THE QCD EFFECTIVE COUPLING CONSTANT IN e^+e^- ANNIHILATION

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The QCD effective coupling constant $\alpha_s(Q^2)$ is determined by comparing the $O(\alpha_s)^2$ jet-distributions with the high-energy e^+e^- data from PETRA. We get $\alpha_s(Q^2 = 1225\ \text{GeV}^2) = 0.125 \pm 0.01$, which corresponds to $\Lambda_{\overline{MS}} = 110^{+70}_{-50}$ MeV with five flavours.

1. The observation of multijet events at PETRA [1] can be interpreted as an evidence in support of quantum chromodynamics, QCD. Detailed comparison of QCD-based models incorporating the $O(\alpha_s)$ and the $O(\alpha_s)^2$ Born diagrams show remarkable consistency with the data, *if* the intrinsic transverse momentum of the hadrons $\langle p_T \rangle \simeq 400$ MeV is taken into account [1,2].

2. A theoretically meaningful determination of the effective QCD coupling constant, $\alpha_s(Q^2)$, is possible only if the complete $O(\alpha_s)^2$ calculations have been performed for the quantity being analysed. The calculations for the total hadronic cross section σ_{tot} were performed by several groups [3] which agree with each other. The result is

$$\sigma_{tot} = \sigma_0 \{1 + \alpha_s(s)/\pi + (1.98 - 0.116 n_f)[\alpha_s(s)/\pi]^2\}, \qquad (1)$$

where the coefficient is specific to a certain regularization prescription, the so-called $\overline{\text{MS}}$-scheme [4]. The coupling constant $\alpha_s(Q^2)$ in (1) is defined as

$$\alpha_s(Q^2) = 2\pi/[b_0 \ln(Q^2/\Lambda^2) + (b_1/b_0) \ln\ln(Q^2/\Lambda^2)], \qquad (2)$$

with

$$b_0 = (33 - 2n_f)/6, \quad b_1 = (153 - 19 n_f)/6, \qquad (3)$$

The coefficient of $(\alpha_s/\pi)^2$ for $n_f = 5$ is 1.4, which is small and hence inspires confidence in using total cross-section measurements to determine $\alpha_s(Q^2)$. However, the demands that such a determination impose on the experimental accuracy are formidable, if not impossible. The variation of σ_{tot} with $\alpha_s(Q^2)$ at $\sqrt{s} = 35$ GeV can be seen in table 1. At low energies the analysis gets involved due to threshold effects and higher twist contributions.

3. The multijet event rates and distributions are ideally suited for the determination of $\alpha_s(Q^2)$. Since $Q^2 \simeq 1300\ \text{GeV}^2$ at the present PETRA/PEP energies, it is possible to analyse a process in a kinematic region where evergy sub-energy and invariant mass is large.

Table 1
Total and multijet cross sections at $\sqrt{s} = 35$ GeV. The three jet cross sections are defined with a thrust cut-off $T_0 < 0.95$ and the four-jets with $T_0 < 0.95$ and the invariant mass M_{ij} cut-off $M_{ij} > 5$ GeV. $n_f = 5$. Typical errors on σ_3 and σ_4 are ±3%.

Λ (GeV) $\overline{\text{MS}}$	$\alpha_s(Q^2)$	$\sigma_{tot}/\sigma_{\mu\mu}$	σ_2/σ_{tot}	σ_3/σ_{tot}	σ_4/σ_{tot}
0.03	0.1033	3.793	0.728	0.243	0.029
0.05	0.1108	3.802	0.702	0.265	0.033
0.075	0.1176	3.809	0.678	0.284	0.038
0.10	0.1229	3.815	0.658	0.301	0.041
0.15	0.1314	3.826	0.627	0.326	0.047
0.20	0.1381	3.835	0.60	0.348	0.052
0.25	0.144	3.842	0.578	0.366	0.056

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Consequently, one could apply perturbation theory with confidence. In perturbative QCD such multijet distributions receive contribution in order α_s or higher. This is what makes them preferred quantities over the inclusive measurement of σ_{tot}. The $O(\alpha_s)^2$ calculations for the multijet rates and distributions have been performed by three independent groups. The calculations of Ellis, Ross and Terrano (ERT) [5] and those of Vermaseren, Gaemers and Oldham (VGO) [6] are numerically in agreement. Calculating $O(\alpha_s)^2$ calculations to the inclusive thrust distribution for $T < T_0$, they obtain

$$f(T_0) \equiv \frac{1}{\sigma_0} \int_{0.5}^{T_0} \frac{d\sigma}{dT} = B_0 [\alpha_s(s)/\pi] [1 + K\alpha_s(s)/\pi] \,, \quad (4)$$

where the coefficient $B_0(T_0 = 0.85) = 1.156$ [7] and

$$K(T_0 = 0.85) = 17.6 \pm 0.3 \quad \text{(ERT)} \,,$$
$$= 17.2 \pm 0.2 \quad \text{(VGO)} \,.$$

the numbers again correspond to the $\overline{\mathrm{MS}}$ scheme. The corrections are not small and disturbing for some tastes, though one should bear in mind that a similar phenomenon is electromagnetic interactions, i.e. the dominance of the $O(\alpha)^2$ effects over the $O(\alpha)$ effects in special kinematic domains, is every day occurrence in e^+e^- annihilation and indeed constitutes a test of the underlying theory of electrons and photons, QED!

Fabricius, Schmidt, Schierholz and Kramer (FSSK) [8] have calculated the analogue of the Sterman-Weinberg formula for three-jets. They find that the "thrust" distribution calculated for three-jet events in well-defined cones in the Sterman–Weinberg variables ϵ and δ [9], $(d/dT')(\epsilon, \delta)$, has a smaller correction compared to the $O(\alpha_s)$ thrust distribution $d\sigma/dT$. In particular the quantity

$$f(\epsilon, \delta) = \int_{0.5}^{T_0} \frac{d\sigma(\epsilon, \delta)}{dT'} \, dT' \quad (5)$$

is stable against $O(\alpha_s)^2$ corrections in their calculations. Note that $f(T_0)$ and $f(\epsilon, \delta)$ are *different* quantities and it is a fallacy to compare them directly and draw conclusions about the accuracy of one against the other, a point missed unfortunately by many.

4. It is worthwhile to discuss the separate pieces which comprise the $O(\alpha_s)^2$ calculation of multijet distributions. These can be classified as follows:

(i) Contribution of $e^+e^- \to q\bar{q}G$ to $O(\alpha_s)$ (Born diagrams) [10].

(ii) Contribution of $e^+e^- \to q\bar{q}GG$, $q\bar{q}q\bar{q}$ to $O(\alpha_s)^2$ (Born diagrams) [11].

(iii) One-loop $O(\alpha_s)^2$ virtual corrections to $e^+e^- \to q\bar{q}G$.

(iv) The soft-part of the process $e^+e^- \to q\bar{q}GG$, $q\bar{q}q\bar{q}$ contributing to the three-jet configuration *alone*.

The processes listed in (i) and (ii) are not new ingredents of the calculations of ERT, VGO and FSSK. They have been previously calculated and verified and hence not controversial. There is agreement on point (iii) among ERT and FSSK, as well as on the cancellation of singular pole terms between (iii) and (iv) [12]. It is then clear that the difference has to be traced to the *finite parts* of the soft process $e^+e^- \to q\bar{q}GG$, $q\bar{q}q\bar{q}$, namely point (iv).

The perturbation theory calculation of the contribution (iv) is unambiguous. The precise form, however, depends on how in practice one defines the variables and cancels divergences between (iii) and (iv). The calculations of ERT, VGO and FSSK differ in these details and hence the various auxiliary quantities pertaining to points (iii) and (iv) cannot be compared directly.

Our aim is to obtain finite density matrix for the three- and four-parton final states in e^+e^- annihilation calculated to $O(\alpha_s)^2$ in QCD. This can be fed into a Monte Carlo integration program generating the distributions of interest relevant for a comparison of perturbative QED with the data. To cancel divergences, we follow the ansatz of ERT [7] and work with invariant masses, as was also done by Kunzst [7]. To recapitulate, the infrared and collinear singularities occur when an invariant mass approaches zero. To be definite let us consider the singularity in y_{13} $(= s_{13}/Q^2)$. One could split the y_{13} integration as follows

$$\int dy_{13} \to \theta(1 - y_{123} - y_{134}) \int_0^{y_0} dy_{13}$$
$$+ \theta(y_{123} + y_{134} - 1) \int_{y_{123}+y_{134}-1}^{y_0} dy_{13} \,, \quad (6)$$

where

$y_{ijk} = s_{ijk}/Q^2$, $s_{ijk} = (p_i + p_j + p_k)^2$,

and $y_0 = y_{123}y_{134}$. One could change the limit of integration y_0 such that $y_0 = \min(y_{123}y_{134}, m_j^2/Q^2)$ where m_j is an invariant mass. The parameter y_0 then serves both as a pure technical device to construct and calculate finite quantities in perturbation theory as well as a convenient way to introduce a jet resolution. That an analogous procedure is used in the studies of bremsstrahlung phenomena in QED is well known.

We have used a method similar to that of Kunszt's in implementing the hadronization of the partons generated in perturbation theory. The resolution dependence in perturbative QCD is a rather delicate matter since it modifies the bare parton calculations mathematically. In particular, one has to define an *equivalent* three-parton system of the original four-parton final state. This necessarily involves additional assumptions about combining the soft quantum. Unfortunately, there is no direct correspondence between distributions in perturbation theory and the experiments because of the confinement aspect of QCD as opposed to QED, where the experimental resolution can be imposed directly on perturbative calculations. Thus, it is not a priori obvious which of the various competing and differing algorithms has the best chance of reproducing the underlying physical configuration faithfully.

Our prescription of introducing a resolution dependence is as follows. In a Monte Carlo calculation which we are using, it is necessary to keep track of which of the invariant masses is falling below the y_0 (or m_j) cut on an event by event basis. We add the Lorentz four-vector of the particular pair of partons whose invariant mass y_{ij} is less than y_0, e.g. y_{13} in eq. (6). This defines for us an equivalent three-parton state for $y_{13} < y_0$ in which one of the partons is necessarily *massive*. We calculate the various distributions for this class of events using the kinematics of the *equivalent* three-parton state. The definition of the shape variables and the associated distributions differ mathematically from their uncombined soft four-parton values. However, we find that for reasonable values of y_0 (or m_j), the equivalent three-parton distributions are very stable with respect to their bare three-parton values. A detailed quantitative study of this point will be presented elsewhere.

We have calculated together with many other quantities, the distributions in the variables C and D introduced by ERT and the thrust distribution calculated by Kunszt [‡1], VGO and by Ellis and Ross. Though our "thrust" variable [‡2] differs from the bare thrust definition for the class of four-parton events with $y_{ij} < y_0$, our results are in approximate agreement with these authors *both* in normalization and shape. Our results for the K-factor corresponding to various thrust cuts are given in table 2. The K-factor depends on T_0 and in particular we find

$$k(T_0 = 0.85) = 16.5 \pm 0.8$$

for the massless quark case with $n_f = 5$. We have also studied the quark mass effects in the Born terms and find them to be small ($\sim$2%) at $\sqrt{s} = 35$ GeV. Thus, the entire $O(\alpha_s)^2$ corrections to $\sigma(T_0)$ vary between 57% (for $T_0 = 0.95$) and 75% (for $T_0 = 0.75$) at PETRA energies for $\Lambda_{\overline{MS}} = 100-150$ MeV .

The K-factor is a good measure of the $O(\alpha_s)^2$ corrections if detailed information about the final hadronic states is not completely available, which for example is the case in the Drell–Yan process. In e^+e^- annihilation, however, the final states can be classified as multijet states with well-defined probabilities for each multijet configuration. Thus, at PETRA energies, there is good evidence for two-, three- and four-jets with the rates in approximate agreement with the perturbative QCD estimates presented in table 1. It is prudent to analyse the final states in e^+e^- annihilation in terms of multijet configurations with *definite* jet multiplicity, and estimate the $O(\alpha_s)^2$ corrections to each

‡1 Our results are not in numerical agreement with Kunszt's results in ref. [7] but are in agreement with the ones in ref. [13].

‡2 We use maximum directed energy to define the thrust for this class of events. The result using directed momenta are very similar.

Table 2
The K-factor defined in eq. (4) for the various thrust cut-off.

T_0	$K(T_0)$
0.95	14.4 ± 0.7
0.90	15.5 ± 0.8
0.85	16.5 ± 1.0
0.80	18.9 ± 1.0
0.75	19.7 ± 1.0

Table 3
$O(\alpha_s)^2$ corrections to the three-jet cross section at $\sqrt{s} = 35$ GeV. Also shown are the hard and soft part of the four-jet cross section. Three and four jets are defined as in table 1.

α_s	$\sigma^{\text{finite}}_{\text{4-parton}} / \sigma_{3\,\text{Born}}$	$\sigma_3 / \sigma_{3\,\text{Born}}$	σ_4 / σ_3
0.103	0.25	1.32	0.12
0.1176	0.284	1.353	0.13
0.1314	0.32	1.40	0.14
0.144	0.35	1.43	0.15

jet multiplicity configuration. This of course needs a definition of a jet and it is here that experimental information could be used to define meaningful multijet configurations which could be compared with data – some day directly.

Since the two-jets first appeared at SPEAR at $\sqrt{s} \simeq 5$–6 GeV and the average invariant mass of a jet at PETRA energies is ~5 GeV, it is not a bad guess to use $M_J = 5$ GeV to define a jet and estimate the jet-multiplicity and the relative probability. With this definition we find that the correction to the genuine three-jet cross section at $\sqrt{s} = 35$ GeV is ~35% – not an alarmingly large number, at the same time $\sigma_{4\,\text{jet}}/\sigma_{\text{tot}} \simeq (3$–$5\%)$ (see table 3). The frequency of four-jets is in agreement with experimental data. So, whereas we agree with the algebraic calculations of ERT and VGO, we do not share their skepticism about the convergence of perturbative theory in jet distributions.

5. Having convinced ourselves of the validity of the parton-level calculations for $f(T_0)_{\text{ERT}}$, the next step is to make a comparison of the $O(\alpha_s)^2$ distributions with the data and determine the value of $\alpha_s(Q^2)$. However, there is a non-trivial step of converting quarks and gluons into hadrons. This was attempted phenomenologically by using an extended Field–Feynman model [14] described in ref. [15]; the model has subsequently been studied by the experimental collaborations at PETRA and PEP and found to be in rather good agreement with their data. The single most important parameter which determines the dominant non-perturbative effects is the intrinsic-p_T of the hadrons. This is assumed to have the form $\exp(-k_T^2/2\sigma_q^2)$ in the Field–Feynman model. Detailed studies of the entire PETRA energy data gives [16] [‡3]

$$\sigma_q = 0.32 \pm 0.04 \text{ GeV} \tag{7}$$

for the intrinsic transverse momentum of the quarks giving $\langle p_T \rangle_{\text{hadron}} \simeq 400$ MeV. We shall use this value to determine the background two-jet events which have a tail in the thrust (or any other related) distribution. Our calculation show that at $\sqrt{s} = 35$ GeV and σ_q in the range of eq. (7) two-jet events *do not* contribute below a thrust $T_0 = 0.82$. This is shown for the value $\sigma_q = 0.32$ GeV in fig. 1 (dashed curve). Thus, the tail of the distribution in $\sigma^{-1}\,d\sigma/dt$ receives contribution only through $O(\alpha_s)$ and higher perturbative QCD diagrams. The fraction of events below T_0, $f(T_0)$, can then be used to determine $\alpha_s(Q^2)$. Using the best value $\sigma_q = 0.32$ GeV, we determine $f(T_0)$. The non-perturbative-p_T *convoluted* $O(\alpha_s)^2$ expression can again be used to express the result as an interpolating function

$$f(T_0) = K_1\,[\alpha_s(s)/\pi]\,[1 + K_2\alpha_s(s)/\pi]\ , \tag{8}$$

‡3 Similar values of σ_q are also obtained by the CELLO, JADE and the PLUTO Collaboration.

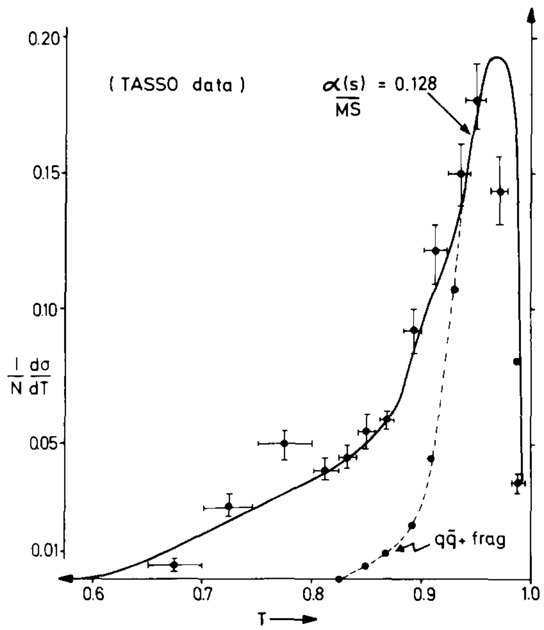

Fig. 1.

and we determine the K-factors to be

$$K_1(T_0 = 0.82) = 1.83 \pm 0.1 \,,$$

$$K_2(T_0 = 0.82) = 16.1 \pm 1.0 \,, \qquad (9)$$

where the errors reflect both variation in σ_q and our Monte Carlo errors. It is remarkable that the factor K_2 has, within errors, the *same* value as in pure parton level $O(\alpha_s)^2$ QCD calculations. We compare eq. (9) using the parameters (10) with the TASSO data, shown in fig. 1. The data has been corrected for acceptance and radiative corrections. Using the TASSO data

$$f(T_0 = 0.82) = 0.124 \pm 0.013 \,,$$

our best determination of $\alpha_s(Q^2)$ (see fig. 2) is:

$$\alpha_s(Q^2) = 0.128 \pm 0.013 \,. \qquad (10)$$

The shape of the distribution $\sigma^{-1}\, d\sigma/dT$ is also in good agreement with the TASSO data, as shown in fig. 1 (solid curve).

We have also analysed the Mark-J data, where we chose to compare the theoretical calculations with the fraction of events chosen by putting a cut on oblateness. This was done to minimise the dependence on σ_q as

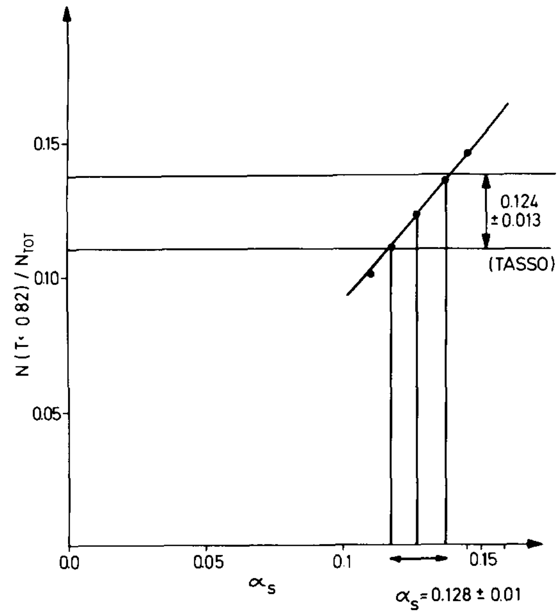

Fig. 2.

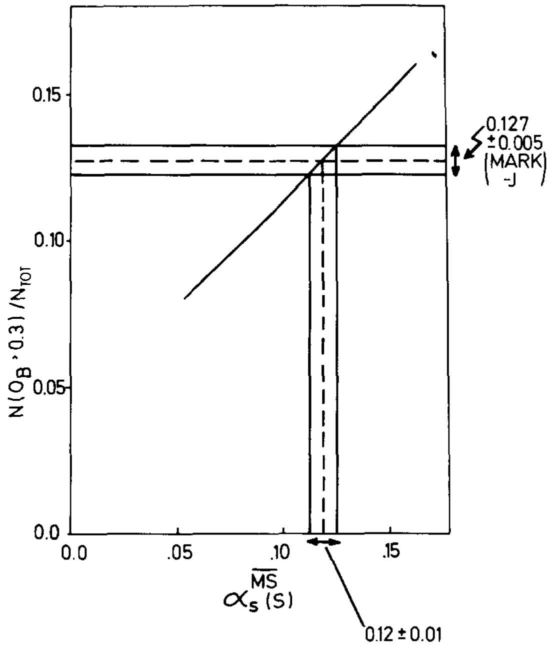

Fig. 3.

studied by the Mark-J collaboration.

Using (O_B = oblateness of the broad jet) the Mark-J data gives $N(O_B > 0.3)/N_{tot} = 0.127 \pm 0.005$. The prediction of the $O(\alpha_s)^2$ corrected QCD calculations is shown in fig. 3. Our best fit is

$$\alpha_s(Q^2 = 1225\ \mathrm{GeV}^2) = 0.122 \pm 0.01 \,. \qquad (11)$$

Taking the average of the TASSO and Mark-J data we find

$$\alpha_s(Q^2 = 1225\ \mathrm{GeV}^2) = 0.125 \pm 0.01 \,. \qquad (12)$$

Corresponding to

$$\Lambda_{\overline{MS}} = 110^{+70}_{-50}\ \mathrm{MeV} \qquad (13)$$

for the QCD scale parameter in the $\overline{MS}$ scheme, with five flavors. The errors do not include the systematic errors on the experimental data. Since the $O(\alpha_s)$ distributions are not significantly changed we expect other data to yield a similar result. The details of the calculations and the comparison with the rest of the data will be published elsewhere.

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