LIGHT COMPOSITE FERMIONS AND ANOMALY MATCHING REVISITED

Carl H. ALBRIGHT [1]
Deutsches Elektronen-Synchrotron DESY, Hamburg, Fed. Rep. Germany

B. SCHREMPP [2] and F. SCHREMPP [2]
II. Institut für Theoretische Physik der Universität Hamburg, Hamburg, Fed. Rep. Germany

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A physically unique and simple solution of 't Hooft's anomaly matching equations in composite models of the type $SU(3)_{MC} \times SU(N)_L \times SU(N)_R$ is found for $N = 6$, if an additional selection criterion for composite "ground states" is introduced.

Two years ago 't Hooft [1] formulated a "naturalness" set of conditions whereby light composite fermions with dimensions of order Λ_{MC}^{-1}, much smaller than their inverse masses, can be formed at high energies by a new "metacolor" gauge interaction which becomes strong at an energy scale Λ_{MC}. Unfortunately the main conditions, triangle anomaly matching for preon and composite fields, the decoupling requirement and metaflavor independence proved to be so rigorous that no non-trivial solutions were found among SU(3) metacolor gauge groups possessing a chiral $SU(N) \times SU(N)' \times U(1)$ metaflavor symmetry.

Since then many authors [2–5] have carried out searches involving many different gauge groups for solutions of the anomaly-matching conditions, if need be relaxing the decoupling and N-independence conditions which are on a weaker footing than the first condition. Meanwhile it has been shown [4,6] that the assumption of no phase transition is required in addition to the Appelquist–Carazzone [7] decoupling theorem in order to justify the decoupling condition. Some of these searches have been carried out in the framework of superalgebras, or in the case of one of the authors, by using a valence-like 3-preon model [3] for composite quarks and leptons. Many solutions have been found but none are particularly attractive [‡1]; either they are too simple or so complex as to admit many composite states which have not been observed.

In this letter we reexamine the anomaly conditions and decoupling requirement and show that by a suitable modification it is possible to obtain a physically unique solution of the anomaly conditions applied to $SU(3)_{MC} \times SU(N) \times SU(N)'$; this solution lies in the physically interesting metaflavour group $SU(6) \times SU(6)'$. Consider the general chirally-symmetric case in a left-handed formulation with N preons in the fundamental $\mathbf{3}$ and N in the $\bar{\mathbf{3}}$ of $SU(3)_{MC}$. The metaflavor group is then $SU(N) \times SU(N)' \times U(1)$ and the two kinds of preons are left-handed Weyl spinors belonging to the representations

$$P = (\mathbf{3}; N, 1) \equiv (\square; \square, 1) ,$$
$$P' = (\bar{\mathbf{3}}; 1, \bar{N}) \equiv (\bar{\square}; 1, \bar{\square}) . \tag{1}$$

To form (left-handed) composite states which are MC-

[1] Permanent address: Department of Physics, Northern Illinois University, DeKalb, Illinois 60115, U.S.A.
[2] Heisenberg Foundation Fellow.

[‡1] Among the most interesting solutions are models reported by Bars and Yankielowicz [4], which satisfy the decoupling condition as well. Albright and Sikivie [5] found a particular example of this type based on $SU(4)_{MC} \times SU(N) \times SU(N)' \times U(1)$ [2].

singlets, we need only consider the 3-preon configurations PPP, $PP'^{\dagger}P'^{\dagger}$ and $P \leftrightarrow P'$. One arrives at the set of spin 1/2 composite fermions equivalent to those listed by 't Hooft [1]

$$
\begin{aligned}
\psi_1 &= (\mathbf{1}; \ydiagram{2,1}, 1) \sim PPP\,, \\
\psi_1' &= (\mathbf{1}; 1, \overline{\ydiagram{2,1}}) \sim P'P'P'\,, \\
\psi_2 &= (\mathbf{1}; \ydiagram{1}, \ydiagram{1,1}) \sim PP'^{\dagger}P'^{\dagger}\,, \\
\psi_2' &= (\mathbf{1}; \overline{\ydiagram{1,1}}, \overline{\ydiagram{1}}) \sim P^{\dagger}P^{\dagger}P'\,, \\
\psi_3 &= (\mathbf{1}; \ydiagram{1,1,1}, 1) \sim PPP\,, \\
\psi_3' &= (\mathbf{1}; 1, \overline{\ydiagram{1,1,1}}) \sim P'P'P'\,, \\
\psi_4 &= (\mathbf{1}; \ydiagram{1}, \ydiagram{2}) \sim PP'^{\dagger}P'^{\dagger}\,, \\
\psi_4' &= (\mathbf{1}; \overline{\ydiagram{2}}, \overline{\ydiagram{1}}) \sim P^{\dagger}P^{\dagger}P'\,, \\
\psi_5 &= (\mathbf{1}; \ydiagram{3}, 1) \sim PPP\,, \\
\psi_5' &= (\mathbf{1}; 1, \overline{\ydiagram{3}}) \sim P'P'P'\,.
\end{aligned}
\tag{2}
$$

In the chirally-symmetric case, to each pair of states ψ_i and ψ_i' is associated the index l_i which can assume integer values.

Anomaly matching on the preon and composite levels leads to the two well-known 't Hooft constraint equations [1]

$$
(N^2-9)l_1 - \tfrac{1}{2}N(N-7)l_2 + \tfrac{1}{2}(N-6)(N-3)l_3 - \tfrac{1}{2}N(N+7)l_4 + \tfrac{1}{2}(N+6)(N+3)l_5 = 3\,, \quad N>2 \tag{3a}
$$

from matching $[\mathrm{SU}(N)]^3$ anomalies and

$$
(N^2-3)l_1 - \tfrac{1}{2}N(N-3)l_2 + \tfrac{1}{2}(N-2)(N-3)l_3 - \tfrac{1}{2}N(N+3)l_4 + \tfrac{1}{2}(N+2)(N+3)l_5 = 1\,, \quad N>1 \tag{3b}
$$

from matching $[\mathrm{SU}(N)]^2 \times \mathrm{U}(1)$ anomalies. There are a number of solutions to these equations for any N with the exception of $N = 3, 6, 9, \ldots$, where there exists none at all. 't Hooft's additional requirement of simultaneous decoupling of preons (if made virtually heavy) and their associated composites proved too strong a constraint leaving only $N = 2$ as a solution. We feel that Preskill and Weinberg's observation [6] mentioned in the introduction justifies dropping the decoupling conditions. The purpose of this paper is to look for alternative constraints and/or modifications of 't Hooft's program as applied to $\mathrm{SU}(3)_{\mathrm{MC}} \times \mathrm{SU}(N) \times \mathrm{SU}(N)' \times \mathrm{U}(1)$.

In the following we discuss two such modifications which, taken separately, do not change matters in a satisfactory way. Only if combined do they lead to a practically unique, simple and appealing solution.

(1) The first modification is to introduce an additional strong constraint on the set of admitted composites. We single out candidates for "ground state" composites by simply combining the spins of the three preons to spin 1/2 and by applying [3,8] the Fermi principle to identical preons in the following restricted sense: we require total antisymmetry with respect to metacolor, metaflavor and Lorentz structure [‡2] (implying spatial symmetry). This reduces the list (2) of composites to the four composites ψ_1, ψ_2, ψ_1' and ψ_2' with non-negative indices only. We immediately lose *all* solutions of 't Hooft's anomaly equations (3a,b) except the one for the trivial case of $N = 2$ where only eq. (3b) applies. It seems rather remarkable that our ground state criteria reduce the plethora of solutions down to that singled out by 't Hooft's decoupling requirement.

(2) As a second modification of 't Hooft's program for $\mathrm{SU}(3)_{\mathrm{MC}} \times \mathrm{SU}(N) \times \mathrm{SU}(N)' \times \mathrm{U}(1)$ we propose to break spontaneously the conserved U(1) symmetry [‡3] (by metacolor forces). The important observation is the following. 't Hooft's idea of keeping composites massless on the scale Λ_{MC} is to leave the full chiral metaflavor group unbroken. This idea is not invalidated by retaining only the chiral part $\mathrm{SU}(N) \times \mathrm{SU}(N)'$. Spontaneous breaking of the U(1) symmetry does not affect the masslessness of the preons, but it obviously has dramatic effects on the composite level, in so far as the $[\mathrm{SU}(N)]^2 \times \mathrm{U}(1)$ anomaly matching condition (3b) gets lost. Of course in dropping this condition we obtain a real proliferation of solutions to the anomaly eq. (3a), including now also solutions for $N = 3, 6, 9, \ldots$. We remark in passing that imposition of the decoupling conditions would again eliminate all solutions except

‡2 The same type of Fermi principle was applied [3] in a search of composite models with the result that known solutions were recovered as well as new solutions found.

‡3 We tacitly assume that the broken U(1) is gauged with a suitable strength such that the appearing Goldstone boson will be eaten up to make the U(1) gauge field massive.

for $N = 2$ as noted from a straightforward computer search.

One realizes that the two modifications of the conventional anomaly-matching scenario, eqs. (2) and (3a,b) have opposite effects: restriction to the "ground state" composites strongly reduces the number of solutions, whereas the spontaneous breaking of the U(1) symmetry increases it. This leads us to combine both modifications, i.e., we consider the SU(N) [3] anomaly equation (3a) for the "ground state" composites ψ_1, ψ_2, ψ_1' and ψ_2' (with non-negative indices). The anomaly equation (3a) then simplifies to

$$(N^2 - 9)l_1 - \tfrac{1}{2}N(N-7)l_2 = 3\,. \tag{4}$$

It is straightforward to see that there is no solution at all for $N = 3, 4$, and 5. For $N = 6$ there is a unique and simple solution

$$SU(6) \times SU(6)' \quad \text{with } l_1 = 0 \quad \text{and } l_2 = 1\,. \tag{5}$$

Within a physically acceptable range of N and l_1, l_2 there is no further solution. To quantify this statement, a computer search for $N \leqslant 100$ and $l_1, l_2 \leqslant 20$ was performed, and only three additional but unacceptable solutions were found, namely

$$\begin{aligned} &SU(8) \times SU(8)' && \text{with } (l_1, l_2) = (1, 13)\,,\\ &SU(10) \times SU(10)' && \text{with } (l_1, l_2) = (3, 18)\,,\\ &SU(20) \times SU(20)' && \text{with } (l_1, l_2) = (3, 9)\,. \end{aligned} \tag{6}$$

Hence we have singled out a *physically unique* solution. Not only is $N = 6$ physically appealing, but also the composite structure is especially simple:

$$\begin{aligned} \psi_2 &= (\mathbf{1}; \square, \begin{smallmatrix}\square\\\square\end{smallmatrix}) = (\mathbf{1}; 6, 15) = PP'^{\dagger}P'^{\dagger}\,,\\ \psi_2' &= (\mathbf{1}; \bar{\begin{smallmatrix}\square\\\square\end{smallmatrix}}, \bar{\square}) = (\mathbf{1}; \overline{15}, \bar{6}) = P^{\dagger}P^{\dagger}P'\,, \end{aligned} \tag{7}$$

both states appearing with unit multiplicity. SU(6) $\times$ SU(6)$'$ is obviously big enough to contain physically interesting subgroups like $SU(3)_c \times [SU(2) \times U(1)]_{W-S}$. In subsequent papers we shall explore spontaneous dynamical symmetry breaking chains such as shown in table 1 arising due to $SU(3)_{MC}$ singlet condensates caused by the strong metacolor force. The left-handed chain represents an interesting laboratory for studying successive decouplings of metaflavor degrees of freedom by giving *dynamical* masses to some of the preons and composites via spontaneous symmetry breaking at each step. Such a mechanism may be contrasted to 't Hooft's original decoupling requirement. The possible connection to reality of the right-hand chain is obvious.

Let us briefly speculate on how a spontaneous breakdown of the U(1) symmetry could be achieved dynamically for $N = 6$. An interesting candidate is a scalar six preon condensate $PPPPPP$ $(=P^6)$ or $P'P'P'P'P'P'$ $(=P'^6)$ which breaks the U(1) symmetry but can be made a singlet under $SU(3)_{MC} \times SU(6) \times SU(6)$ due to the fact that 6 is a multiple of 3.

Breaking the U(1) implies preon number violation. The term "preon" then takes on the meaning of "*valence preon*"; it represents an indeterminate superposition of states $P(P^6)^n$, $n = 0, 1, 2, \ldots$, all having spin 1/2 and the same transformation properties under $SU(3)_{MC} \times SU(6) \times SU(6)'$ as the preon in the presence of an unbroken preon number U(1). The usual $U(1)_{B-L}$ or $U(1)_{e.m.}$ will have to be contained in the group SU(6) $\times$ SU(6)$'$ in our case.

In closing let us discuss the following important point. At first sight one might argue that from the experimental information on QCD it is known that any gauge theory with gauge group SU(3) and six flavors should be spontaneously broken as

Table 1

$$SU(6)_L \times SU(6)_R$$
$$\downarrow$$
$$SU(5)_L \times SU(5)_R \times U(1)_{L+R}$$
$$\swarrow \qquad\qquad \searrow$$
$$SU(4)_L \times SU(4)_R \times U(1)^2_{L+R} \qquad SU(3)_{L+R} \times SU(2)_L \times SU(2)_R \times U(1)^2_{L+R}$$
$$\vdots$$
$$U(1)^5_{L+R}$$

$$\mathrm{SU}(6) \times \mathrm{SU}(6)' \times \mathrm{U}(1)_V \to \mathrm{SU}(6)_V \times \mathrm{U}(1)_V ,$$

which would be at variance with our solution for SU(3) metacolor.

However, one easily convinces oneself that there are yet enough internal degrees of freedom in such theories as to allow different realizations on the QCD and metacolor levels.

Intrinsic quark masses may, for instance, have a crucial influence on the pattern of spontaneous breaking of chiral flavor symmetries as discussed recently by Preskill and Weinberg [6] and Bars and Yankielowicz [4]. Following these authors, let us suppose that in QCD one or more quarks have an intrinsic mass m_0 bigger than a certain critical mass. Then consistency with the Appelquist–Carazzone theorem [7] for $m_0 \to \infty$ enforces the residual chiral symmetry to break down spontaneously, e.g. to the vectorial subgroup (in agreement with experiment!). In contrast, for SU(3) metacolor and *massless preons* the chiral SU(6) $\times$ SU(6)$'$ symmetry may well be unbroken and the U(1) broken instead (if – as stated earlier – 't Hooft's stronger "decoupling condition" is given up).

After completion of this paper we received a preprint by Casalbuoni and Gatto [9], which also discusses a possible spontaneous breaking of the preon number U(1) symmetry.

Composite quarks and leptons, resulting from **(6, 15)** and **(15, 6)** composites belonging to a broken $\mathrm{SU}(6)_L \times \mathrm{SU}(6)_R$ flavor group have been discussed by Squires [10].

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