrix} m_u = 0.005\ \text{GeV} & m_t = 26\ \text{GeV} & m_h = 18\ \text{TeV} \\ m_d = 0.007\ \text{GeV} & m_s = 0.15\ \text{GeV} & m_g = 4.1\ \text{TeV} \end{matrix} \tag{20}$$

and

$$\begin{matrix} \theta_L = 0.059 & \beta_L = 0.0015 & \gamma_L = 0.0087 \\ \theta'_L = 0.24 & \beta'_L = 0.0027 & \gamma'_L = 0.066 \end{matrix} \tag{21}$$

These mixing angles are defined for the two charges separately. The experimentally measured relative mixing is given by the Cabibbo–Kobayashi–Maskawa matrix $\mathscr{C} = \mathscr{U}_L(Q = \frac{2}{3})\, \mathscr{U}_L(Q = -\frac{1}{3})^{-1}$. This involves the relative phases for the two charges in (5) too. Depending on these phases, e.g. the Cabibbo angle Θ_c is in the range $|\theta_L - \theta'_L| < \Theta_c < |\theta_L + \theta'_L|$ which according to (21) yields $0.18 < \Theta_c < 0.30$ which is in the range of the experimental value $\Theta_c = 0.24$.

One might wonder whether the heavy family could be identified with the t and b quark. In this case the horizontal symmetry is $SU(3)_{LH} \otimes SU(3)_{RH}$. We found that with mass values for m_u, m_d, m_s, m_c, m_b and m_t as written above all mixing angles come out very small.

For example, the Cabibbo angle is obtained from

$$-\theta'_L = \frac{m_d}{m_c} \frac{y'_1 y'_2}{y'_3}, \tag{22}$$

which yields very small values for θ'_L if $\alpha_2 \cong 10^{-2}$.

This shows that a fourth heavy quark family is really necessary within the scenario described in this paper.

Up to now we neglected the off-diagonal terms u_1, u_2 and u_3 in the mass matrix (1). They can also make some small contributions to the light family masses. Their general effect is to lower the fourth family mass if $\alpha_{1,2,3}$ are held fixed. As already mentioned $u_{1,2,3} \neq 0$ would still lead to two massless families with radiative corrections neglected.

The mass matrix (1) can also be obtained in a model based on the horizontal group $SU(3)_{LH} \otimes SU(3)_{RH}$ with the fourth family being a horizontal singlet and L and R intercharged ("vertically mirror"). Then the $u_4 + v_4$ term is given by the Yukawa coupling to the σ_V field and the off-diagonal terms $u_{1,2,3}$ and $v_{1,2,3}$ arise from Yukawa couplings to new type of Higgs-scalars $\chi^{\alpha a}_{L\beta}$ and $\chi^{\rho r}_{R\sigma}$. The term $\alpha_3(u_4 + v_4)$ is replaced by the vacuum expectation value v of the Higgs-field ϕ, whereas the remaining diagonal elements $\alpha_1 v$ and $\alpha_2 v$ are still the result of radiative corrections. The main difference to the other model above is that α_3 is here of order one, so that the heavy family masses are now in the 100 GeV to 1 TeV range.

Let us note, that instead of the horizontal group $SU(4)_{LH} \otimes SU(4)_{RH}$, it is also possible to take $O(4)_{LH} \otimes O(4)_{RH}$, replacing the $SU(4)$ quartets by $O(4)$ vectors. This group is anomaly free. Hence there is no need for the heavy mirror families. It also opens up the possibility of an $O(10)_V \otimes O(10)_H$ grand unification [17] of the vertical $O(10)_V \supset SU(2)_{LV} \otimes SU(2)_{RV} \otimes U(1)_{B-L}$ with an $O(10)_H$ containing the horizontal group $O(4)_{LH} \otimes O(4)_{RH}$.

It is remarkable that the masses of the fourth family which we obtained lie near to the "technicolour" scale [18, 19] which is required for the dynamical symmetry breaking in $SU(2)_L \otimes U(1)$. One could imagine that the heavy family has an additional unbroken colour-like quantum number. For example, one can start with $SU(3+n)_{LH} \otimes SU(3+n)_{RH}$ and can leave a vector-like $SU(n)$ technicolour unbroken for the n heavy families [20]. In fact, after finishing this paper we received the work of Ellis and Sikivie [21] where a similar model was studied in the context of extended technicolour models. The main difference to our model is that extended technicolour is usually assumed to be vector-like. A left-right symmetric chiral extended technicolour scheme seems at first sight more complicated. But the graph in Fig. 1 has the advantage of depending only on the gauge boson mass ratios, so that the horizontal gauge bosons can be heavy enough. This solves the problem of flavour changing neutral currents in extended technicolour models.

Finally let us note that the following two main features of the chiral horizontal gauge model were important in order to come to the mass matrix (1):

(*i*) the existence of mixed left- and right-handed flavour-changing horizontal gauge bosons,
(*ii*) the simple Yukawa structure (15) with the result that the non-zero vacuum expectation value of every Higgs-field component gives rise to just one single non-zero element in the quark mass matrix.

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