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SUPERWEAK *CP* VIOLATION AND RIGHT-HANDED HORIZONTAL INTERACTIONS

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A horizontal extension of the Weinberg-Salam electroweak theory by a right-handed $O(3)_{RH}$ gauge symmetry for the three-fermion families is studied. By an appropriate choice of the Higgs scalar fields the *CP* symmetry of the lagrangian is spontaneously broken, but the mixing of the left-handed fermion states, and hence the Kobayashi-Maskawa mixing matrix, remains real. The *CP* violation is manifested in the superweak horizontal gauge interactions, which are suppressed by the large mass of the corresponding gauge bosons. It is, however, possible that the horizontal boson acting on the second and third families can be considerably lighter than the other two, implying an interesting phenomenology of the related *CP* violation effects and flavour-changing neutral currents.

1. Introduction

17 years after its discovery [1] the reason for *CP* violation is still mysterious. Up to now it has been experimentally observed only in the subtleties of K-meson decays. This fact follows from its smallness compared to ordinary weak interactions. Theoretically it is, in fact, not difficult to implement *CP* (or equivalently *T*) violation. It is, however, harder to understand that if there is *CP* violation at all why is it so small. This applies, in particular, to most of the currently popular models where the *CP* violation is part of the weak interactions described by spontaneously broken gauge theories (see, e.g. [2, 3]). There exists also the danger of large *CP* (and *P*) violations in strong interactions, induced by non-perturbative QCD effects [4-6], if the quark mass matrix obtained from spontaneous symmetry breaking is complex.

A possible way to explain the smallness of *CP* violation is to associate it to a new superweak interaction due to the repetition of lepton-quark families. Such "horizontal" weak interactions [7-9] have to be superweak anyway because of the observed accuracy of separate electron-, muon-, and τ-lepton number conservation and because of the small upper limits on flavour-changing neutral currents [10, 11]. The horizontal interactions are associated to a spontaneously broken gauge symmetry; therefore, this is a field theoretic realization [12] of the superweak model of *CP* violation [13]. As shown recently by Davidson and Wali [14], this *CP*

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violation scenario can be implemented for any number of fermion families with an SU(2) horizontal symmetry. In particular, they pointed out the importance of the chiral structure of horizontal interactions for obtaining realistic fermion mass matrices. An important property of their model is the vanishing of the tree level strong and weak *CP* violation, leaving the horizontal interaction as the main source of *CP* violation. (For different, recent attempts with horizontal *CP* violation, see also [15–17].)

In this paper we present a new *CP*-violation model of "horizontal" type, based on the gauge symmetry group $SU(2)_{LV} \otimes U(1) \otimes O(3)_{RH}$. Here $SU(2)_{LV} \otimes U(1)$ is the Weinberg–Salam electroweak symmetry [18, 19] and $O(3)_{RH}$ is a right-handed "horizontal" symmetry for the three observed families. (We assume the existence of the t-quark completing the third family.) The main difference compared to the model of Davidson and Wali [14] (and also compared to the other models in refs. [15–17]) is in the chiral structure of the horizontal interactions: in our case only the right-handed fermions carry non-zero $O(3)_{RH}$ quantum numbers and the left-handed fermions are horizontal scalars. The Davidson–Wali model is based on an axial-vector-like ("flavour-chiral") horizontal symmetry with opposite horizontal isospin for left- and right-handed fermions. The change in the chiral structure implies, besides the difference in the horizontal interaction, a completely different Higgs sector and therefore different structures for fermion and vector–boson mass matrices. In addition to the questions studied in refs. [14–17] we also explicitly construct and study the Higgs sector of our model. This allows a more precise specification of the phenomenological consequences. From the study of the Higgs potential we show that in our model *CP* violation can arise from the spontaneous breaking of the original *CP* symmetry of the lagrangian by the minimum of the Higgs potential. At the tree level the mixing angles of the left-handed quarks and leptons are real due to the choice of the Higgs fields even in the case of three families. (Therefore, at least in the tree approximation, the Kobayashi–Maskawa mechanism [20] of *CP* violation in ordinary weak interactions is not operating.) The strong *CP* violation also vanishes at the tree level. The right-handed mixing is, however, complex, inducing *CP* violation in the superweak horizontal interactions, which act only on the right-handed fermion components. The $SU(2)_{LV} \otimes U(1) \otimes O(3)_{RH}$ model is a modest extension of the Weinberg–Salam electroweak theory, as the $O(3)_{RH}$ horizontal symmetry affects only the right-handed fermion components, which are scalar under the "vertical" $SU(2)_{LV}$.

In sect. 2, after definining the model precisely, we construct and diagonalize the mass matrices of fermions and gauge bosons. In sect. 3 we first show (in part referring to the appendix) how *CP*-violating minima of the Higgs potential arise and then determine the low-energy, effective, four-fermion interactions containing the *CP*-violating "horizontal" neutral currents. In sect. 4 we briefly discuss the phenomenological implications of the model and make a few concluding remarks.

3. The $O(3)_{RH}$ family group

We assume in this paper that there are three lepton–quark families spanning a three-dimensional (fundamental, vector) representation of $O(3)_{RH}$ with the right-handed field components $\psi_{Rr}(x)_q$ ($r = 1, 2, 3$; $q = 1, \frac{2}{3}, \frac{1}{3}$). Of course, the Lie algebra of O(3) is isomorphic to SU(2); therefore, we could call the horizontal symmetry $SU(2)_{RH}$ as well. By using the name $O(3)_{RH}$ we want to emphasize that representations with half-integer SU(2) spin (doublets, quartets, etc.) will not be considered. Besides, for a larger number $n > 3$ of families we would take the generalization $O(n)_{RH}$ and its n-dimensional representation. The index q above refers to the electric charge: $q = 1$ stands for charged leptons, $q = \frac{2}{3}$ for up-quarks and $q = \frac{1}{3}$ for down-quarks. Right-handed neutrinos will not be considered in the present paper because of their exceptional status.

The left-handed fermion field components $\psi_L^\alpha(x)_h$ ($h = 1, 2, 3$) are doublets (index α) under $SU(2)_{LV}$. The index h stands for the three different families. There are, of course, separate left-handed doublet fields for quarks and leptons but in most cases it will not be necessary to specify whether quarks or leptons are being considered.

The only difference is in the values of the weak hypercharge Y belonging to U(1): $Y = \frac{1}{3}$ for the quark doublet and $Y = -1$ for the lepton doublet. (The Y values for the right-handed fields are also determined by the familiar expression for the electric charge: $Q = T_{LV3} + \frac{1}{2}Y$.)

The simplest possible set of Higgs scalar fields consists of a complex $SU(2)_{LV}$ doublet, $Y = 1$ and $O(3)_{RH}$ triplet: $\phi_r^\alpha(x)$ and three real $SU(2)_{LV}$ singlet, $Y = 0$, $O(3)_{RH}$ triplets: $\eta_r(x)_J$ ($J = 1, 2, 3$). These latter do not have Yukawa coupling to fermions. They are needed in order to give large masses to the horizontal gauge bosons. The Yukawa coupling, e.g. to quarks has the form

$$\mathscr{L}_Y = -\sum_h \{G_h \bar{\psi}_{Rr}(x)_{1/3} \phi^+_{\alpha r}(x) \psi_L^\alpha(x)_h + \bar{G}_h \bar{\psi}_{Rr}(x)_{2/3} \tilde{\phi}^+_{\alpha r}(x) \psi_L^\alpha(x)_h\} + \text{h.c.}, \quad (2.1)$$

where the $Y = -1$ field $\tilde{\phi}$ is defined as usual by $\tilde{\phi}^\alpha \equiv \varepsilon_{\alpha\beta}\phi^+_\beta$. The Yukawa coupling constants are real in accordance with the CP-invariance of the lagrangian. The zeroth-order quark mass matrices are given by the vacuum expectation value of the ϕ field, which is of the form

$$\langle 0|\phi_r^\alpha(x)|0\rangle = \delta_{\alpha 2}\sqrt{\tfrac{1}{2}}(u_r + iv_r) . \quad (2.2)$$

This gives the following mass matrix, for instance, for the down-quarks:

$$m_{rh} = \sqrt{\tfrac{1}{2}}(u_r - iv_r)G_h . \quad (2.3)$$

For the charged leptons we have the same form as this, only with different Yukawa coupling constants G'_h. For the up-quarks, on the other hand, G_h is replaced by $\bar{G}_h$ and the complex conjugate is taken. In order to obtain the physical fermion states with definite mass the mass matrix m has to be diagonalized by a biunitary transformation:

$$
\begin{aligned}
U_{\mathrm{R}} m U_{\mathrm{L}}^{-1} &= \mathrm{diag}\,(m)\,, \\
U_{\mathrm{R}} m m^{+} U_{\mathrm{R}}^{-1} &= U_{\mathrm{L}} m^{+} m U_{\mathrm{L}}^{-1} = \mathrm{diag}\,(m^2)\,.
\end{aligned}
\tag{2.4}
$$

(In what follows we shall explicitly consider only the down-quark sector, as the results can be easily transcribed also for the up-quarks and charged leptons.) U_{R} and U_{L} are, in general, 3×3 unitary matrices. The specific form of eq. (2.3) implies

$$
\begin{aligned}
(m^{+}m)_{hh'} &= \tfrac{1}{2}\rho^2 G_h G_{h'}\,, \qquad (\rho^2 \equiv \rho_r \rho_r)\,, \\
(m m^{+})_{rr'} &= \tfrac{1}{2} G^2 \rho_r \rho_{r'}\, \mathrm{e}^{i(\varphi_{r'}-\varphi_r)}\,, \qquad \Big(G^2 \equiv \sum_h G_h^2\Big)\,,
\end{aligned}
\tag{2.5}
$$

where we used the notation

$$
u_r + i v_r \equiv \rho_r\, \mathrm{e}^{i\varphi_r}\,. \tag{2.6}
$$

From eq. (2.5) it follows that

$$
U_{\mathrm{L}} = O_L\,, \qquad U_{\mathrm{R}} = O_{\mathrm{R}} F\,, \tag{2.7}
$$

where O_{L} and O_{R} are real orthogonal matrices and F is a diagonal matrix containing the phases

$$
F = \begin{pmatrix} \mathrm{e}^{i\varphi_1} & 0 & 0 \\ 0 & \mathrm{e}^{i\varphi_2} & 0 \\ 0 & 0 & \mathrm{e}^{i\varphi_3} \end{pmatrix}. \tag{2.8}
$$

From the remarks made after eq. (2.3), it follows that O_{L} is, in general, different for different electric charges, whereas O_{R} is the same and the phases in F are the same for down-quarks and charged leptons and they are opposite for the up-quarks.

It can be shown, that in the parameterization introduced for the three family mixing problem by Maiani [21], we have for the right-handed orthogonal mixing matrix O_{R}:

$$
O_{\mathrm{R}} = \begin{pmatrix} \cos\beta_{\mathrm{R}} & -\sin\gamma_{\mathrm{R}}\sin\beta_{\mathrm{R}} & -\cos\gamma_{\mathrm{R}}\sin\beta_{\mathrm{R}} \\ 0 & \cos\gamma_{\mathrm{R}} & -\sin\gamma_{\mathrm{R}} \\ \sin\beta_{\mathrm{R}} & \sin\gamma_{\mathrm{R}}\cos\beta_{\mathrm{R}} & \cos\gamma_{\mathrm{R}}\cos\beta_{\mathrm{R}} \end{pmatrix}, \tag{2.9}
$$

where the angles are given by

$$
\sin\beta_{\mathrm{R}} = \frac{\rho_1}{\rho}\,, \qquad \sin\gamma_{\mathrm{R}} = \frac{\rho_2}{\sqrt{\rho^2-\rho_1^2}}\,. \tag{2.10}
$$

The third Maiani-angle θ_{R} is zero. The horizontal interaction of the physical quarks depends on the right-handed mixing matrix U_{R} given by eqs. (2.7)–(2.9).

It can be easily shown that the mass matrix m in eq. (2.3) has two zero eigenvalues (the third, non-zero eigenvalue is $m_3=\sqrt{\frac{1}{2}}G\rho$). This means that the masses of the first two families are zero at the tree level. In order to obtain non-zero tree-level masses, one has to introduce three Yukawa-coupled ϕ-like Higgs field: $\phi_r^\alpha(x)_J$ ($J=1,2,3$). (For the possibility of radiative generation of the masses of the first and second family see the remarks in sect. 4.) The Higgs potential with three η-like and three ϕ-like fields is, however, rather complicated, because of the large number of independent invariants. Therefore, in sect. 3 we shall consider in detail only the simpler case with one ϕ-field. Note, that if the three ϕ-fields have the vacuum expectation values

$$\langle 0|\phi_r^\alpha(x)_J|0\rangle=\delta_{\alpha 2}\delta_{rJ}\sqrt{\tfrac{1}{2}}(u_r+iv_r)\,, \tag{2.11}$$

then both the gauge boson matrix and the structure of the fermion mixing matrices in (2.7) remain the same. As a consequence, from the point of view of CP violation nothing essential is changed. The down-quark mass matrix, for instance, following from eq. (2.11) is

$$m_{rh}=\sqrt{\tfrac{1}{2}}(u_r-iv_r)G_{J-r,h}\,. \tag{2.12}$$

This implies

$$\begin{aligned}(m^+m)_{hh'}&=\tfrac{1}{2}\sum_{J=1}^{3}\rho_J^2G_{Jh}G_{Jh'}\,,\\ (mm^+)_{rr'}&=\tfrac{1}{2}\sum_{h=1}^{3}G_{rh}G_{r'h}\rho_r\rho_{r'}\,e^{i(\varphi_{r'}-\varphi_r)}\,.\end{aligned} \tag{2.13}$$

The relations (2.7), (2.8) are valid as before, but the explicit form of O_R is more complicated than (2.9). In fact, O_R is now different for different electric charges (like O_L, too). The reality of the left-handed mixings implies that the Cabibbo matrix entering the left-handed, vertical interactions

$$C_L\equiv U_L(q=\tfrac{2}{3})U_L(q=\tfrac{1}{3})^{-1} \tag{2.14}$$

is real. Therefore the Kobayashi–Maskawa CP-violation phase [20] vanishes. There is, however, CP-violation in the right-handed horizontal couplings because of the phases in U_R. These phases are opposite for the up-quarks and down-quarks. This means that det m has opposite phase for up- and down-quarks, therefore the total phase of the determinant of the quark mass matrix on the whole flavour space is zero at the tree-level. As a consequence, there is probably no problem with the induced strong CP (and P) violation as it can arise only through the radiative corrections, similarly to the model in [22, 23].

Let us consider the mass matrix of gauge bosons following from the vacuum expectation values in (2.11). [This also contains eq. (2.2) with a single ϕ-field as a special case.] Corresponding to the symmetry group $SU(2)_{LV}\otimes U(1)\otimes O(3)_{RH}$ there are altogether 7 gauge bosons: $W_{1,2,3}(x)_\mu$ for $SU(2)_{LV}$, $B(x)_\mu$ for U(1) and

$H_{1,2,3}(x)_\mu$ for $O(3)_{RH}$. The experimental upper limits on the flavour-changing (in fact, generally "family-changing") neutral currents [10, 11] tell us that the masses of the horizontal gauge bosons are at least a factor 10^2 larger than the masses of the Weinberg–Salam gauge bosons. These high masses are provided by the vacuum expectation values of the Higgs fields $\eta_r(x)_J$ $(J=1,2,3)$, which are scalar with respect to $SU(2)_{LV} \otimes U(1)$. Therefore, our model has two different intrinsic mass scales set by the magnitudes of the vacuum expectation values of ϕ-fields and η-fields, respectively, the latter being at least a factor 10^2 larger than the former.

In order to break down the $O(3)_{RH}$ symmetry completely we need at least two η-fields. The third one is needed for the spontaneous breaking of the CP symmetry. As we shall see in the appendix, it seems to be impossible to have a CP-violating minimum of the Higgs potential with two η-fields. By the $O(3)_{RH}$ symmetry transformation freedom the vacuum expectation values of the three η-fields can always be transformed into the position:

$$\begin{aligned}
\langle 0|\eta_r(x)_1|0\rangle &= \delta_{r1} z_1 \equiv z_{1r}\,,\\
\langle 0|\eta_r(x)_2|0\rangle &= (\delta_{r1}\cos\alpha + \delta_{r2}\sin\alpha) z_2 \equiv z_{2r}\,,\\
\langle 0|\eta_r(x)_3|0\rangle &= [(\delta_{r1}\cos\alpha' + \delta_{r2}\sin\alpha')\sin\beta' + \delta_{r3}\cos\beta'] z_3 \equiv z_{3r}\,,
\end{aligned} \tag{2.15}$$

where the absolute values are ordered according to $|z_1| \geqslant |z_2| \geqslant |z_3|$. The gauge boson H_a $(a=1,2,3)$ belongs to the rotation in the $O(3)_{RH}$ space around the ath axis, and hence to the generator T_a, where

$$T_1 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & -i \\ 0 & i & 0 \end{pmatrix}, \qquad T_2 = \begin{pmatrix} 0 & 0 & i \\ 0 & 0 & 0 \\ -i & 0 & 0 \end{pmatrix}, \qquad T_3 = \begin{pmatrix} 0 & -i & 0 \\ i & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}. \tag{2.16}$$

This implies that for a generic vacuum expectation value z_r of an η-like Higgs field the mass-squared matrix of the horizontal bosons is (with g_H the $O(3)_{RH}$ coupling constant)

$$M^2_{ab} = \tfrac{1}{4} g_H^2 (z_r z_r \delta_{ab} - z_a z_b)\,. \tag{2.17}$$

Adding together the contributions of the three η-like fields in eq. (2.15) and the similar contributions of the (complex) fields $\phi_r^\alpha(x)_J$, having the vacuum expectation values (2.11), we have

$$M^2_{ab} = \tfrac{1}{4} g_H^2 \Big[\sum_{J=1}^{3} (z_{Jr} z_{Jr} \delta_{ab} - z_{Ja} z_{Jb}) + (u_r u_r + v_r v_r)\delta_{ab} - u_a u_b - v_a v_b \Big]\,. \tag{2.18}$$

The last terms due to the ϕ-fields are, of course, only small corrections to the contribution of the η-fields.

The Weinberg–Salam $SU(2)_{LV} \otimes U(1)$ gauge bosons get their masses from the vacuum expectation values of the $SU(2)_{LV}$ doublet fields $\phi_r^\alpha(x)_J$ $(J=1,2,3)$; therefore, the mass of the charged W-bosons is the standard one:

$$M_{\mathrm{W}}^2=\tfrac{1}{4}g_{\mathrm{V}}^2\rho^2\,,\qquad (\rho^2\equiv\rho_r\rho_r=u_ru_r+v_rv_r)\,. \tag{2.19}$$

Here g_{V} denotes the "vertical" $\mathrm{SU(2)_{LV}}$ coupling constant. The coupling constant belonging to the weak-hypercharge group U(1) will be denoted below by g'. The photon and Z-fields are the usual linear combinations of W_3 and B with the Weinberg-angle θ_{W}:

$$\begin{aligned} A_\mu &=\sin\theta_{\mathrm{W}}W_{3\mu}+\cos\theta_{\mathrm{W}}B_\mu,\\ Z_\mu &=-\cos\theta_{\mathrm{W}}W_{3\mu}+\sin\theta_{\mathrm{W}}B_\mu\,,\end{aligned}\qquad \left(\sin\theta_{\mathrm{W}}\equiv\frac{g'}{\sqrt{g_{\mathrm{V}}^2+g'^2}}\right). \tag{2.20}$$

The photon field A_μ is, of course, massless, whereas the diagonal element in the mass-squared matrix belonging to Z is

$$M_{ZZ}^2=\tfrac{1}{4}(g_{\mathrm{V}}^2+g'^2)\rho^2\,. \tag{2.21}$$

This is, however, not the physical mass-squared of the Z-boson, because the vacuum expectation value of the ϕ-fields induces mixing terms between the horizontal gauge bosons and the Z-field in (2.20), too. Using the generators in eq. (2.16), together with the generators of $\mathrm{SU(2)_{LV}}\otimes\mathrm{U(1)}$, it can be easily shown that the H_a-Z mixing term is ($a=1,2,3$):

$$M_{a\mathrm{Z}}^2=M_{\mathrm{Z}a}^2=\tfrac{1}{2}g_{\mathrm{H}}\sqrt{g_{\mathrm{V}}^2+g'^2}\rho_b\rho_c\sin(\varphi_c-\varphi_b)\,, \tag{2.22}$$

abc being a cyclic permutation of 123.

The diagonalization of the 4×4 mass-squared matrix of the neutral (Z and H) gauge bosons is facilitated by the hierarchy of the vacuum expectation values. Because of $|z_J|\gg\rho$ we can use, e.g., ordinary time-independent perturbation theory [24] for the determination of the eigenvalues and eigenvectors, taking the contributions of the ϕ-fields (proportional to ρ) as the perturbation. The diagonalization of the "unperturbed" matrix M_{ab}^2 ($a,b=1,2,3$) is possible analytically. The general formulae are, however, rather lengthy and later on we do not need them, as we shall assume that one of the horizontal gauge bosons (namely H_1) is much lighter than the other two. This means that it is enough to consider the special case $|z_1|\gg|z_2|\gg|z_3|$. In this case one can neglect z_3 completely in the mass matrix (not, however, in minimizing the Higgs-potential: see sect. 3). Putting $z_3=0$, H_3 becomes diagonal and the small mixing angle θ_{H} among H_1 and H_2 is given by:

$$\operatorname{tg}2\theta_{\mathrm{H}}=\frac{-z_2^2\sin2\alpha}{z_1^2+z_2^2\cos2\alpha}\,. \tag{2.23}$$

The "unperturbed" (i.e. $\rho=0$) masses of the neutral gauge bosons are the following:

$$\begin{aligned} M_{\mathrm{Z}}^2 &=0\,, & M_{\mathrm{H1}}^2 &=\tfrac{1}{4}g_{\mathrm{H}}^2w_2^2\,,\\ M_{\mathrm{H2}}^2 &=\tfrac{1}{4}g_{\mathrm{H}}^2w_1^2\,, & M_{\mathrm{H3}}^2 &=\tfrac{1}{4}g_{\mathrm{H}}^2(z_1^2+z_2^2)\,,\end{aligned} \tag{2.24}$$

where we introduced the notation

$$w_1 = \tfrac{1}{2}[z_1^2 + z_2^2 + \sqrt{(z_1^2 - z_2^2)^2 + 4z_1^2 z_2^2 \cos^2\alpha}\,],$$
$$w_2 = \tfrac{1}{2}[z_1^2 + z_2^2 - \sqrt{(z_1^2 - z_2^2)^2 + 4z_1^2 z_2^2 \cos^2\alpha}\,]. \tag{2.25}$$

This shows, that $|z_1| > |z_2| > |z_3|$ implies, in general, $M_{\rm H3} > M_{\rm H2} > M_{\rm H1}$.

Taking into account the first-order perturbation due to the terms of order ρ we have:

$$M_Z^2 = \tfrac{1}{4}(g_V^2 + g'^2)\rho^2, \qquad M_{\rm H3}^2 = \tfrac{1}{4}g_{\rm H}^2(z_1^2 + z_2^2 + \rho_1^2 + \rho_2^2),$$
$$M_{\rm H1}^2 = \tfrac{1}{4}g_{\rm H}^2[w_2^2 + \rho_3^2 + \rho_2^2\cos^2\theta_{\rm H} + \rho_1^2\sin^2\theta_{\rm H} + \rho_1\rho_2\sin 2\theta_{\rm H}\cos(\varphi_1 - \varphi_2)], \tag{2.26}$$
$$M_{\rm H2}^2 = \tfrac{1}{4}g_{\rm H}^2[w_1^2 + \rho_3^2 + \rho_2^2\sin^2\theta_{\rm H} + \rho_1^2\cos^2\theta_{\rm H} - \rho_1\rho_2\sin 2\theta_{\rm H}\cos(\varphi_1 - \varphi_2)].$$

It can be seen that the Z-boson mass remains up to first order at its canonical value $M_Z = M_W/\cos\theta_W$. In the next order it is shifted slightly upwards due to the mixing with H-bosons. The situation is phenomenologically similar to the case of neutral SU(2), which was considered in a different context recently by Claudson, Georgi and Yildiz [25]. The eigenvectors corresponding to the eigenvalues in (2.26) can also be determined by perturbation theory. Transforming this information, also up to first order, into a rotation matrix with respect to the unperturbed coordinate system, one can obtain the inverse mass-squared matrix M^{-2} of the HZ system, which enters the low-energy effective neutral current interactions. The final result is given in table 1. It is remarkable, that the off-diagonal terms in the $\mathrm{H}_{1,2,3}$ subspace are a factor ρ^2/z^2 smaller than the diagonal ones. Similarly, the HZ mixing terms are smaller than $M_Z^{-2} \gg M_{\mathrm{H}a}^{-2}$ by the same factor.

3. *CP*-violating effective four-fermion interactions

As we discussed in sect. 2, from the point of view of *CP* violation it is enough to consider the simpler case with a single Yukawa-coupled field $\phi_r^\alpha(x)$. A necessary condition for spontaneous *CP* violation is that $\phi_r^\alpha(x)$ develops complex vacuum expectation values in the minimum of the Higgs potential. This condition is, however, not always sufficient. The overall phase in eq. (2.2) can be arbitrarily adjusted by the $\mathrm{SU(2)_{LV}}$ freedom left over after transforming the vacuum expectation values in the $\alpha = 2$ direction. As we shall see below, even a non-zero relative phase between the different components does not always imply *CP* violation.

A convenient complete set of independent invariants for a single ϕ and three different η-fields is the following:

$$B_J = \eta_{rJ}\eta_{rJ} - z_J^2 \equiv \eta_J \cdot \eta_J - z_J^2, \qquad (J = 1, 2, 3),$$
$$B_{3+J} = \eta_K \cdot \eta_L - z_K z_L \cos\alpha_{KL}, \qquad (K \neq L \neq J;\, J, K, L = 1, 2, 3),$$
$$B_7 = \phi_{\alpha r}^+\phi_r^\alpha - \tfrac{1}{2}\rho^2 \equiv \phi^+ \cdot \phi - \tfrac{1}{2}\rho^2,$$

TABLE 1

The inverse mass-squared matrix M^{-2} of neutral gauge bosons up to the first order in ρ^2/z^2

M_{H1}^{-2}	$(M_{H1}^{-2} - M_{H2}^{-2})[\frac{1}{2}(\rho_1^2-\rho_2^2)\sin 2\theta_H + \rho_1\rho_2 \cos 2\theta_H \cos(\varphi_1-\varphi_2)](w_1^2-w_2^2)^{-1}$	$(M_{H1}^{-2} - M_{H3}^{-2})[\rho_1\rho_3 \cos\theta_H \cos(\varphi_3-\varphi_1) - \rho_2\rho_3 \sin\theta_H \cos(\varphi_2-\varphi_3)]w_1^{-2}$	$(M_Z^{-2} - M_{H1}^{-2})[\rho_2\rho_3 \cos\theta_H \sin(\varphi_2-\varphi_3) + \rho_1\rho_3 \sin\theta_H \sin(\varphi_3-\varphi_1)]\dfrac{2\sqrt{g_V^2+g'^2}}{g_H w_2^2}$
	M_{H2}^{-2}	$(M_{H2}^{-2} - M_{H3}^{-2})[\rho_2\rho_3 \cos\theta_H \cos(\varphi_2-\varphi_3) + \rho_1\rho_3 \sin\theta_H \cos(\varphi_3-\varphi_1)]w_2^{-2}$	$(M_Z^{-2} - M_{H2}^{-2})[\rho_1\rho_3 \cos\theta_H \sin(\varphi_3-\varphi_1) + \rho_2\rho_3 \sin\theta_H \sin(\varphi_2-\varphi_3)]\dfrac{2\sqrt{g_V^2+g'^2}}{g_H w_1^2}$
		M_{H3}^{-2}	$(M_Z^{-2} - M_{H3}^{-2})\rho_1\rho_2 \sin(\varphi_1-\varphi_2)\dfrac{2\sqrt{g_V^2+g'^2}}{g_H(z_1^2+z_2^2)}$
			M_Z^{-2}

M^{-2} is symmetric; therefore, only the upper part is filled in. M_{Ha}^{-2} and M_Z^{-2} are given in eq. (2.26); for other notations see the text.

$$
\begin{aligned}
&Q_J = 2\eta_{rJ}\eta_{sJ}\phi^{+}_{\alpha r}\phi^{\alpha}_{s}\,, \qquad (J=1,2,3)\,, \\
&Q_{3+J} = (\eta_{rK}\eta_{sL} + \eta_{sK}\eta_{rL})\phi^{+}_{\alpha r}\phi^{\alpha}_{s}\,, \qquad (K \neq L \neq J;\, J, K, L = 1,2,3)\,, \\
&Q_7 = (\phi^{+}\cdot\phi)^2 - \phi^{+}_{\alpha r}\phi^{\alpha}_{s}\phi^{+}_{\beta s}\phi^{\beta}_{r}\,, \\
&Q_8 = \phi^{+}_{\alpha r}\phi^{\alpha}_{s}\phi^{+}_{\beta r}\phi^{\beta}_{s}\,.
\end{aligned}
\tag{3.1}
$$

The most general renormalizable invariant potential is therefore:

$$
V(\eta,\phi) = \sum_{j,k=1}^{7} \lambda_{jk} B_j B_k + \sum_{j=1}^{8} \kappa_j Q_j\,. \tag{3.2}
$$

Here λ_{jk} and κ_j are dimensionless coupling constants, such that the first term with λ_{jk} is a positive definite quadratic form of the B's and κ_j ($j = 1, \dots, 8$) is positive. The parameters with the dimension of a mass ($z_{1,2,3}$ and ρ) are put into the definition of the bilinear invariants B_j ($j = 1, \dots, 7$).

It is shown in the appendix that for a finite range of the parameters this potential has its absolute minimum at the point of the vacuum expectation values given by eqs. (2.15) and (2.2). The angle α in (2.15) is equal to α_{12} contained in B_6 and α' and β' can also be expressed by α_{12}, α_{13} and α_{31}. In the present paper we shall not need α' and β' explicitly; therefore, this relation is omitted. (In general, α_{KL} is the angle between the directions of the vacuum expectation values z_{Kr} and z_{Lr}.) The important point is that the phases $\varphi_{1,2,3}$ in the vacuum expectation value of ϕ are completely general and none of the $\rho_{1,2,3}$ vanishes. As we shall see below, this insures CP non-conservation in the horizontal interactions.

Our next task is to determine the low-energy effective four-fermion interactions among the physical fermions. The charged current is the standard one with a real Cabibbo–Kobayashi–Maskawa matrix; therefore, we shall concentrate on the neutral current interactions. As was shown in the previous section, the four neutral gauge bosons $H_{1,2,3}$ and Z are mixed by the spontaneous symmetry breaking. The corresponding inverse mass-squared matrix is given by table 1 in the case $M_Z^{-2} \gg M_{H1}^{-2}, M_{H2}^{-2}, M_{H3}^{-2}$ (equivalent to $\rho^2 \ll z_j^2$). The neutral currents have to be given in terms of the physical fermion fields $\chi_{L,R}(x)$ obtained from $\varphi_{L,R}(x)$ by the unitary transformations in (2.7), which diagonalize the mass matrix:

$$
\chi_{L,R}(x) \equiv U_{L,R}\psi_{L,R}(x)\,. \tag{3.3}
$$

The "family-changing" (horizontal) neutral current belonging to H_a ($a = 1, 2, 3$) is then:

$$
J_{a,q}(x)_\mu = (U_{Rq}T_aU_{Rq}^{-1})_{rs}\bar{\chi}_{Rr}(x)_q\gamma_\mu\chi_{Rs}(x)_q\,. \tag{3.4}
$$

Here T_a is the horizontal generator given by eq. (2.16) and the index $q = 1, \frac{2}{3}, \frac{1}{3}$ is also included in order to distinguish the different electric charges. The matrices $U_RT_aU_R^{-1}$ are displayed in table 2 for U_R given by eqs. (2.7)–(2.9). The neutral

TABLE 2

The matrices $\hat{T}_a = U_{\mathrm{R}} T_a U_{\mathrm{R}}^{-1}$ in the horizontal currents (3.4) for the electric charge $q = 1$ and $\frac{1}{3}$ (charged leptons and down-quarks)

$$\hat{T}_1 = \begin{pmatrix} \sin(\varphi_2-\varphi_3)\sin 2\gamma_{\mathrm{R}}\sin^2\beta_{\mathrm{R}} & -i\cos(\varphi_2-\varphi_3)\sin\beta_{\mathrm{R}} - \sin(\varphi_2-\varphi_3)\sin\beta_{\mathrm{R}}\cos 2\gamma_{\mathrm{R}} & -\frac{1}{2}\sin(\varphi_2-\varphi_3)\sin 2\beta_{\mathrm{R}}\sin 2\gamma_{\mathrm{R}} \\ i\cos(\varphi_2-\varphi_3)\sin\beta_{\mathrm{R}} - \sin(\varphi_2-\varphi_3)\sin\beta_{\mathrm{R}}\cos 2\gamma_{\mathrm{R}} & -\sin(\varphi_2-\varphi_3)\sin 2\gamma_{\mathrm{R}} & -i\cos(\varphi_2-\varphi_3)\cos\beta_{\mathrm{R}} + \sin(\varphi_2-\varphi_3)\cos\beta_{\mathrm{R}}\cos 2\gamma_{\mathrm{R}} \\ -\frac{1}{2}\sin(\varphi_2-\varphi_3)\sin 2\gamma_{\mathrm{R}}\sin 2\beta_{\mathrm{R}} & i\cos(\varphi_2-\varphi_3)\cos\beta_{\mathrm{R}} + \sin(\varphi_2-\varphi_3)\cos\beta_{\mathrm{R}}\cos 2\gamma_{\mathrm{R}} & \sin(\varphi_2-\varphi_3)\cos^2\beta_{\mathrm{R}}\sin 2\gamma_{\mathrm{R}} \end{pmatrix}$$

$$\hat{T}_2 = \begin{pmatrix} -\sin(\varphi_3-\varphi_1)\cos\gamma_{\mathrm{R}}\sin 2\beta_{\mathrm{R}} & [-i\cos(\varphi_3-\varphi_1) - \sin(\varphi_3-\varphi_1)]\sin\gamma_{\mathrm{R}}\cos\beta_{\mathrm{R}} & i\cos(\varphi_3-\varphi_1)\cos\gamma_{\mathrm{R}} + \sin(\varphi_3-\varphi_1)\cos\gamma_{\mathrm{R}}\cos 2\beta_{\mathrm{R}} \\ [i\cos(\varphi_3-\varphi_1) - \sin(\varphi_3-\varphi_1)]\sin\gamma_{\mathrm{R}}\cos\beta_{\mathrm{R}} & 0 & [i\cos(\varphi_3-\varphi_1) - \sin(\varphi_3-\varphi_1)]\sin\beta_{\mathrm{R}}\sin\gamma_{\mathrm{R}} \\ -i\cos(\varphi_3-\varphi_1)\cos\gamma_{\mathrm{R}} + \sin(\varphi_3-\varphi_1)\cos\gamma_{\mathrm{R}}\cos 2\beta_{\mathrm{R}} & [-i\cos(\varphi_3-\varphi_1) - \sin(\varphi_3-\varphi_1)]\sin\beta_{\mathrm{R}}\sin\gamma_{\mathrm{R}} & \sin(\varphi_3-\varphi_1)\cos\gamma_{\mathrm{R}}\sin 2\beta_{\mathrm{R}} \end{pmatrix}$$

$$\hat{T}_3 = \begin{pmatrix} -\sin(\varphi_1-\varphi_2)\sin\gamma_{\mathrm{R}}\sin 2\beta_{\mathrm{R}} & [-i\cos(\varphi_1-\varphi_2) + \sin(\varphi_1-\varphi_2)]\cos\gamma_{\mathrm{R}}\cos\beta_{\mathrm{R}} & -i\cos(\varphi_1-\varphi_2)\sin\gamma_{\mathrm{R}} + \sin(\varphi_1-\varphi_2)\sin\gamma_{\mathrm{R}}\cos 2\beta_{\mathrm{R}} \\ [i\cos(\varphi_1-\varphi_2) + \sin(\varphi_1-\varphi_2)]\cos\gamma_{\mathrm{R}}\cos\beta_{\mathrm{R}} & 0 & [i\cos(\varphi_1-\varphi_2) + \sin(\varphi_1-\varphi_2)]\cos\gamma_{\mathrm{R}}\sin\beta_{\mathrm{R}} \\ i\cos(\varphi_1-\varphi_2)\sin\gamma_{\mathrm{R}} + \sin(\varphi_1-\varphi_2)\sin\gamma_{\mathrm{R}}\cos 2\beta_{\mathrm{R}} & [-i\cos(\varphi_1-\varphi_2) + \sin(\varphi_1-\varphi_2)]\cos\gamma_{\mathrm{R}}\sin\beta_{\mathrm{R}} & \sin(\varphi_1-\varphi_2)\sin\gamma_{\mathrm{R}}\sin 2\beta_{\mathrm{R}} \end{pmatrix}$$

For $q = \frac{2}{3}$ (up-quarks) everything remains the same only the phases have to be reversed: $\varphi_i \to -\varphi_i$.

current belonging to Z is the standard one:

$$J_Z(x)_\mu = \sin^2\theta_W J_{em}(x)_\mu - \sum_{h=1}^{3} \bar{\chi}_L(x)_h \gamma_\mu \frac{\tau_3}{2} \chi_L(x)_h , \tag{3.5}$$

where h is the family index and J_{em} is the electromagnetic current.

The full low-energy effective neutral current interaction hamiltonian is

$$\mathcal{H}(x)_{eff} = \frac{(g_V^2 + g'^2)}{2M_Z^2} J_Z(x)_\mu J_Z(x)^\mu + \tfrac{1}{2} g_H \sqrt{g_V^2 + g'^2} J_Z(x)_\mu \sum_{a=1}^{3} M_{Za}^{-2} \sum_q J_{a,q}(x)^\mu$$

$$+ \tfrac{1}{8} g_H^2 \sum_{a,b=1}^{3} M_{ab}^{-2} \sum_{qq'} J_{a,q}(x)_\mu J_{b,q'}(x)^\mu . \tag{3.6}$$

The first term is the standard Weinberg–Salam neutral current interaction. The third term is the "family-changing" (flavour-changing) neutral current interaction between two horizontal currents, and the second one is a mixed interaction involving the Z neutral current and the horizontal currents. The inverse gauge boson masses are given by table 1. Because of $M_Z^{-2} \gg M_{ab}^{-2}, M_{Za}^{-2}$ the family-changing pieces are in general small. As shown by table 1, the off-diagonal ($a \neq b$) pieces in the last term of (3.6) have an extra suppression factor compared to the diagonal ($a = b$) terms, which are similar in order of magnitude to the mixed ZH terms.

The CP violation in $\mathcal{H}_{eff}$ is caused by the matrices $U_R T_a U_R^{-1}$ (see table 2) appearing in the horizontal currents. It can be seen from eq. (2.10), that for general non-zero $\rho_{1,2,3}$ the mixing angles β_R and γ_R do not take on any special value; therefore, in general neither $\sin\beta_R$, $\sin\gamma_R$ nor $\cos\beta_R$, $\cos\gamma_R$ vanish. Similarly, as is shown in the appendix, the phase differences $(\varphi_i - \varphi_k)$ are also arbitrary; hence, the off-diagonal elements in $U_R T_a U_R^{-1}$ are general complex numbers, which appear in $\mathcal{H}_{eff}$. This violates the CP symmetry (and time-reversal symmetry), which would require the coefficients of the current–current terms in $\mathcal{H}_{eff}$ to be real.

4. Discussion and concluding remarks

In the previous sections we have seen, that in the $SU(2)_{LV} \otimes U(1) \otimes O(3)_{RH}$ model the spontaneous breaking of CP invariance is manifested at the tree level only in the right-handed horizontal interactions suppressed by the large masses of the horizontal gauge bosons. This is the consequence of the peculiar chiral structure of interactions (only left-handed "vertically" and only right-handed "horizontally") and of the form of vacuum expectation values in eq. (2.2) or (2.11). This form is achieved in the simpler case (2.2), considered in detail in sect. 3, by taking a single ϕ-field. In this case the masses of the first two families vanish at the tree level. These masses have to be given by the loop corrections. An interesting possibility for this is to embed the $O(3)_{RH}$ horizontal symmetry in a left-right symmetric $O(3)_{LH} \otimes O(3)_{RH}$, similarly to a recent paper on radiative quark mass generation

[26]. The radiative corrections generally do not preserve the specific form of the fermion mass matrix m given by (2.3); therefore, the Kobayashi–Maskawa phase δ in the left-handed mixing matrix (2.14) is expected to get a small, non-zero value at the one-loop level. This implies a small CP violation also in vertical interactions, where the smallness is explained by the fact that $\delta \neq 0$ comes from radiative corrections. The presence of CP violations in both horizontal and vertical interactions, of course, complicates the phenomenology. In the discussion below we shall assume, for simplicity, that the horizontal CP violation dominates. (If the vertical CP violation dominates, the phenomenology is of the conventional Kobayashi–Maskawa type [20, 27].)

The CP non-conserving, low-energy effective four-fermion interaction containing the horizontal neutral currents was determined in sect. 3. The dominant piece of the dd→ss transition relevant for the neutral kaon system is, from eqs. (3.6), (3.4) and table 2, the following:

$$\begin{aligned}\mathcal{H}_{\mathrm{dd}\to\mathrm{ss}}(x) \simeq \tfrac{1}{8} g_{\mathrm{H}}^2 \bar{s}_{\mathrm{R}}(x)\gamma_\mu d_{\mathrm{R}}(x)\bar{s}_{\mathrm{R}}(x)\gamma^\mu d_{\mathrm{R}}(x) \\ \times\{M_{\mathrm{H1}}^{-2} \sin^2\beta_{\mathrm{R}}[\sin^2(\varphi_2-\varphi_3)\cos^2 2\gamma_{\mathrm{R}} - \cos^2(\varphi_2-\varphi_3) \\ - i\sin 2(\varphi_2-\varphi_3)\cos 2\gamma_{\mathrm{R}}] \\ + M_{\mathrm{H2}}^{-2}\sin^2\gamma_{\mathrm{R}}\cos^2\beta_{\mathrm{R}}[-\cos 2(\varphi_3-\varphi_1) - i\sin 2(\varphi_3-\varphi_1)] \\ + M_{\mathrm{H3}}^{-2}\cos^2\gamma_{\mathrm{R}}\cos^2\beta_{\mathrm{R}}[-\cos 2(\varphi_1-\varphi_2) + i\sin 2(\varphi_1-\varphi_2)]\}\,. \end{aligned} \tag{4.1}$$

According to eq. (2.26) the horizontal gauge boson masses satisfy $M_{\mathrm{H1}} < M_{\mathrm{H2}} < M_{\mathrm{H3}}$. This favours the first term, but the mixing angles β_{R} and γ_{R} can be small, typically one can take, e.g., $\beta_{\mathrm{R}} = 10^{-2}$ and $\gamma_{\mathrm{R}} = 10^{-1}$. [This is satisfied if in eq. (2.10) $\rho_1/\rho = 10^{-2}$ and $\rho_2/\rho = 10^{-1}$. Note that β_{R} is the mixing angle between the first and third families, whereas γ_{R} is the one between the second and third families; therefore, it is plausible to take β_{R} smaller.] The effect of these mixing angles in (4.1) is that if all the three terms in the curly brackets are roughly the same, then $M_{\mathrm{H1}} \cong 10^{-1} M_{\mathrm{H2}} \cong 10^{-2} M_{\mathrm{H3}}$. This leaves open the interesting possibility, that the gauge boson H_1 is considerably lighter than H_3. Consequently, the horizontal transitions between the second and third family are larger than those between the first and second one [11]. Replacing $\cos\beta_{\mathrm{R}}$ and $\cos\gamma_{\mathrm{R}}$ by 1, and considering the three contributions in (4.1) separately, the measured value of the imaginary part of the $\mathrm{K}_{\mathrm{L}}^0, \mathrm{K}_{\mathrm{S}}^0$ mass difference (or the conventional ε-parameter of CP violation) gives, for $\sin 2\Delta\varphi$ of order 1,

$$\begin{aligned} \sin^2\beta_{\mathrm{R}} M_{\mathrm{W}}^2/M_{\mathrm{H1}}^2 &\leq 10^{-9}\,, \\ \sin^2\gamma_{\mathrm{R}} M_{\mathrm{W}}^2/M_{\mathrm{H2}}^2 &\leq 10^{-9}\,, \\ M_{\mathrm{W}}^2/M_{\mathrm{H3}}^2 &\leq 10^{-9}\,. \end{aligned} \tag{4.2}$$

Here one of the inequalities has to be an equality. For the above values of the

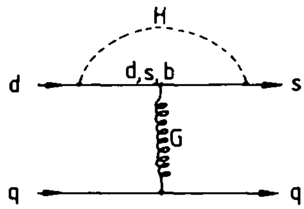

Fig. 1. The "penguin" graph induced by the horizontal gauge boson H (G is a gluon).

mixing angles ($\beta_R = 10^{-2}$, $\gamma_R = 10^{-1}$) this means $M_{H1} \geqslant 30$ TeV, $M_{H2} \geqslant 300$ TeV, $M_{H3} \geqslant 3000$ TeV. Therefore, as already noted in another horizontal *CP*-violation model by Davidson and Wali [14], the mass of the horizontal gauge boson H_3 comes out from the *CP* violation about a factor 10^2 larger than the lower limits obtained from the absence of the family-changing decays in the first two families [10].

The prediction for the *CP*-violating ε' in our model is smaller than the one in the Kobayashi–Maskawa model [28]. ε' is proportional to the imaginary part of the so-called "penguin graphs" [29]. In the standard Kobayashi–Maskawa scheme the penguin graphs induced by the W-boson contribute to ε' and give $\varepsilon'/\varepsilon \simeq 1\%$ [28]. In our model these graphs are purely real since the Kobayashi–Maskawa phase is zero. However, the horizontal bosons induce extra "penguin graphs" (fig. 1), which are non-real and hence contribute to ε'. The graphs are finite because the analogue of GIM mechanism is operative. (This is a consequence of the relation $\sum_a \hat{T}_a \hat{T}_a = 2$ satisfied by the matrices $\hat{T}_a$ of table 2.) The detailed prediction depends upon what we assume for various mixing angles. Roughly, we find the ratio

$$\left| \frac{\varepsilon'_H}{\varepsilon'_{KM}} \right| \cong 10^{-4} - 10^{-6} \tag{4.3}$$

between ε' in our scheme (ε'_H) and in the Kobayashi–Maskawa scheme (ε'_{KM}). Within the present experimental accuracy this prediction is indistinguishable from the prediction of the superweak model [13].

The electric dipole moment of the neutron is in our model smaller than the "usual" superweak value $D_n = 10^{-29} e \cdot$ cm [30]. This is due to the extra suppression factor in the imaginary part of the graph in fig. 2. Namely, the diagonal part of the H-boson inverse mass-squared matrix gives a pure real contribution (see table 2). As already noted previously, the off-diagonal terms of M^{-2} in table 1 have additional suppression factors like $\rho_i \rho_k / z_l^2$.

Another phenomenological consequence of the $SU(2)_{LV} \otimes U(1) \otimes O(3)_{RH}$ model is the small mixing of the Z-boson with the three horizontal gauge bosons. According to eq. (3.6) and table 1 the corresponding decays of the Z are suppressed, compared to the normal neutral current decays, by factors like $\rho_i \rho_k / z_l^2$. Again, the H_3 channels are suppressed by the smallest factor: $\rho_1 \rho_2 / z_1^2$ and the H_1 channels by the least

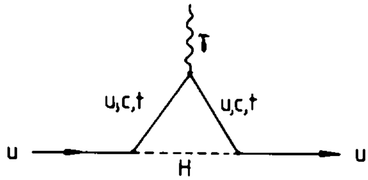

Fig. 2. The graph with a horizontal gauge boson H contributing to the electric dipole moment of the u-quark. (The wavy line γ is a photon.)

small one: $\rho_2\rho_3/w_2^2$. Hence there are small *CP* non-conserving, flavour-changing decays of the Z to the first two families at the level of 10^{-12} and to the second and third families at the level of 10^{-6} (in the amplitude, for the above values of β_R and γ_R). These are very small, but perhaps the mixing angles are different. For $\sin\gamma_R = O(1)$ and $\beta_R = 10^{-3}$ the suppression factor $\rho_2\rho_3/w_2^2$ is only 10^{-3} and then the observation of the *CP*-violating, flavour-changing decay $Z \to \mu\tau$ is not hopeless.

As mentioned above, the possibility of a relatively light horizontal gauge boson H_1 acting on the second and third families is not ruled out by the present data. $M_{H1} \simeq 3$ TeV, for instance, would require in eq. (4.2) $\beta_R \simeq 10^{-3}$. [An even smaller value of M_{H1} is allowed if in eqs. (4.1), (4.2) $\sin 2(\varphi_2 - \varphi_3) < 1$ is taken into account.] Perhaps the best place to search for such effects is offered by possible rare τ-decays [11], but rare decays of the b-quark may also give interesting information. The *CP*-violating character of the horizontal interactions implies that these rare decay channels would show large *CP*-violating effects.

In summary: the predictions of the $SU(2)_{LV} \otimes U(1) \otimes O(3)_{RH}$ model for the conventional *CP*-violating parameters are typically of "superweak" magnitude (or even smaller). This is due to the large mass ($\simeq 10^6$ GeV) of the horizontal gauge boson acting on the first two families. It is, however, possible that the horizontal gauge boson acting on the heavy families is much lighter ($\simeq 10^{3}$ GeV). Therefore, the *CP* violation in the third family can be of "milliweak" strength.

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Appendix

In this appendix we show that for an appropriate range of parameter values the Higgs potential (3.2) has its absolute minimum in the point of the vacuum expectation values given by eq. (2.2) and (2.15). Let us first ignore the "mixed" quadrilinear couplings $Q_1, \dots, Q_6$ connecting the ϕ-field with the η-fields. We first determine the minimum in this case and then "switch on" the couplings $Q_1, \dots, Q_6$ keeping the corresponding parameters $\kappa_1, \dots, \kappa_6$ small compared to the others. We shall see that the absolute minimum obtained by this "perturbative" procedure fulfills all our requirements.

The parameters λ_{jk} determine, by assumption, a positive definite quadratic form which has its (zero) absolute minimum if all B_j's ($j = 1, \ldots, 7$) are zero. This can be fulfilled only if $\eta_r(x)_J$ ($J = 1, 2, 3$) is equal to the three-vector with r-component z_{Jr}, having length z_J and relative angles α_{JK} to the other z_{Kr}'s. By an appropriate $O(3)_{RH}$ transformation z_{Jr} can be brought to the form given in (2.15) where $\alpha = \alpha_{12}$ and α', β' can also be expressed by α_{12}, α_{23} and α_{31}. The requirement $B_7 = 0$ implies that the ϕ-field has a length $\sqrt{\frac{1}{2}}\rho$.

Using the Cauchy–Schwarz inequality it can be easily shown that the invariants Q_7 and Q_8 are non-negative. By explicit calculation or, more elegantly, with the help of the hermitian matrix $V_{rs} = \phi^+_{\alpha r}\phi^\alpha_s$, one can show that

$$Q_7 = 2\{\phi^+_{1r}\phi^1_r\phi^+_{2s}\phi^2_s - \phi^+_{2r}\phi^1_r\phi^+_{1s}\phi^2_s\} \geqslant 0 \,. \tag{A.1}$$

This is zero if and only if $\phi^1_r = \lambda\phi^2_r$ (λ independent of r). By an appropriate $SU(2)_{LV}$ transformation it is always possible to achieve, e.g., $\phi^{\alpha=1}_{r=1} = 0$, implying $\lambda = 0$ and hence $\phi^1_r = 0$ ($r = 1, 2, 3$). This insures that the photon remains massless (the electric charge will be conserved) and the minimum in ϕ has the form required by eq. (2.2). In order to fulfill also the previous condition we have to put

$$\rho^2 = \rho_r\rho_r = u_r u_r + v_r v_r \,. \tag{A.2}$$

The invariant Q_8 has in this minimum the form

$$Q_8 = \tfrac{1}{4}(u_r u_r - v_r v_r)^2 + (u_r v_r)^2 \geqslant 0 \,. \tag{A.3}$$

This shows that the absolute minimum of the potential V without the mixed quadrilinear terms is equal to zero and it is in the set of points satisfying

$$u_r u_r = v_r v_r = \tfrac{1}{2}\rho^2 \,, \qquad u_r v_r = 0 \,. \tag{A.4}$$

As far as the ϕ-dependent part is concerned this is consistent with a result in ref. [31].

The relative orientation of the equal length orthogonal vector pair u, v to the three vectors $z_{1,2,3}$ is up to now undetermined. There is also the freedom of choosing the overall phase left over from the $SU(2)_{LV}$ transformation (resulting in $\phi^1_r = 0$). A common phase change $\varphi_r \to \varphi_r + \varphi$ in the three components of ϕ^2_r is equivalent to the transformation

$$u'_r = u_r \cos\varphi - v_r \sin\varphi \,, \qquad v'_r = u_r \sin\varphi + v_r \cos\varphi \,. \tag{A.5}$$

This leaves the plane of u, v always invariant, and it is a rigid rotation of the u, v pair if eq. (A.4) holds. In other words, the whole potential V can only depend on the normal vector n of the u, v plane because it is independent of φ.

The effect of the mixed terms $Q_1, \ldots, Q_6$ is to fix the relative orientation of the normal vector n with respect to the three vectors $z_{1,2,3}$. Explicitly, the "mixed" part of the V in the minimum obtained above is

$$V_{\text{mix}} = \sum_{j=1}^{3} \kappa_j[(u_r z_{jr})^2 + (v_r z_{jr})^2]$$

$$+ \sum_{\substack{j=1 \\ k \neq l \neq j}}^{3} \kappa_{3+j}[u_r z_{kr} u_s z_{ls} + v_r z_{kr} v_s z_{ls}] . \tag{A.6}$$

If the parameters $\kappa_1, \ldots, \kappa_6$ are small compared to λ_{jk} and κ_7, κ_8, then in first approximation the vectors $z_{1,2,3}$ remain unchanged and u, v still satisfy eq. (A.4). After some trigonometric work one can show that in this case

$$V_{\text{mix}} = \sum_{j=1}^{3} \Gamma_j \sin^2 \omega_j + \sum_{kl=12,23,31} \Gamma_{kl} \sin \omega_k \sin \omega_l \cos (\delta_k - \delta_l) , \tag{A.7}$$

where

$$\begin{aligned} \Gamma_j &= \tfrac{1}{2}\rho^2 \kappa_j z_j^2 , \qquad (j = 1, 2, 3) , \\ \Gamma_{kl} &= \kappa_{3+j} z_k z_l \tfrac{1}{2}\rho^2 , \quad (k \neq l \neq j) . \end{aligned} \tag{A.8}$$

ω_j is the angle between n and z_j and $\delta_k - \delta_l$ is the angle between the projections of the vectors z_k and z_l onto the u, v plane with normal vector n. It follows from a spherical triangle that we also have

$$\cos (\delta_k - \delta_l) = \frac{\cos \alpha_{kl} - \cos \omega_1 \cos \omega_2}{\sin \omega_1 \sin \omega_2} , \tag{A.9}$$

where α_{kl} is the angle between z_k and z_l.

Combining (A.7)–(A.9), one can see that the minimum of V_{mix} as a function of the orientation of n relative to the vectors z_j $(j = 1, 2, 3)$ is at general values of the angles $\omega_{1,2,3}$. For instance, taking $\cos \alpha_{kl} = 0$ (z_k orthogonal to z_l) for all k, l and $\Gamma_{12} = \Gamma_{23} = \Gamma_{31} > \Gamma_{1,2,3}$, the minimum is somewhere in the middle, between the three orthogonal directions of z_j's. This means that the phases φ_j given by

$$\text{tg}\, \varphi_j = v_j / u_j \tag{A.10}$$

are general, resulting in *CP* violation. This conclusion is not changed if the small changes in the vectors z_j and u, v due to the small mixed couplings are also taken into account. One can, for instance, show that in the plane perpendicular to n the vectors u and v will not remain exactly orthogonal. The deviation of their angle from $\frac{1}{2}\pi$ will be of the order $\kappa_j/\kappa_8 \ll 1$ $(j = 1, \ldots, 6)$. The position of the absolute minimum of V is an analytic function of $\kappa_1, \ldots, \kappa_6$ in some neighbourhood of the point $\kappa_1 = \cdots = \kappa_6 = 0$. This insures that within this neighbourhood the minimum constructed above remains the absolute minimum. In the whole parameter space there can be, of course, also other regions with different absolute minima. There might be *CP*-conserving minima and also other minima with *CP* violation.

From the above construction it is also clear, that with two η-fields it is impossible to obtain, at least by this perturbative method, a *CP*-violating minimum. Namely, in the form (A.7) of V_{mix} there are then only the terms with Γ_1, Γ_2 and Γ_{12}. In this case one can show that in the minimum the n-vector is always in the z_1, z_2 plane. By the use of the phase freedom (A.5) one can always rotate u or v in the z_1, z_2 plane. Taking, for instance, v we have $v_3 = 0$ and $u_1 = u_2 = 0$. Therefore, the phase differences $\Delta\varphi_{jk} = \varphi_j - \varphi_k$ given by eq. (A.10) are either 0 or $\frac{1}{2}\pi$. Due to the specific structure of the matrices in table 2 the *CP* violation in the horizontal neutral current interaction is proportional to $\sin 2\Delta\varphi_{jk}$; therefore, it vanishes for $\Delta\varphi_{jk} = 0$ and $\Delta\varphi_{jk} = \frac{1}{2}\pi$.

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