Z. Phys. C - Particles and Fields 13, 321-324 (1982)

Zeitschrift für Physik C Particles and Fields

Some Rare Processes in a Model of Composite Quarks and Leptons

Z.Y. Zhu*
Deutsches Elektronen-Synchrotron DESY, Hamburg 52, Federal Republic of Germany
Received 11 December 1981

Abstract. The branching ratios of some rare processes, i.e. meson $\to \mu e$, meson $\to$ meson μe, baryon $\to$ baryon μe, are calculated in a composite quark and lepton model. Use is made of an effective Hamiltonian which originates from hypercolor singlet 0^- and 1^- bound state exchange mechanism, acting at a mass scale M_P and M_V respectively. Under the reasonable assumption $M_P = M_V$ only $K_L \to \mu e$ puts a significant limit on the composite scale.

The proliferation of the number of quarks and leptons, the existence of the generation structure have led many authors to consider quarks and leptons as composite states of some more fundamental particles. Many models have been suggested, e.g. see (1). In this paper I would like to discuss some effects in a class of model in which each quantum number (flavour, generation, color) corresponds to a carrier. This structure was also the basic idea of the Sakata model and of the quark model. Although it is a very simple idea, it seems to me still worthwhile to give some attention to it.

For the sake of definiteness I take a model suggested by Pati et al. [1] in which the fundamental blocks are

$$(f_u, f_d), (\xi_e, \xi_\mu, \xi_\tau \ldots), (r, b, y, l) \equiv (c_i, c_0) \quad i = 1, 2, 3 \tag{1}$$

They are carriers of flavor, generation and color and, following Pati [1], are called flavons, spinons and chromons respectively. Each quark or lepton is supposed to be composed of one flavon, one spinon and one chromon. Their structure is

$$\nu_e = (f_u \xi_e c_0), \quad \nu_\mu = (f_u \xi_\mu c_0), \quad \nu_\tau = (f_u \xi_\tau c_0)$$
$$e = (f_d \xi_e c_0), \quad \mu = (f_d \xi_\mu c_0), \quad \tau = (f_d \xi_\tau c_0)$$
$$u_i = (f_u \xi_e c_i), \quad c_i = (f_u \xi_\mu c_i), \quad t_i = (f_u \xi_\tau c_i)$$
$$d_i = (f_d \xi_e c_i), \quad s_i = (f_d \xi_\mu c_i), \quad b_i = (f_d \xi_\tau c_i) \tag{2}$$

Obviously, there must exist many kinds of preon exchange reactions between quarks and leptons, including some rare reactions which have not been discovered so far. For example, see Fig. 1. I am going to discuss them here.

The problem is that we have no experimental hint on the preon dynamics which is responsible for binding preons into quarks and leptons. However, it seems reasonable to assume that it is a superstrong interaction with a large energy scale Λ_s. Since so far we have not discovered its signature and preons, it is natural to guess it possesses confinement properties. In fact it is by now rather popular to assume that it is a nonabelian gauge theory like QCD. In this note I also make this assumption and, rather arbitrarily, call it the hypercolor interaction. Thus, preons are supposed to have some hypercolor index. Quarks and leptons are supposed to be hypercolor singlets. Only as the energy becomes of order Λ_s, the interaction between quarks and leptons become superstrong. For low energy reactions, the derived effective interaction would be rather weak because of their hypercolor neutrality. In order to write down the form of the effective interaction of quarks and leptons we can use the analogy to strong hadron interactions because of the similarity of hypercolor interaction to QCD. As we know, for strong hadron interaction single gluon exchange is prohibited because hadrons are color singlets. Two gluon exchange results in Van der Waals force between hadrons. But so far no evidence is found [2] to support this mechanism. So it is rather suggestive that the exchange of color singlet bound states, such as pion, kaon, ... between hadrons play the dominant role in very low energy strong hadron interaction. Applying similar arguments to the case of composite quarks and leptons,

* Permanent address: Institute of Theoretical Physics, Academia Sinica, P.O. Box 2735, Beijing, People's Republic of China

0170-9739/82/0013/0321/$01.00

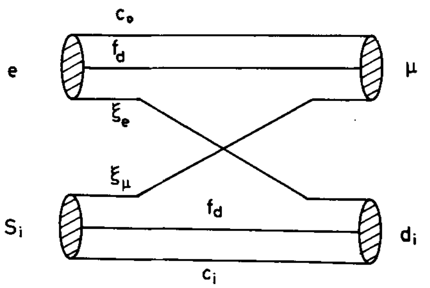

Fig. 1. See text

we can write down the following low energy effective interaction Hamiltonian between quarks and leptons

$$\mathscr{H}_{\text{eff}} = \sum_i \frac{g^2}{m_{pi}^2} \bar{\psi}\gamma_5\psi\bar{\psi}\gamma_5\psi + \sum_i \frac{g^2}{m_{vi}^2} \bar{\psi}\gamma_\mu\psi\bar{\psi}\gamma^\mu\psi + \sum_i \frac{g^2}{m_{A_i}^2} \bar{\psi}\gamma_\mu\gamma_5\psi\bar{\psi}\gamma^\mu\gamma_5\psi + \dots \quad (3)$$

where m_{p_i}, m_{v_i}, m_{A_i} ... are the masses of hypercolor singlet pseudoscalar, vector, axial vector ... bound state mesons formed by suitable preons. Generally one expects their order of magnitude to be Λ_s unless there is some mechanism which prevents this. Furthermore, since in ordinary hadron spectrum all excited states and multiquark states (if they exist) are heavier than the pseudoscalar and vector ground state mesons we can omit the excited state contribution in (3) and obtain

$$\mathscr{H}_{\text{eff}} = \frac{g^2}{M_P^2} \bar{\psi}\gamma_5\psi\bar{\psi}\gamma_5\psi + \frac{g^2}{M_V^2} \bar{\psi}\gamma_\mu\psi\bar{\psi}\gamma^\mu\psi \quad (4)$$

Now I will use (4) to calculate various preon exchange reactions.

1. Chromon Exchange Reactions

There is an interesting possibility that for some unknown reason $c_i\bar{c}_j$ vector bound states possess zero mass and the effective interactions describes QCD. Another possibility is that these $c_i\bar{c}_j$ mesons are very heavy and QCD comes from another unknown source. In this case the heavy exchange effect is a very small correction. These must be difficult to detect. Since I cannot say anything interesting, so I will not discuss this possibility here. By the same reason I exclude the discussion of $c_0\bar{c}_0$ exchange.

For c_0 and c_i exchange reactions between a quark and a lepton, the effective $\mathscr{H}$ comes from the exchange of a color triplet bound state. So it cannot give contribution to very low energy hadron–lepton interactions, but it may show some effects in lepton–hadron deep inelastic scattering where colored quarks are effective. It will result in the following types of effective $\mathscr{H}$

$(\bar{\nu}_\mu\Gamma c)(\bar{u}\Gamma\nu_e)$, $(\bar{\mu}\Gamma s)(\bar{u}\Gamma\nu_e)$, $(\bar{\mu}\Gamma s)(\bar{d}\Gamma e)$
$(\bar{\nu}_\mu\Gamma c)(\bar{d}\Gamma e), \dots$

In this paper I am not going to discuss this possibility further.

2. Flavon Exchange Reactions

The relation between flavor exchange reactions and electro-weak interaction is similar to the relation between colored chromon exchange reactions and QCD. So I also will not discuss them here.

3. Spinon Exchange Reactions

These reactions can lead to many flavor changing neutral processes. Taking two generations as examples, we have

$$(\bar{\nu}_e\Gamma\nu_\mu)(\bar{\mu}\Gamma e) + \text{h.c.};\ (\bar{\nu}_e\Gamma\nu_\mu)(\bar{c}\Gamma u) + \text{h.c.};\ (\bar{e}\Gamma\mu)(\bar{c}\Gamma u) + \text{h.c.};\ (\bar{u}\Gamma c)(\bar{s}\Gamma d) + \text{h.c.};\ (\bar{\nu}_e\Gamma\nu_\mu)(\bar{s}\Gamma d) + \text{h.c.};\ (\bar{e}\Gamma\mu)(\bar{s}\Gamma d) + \text{h.c.}, \quad (5)$$

where the 2nd, 3rd, 4th and 6th part of the effective $\mathscr{H}$ can lead to rare processes which experimentally have not been discovered so far, e.g.

$$D^0 \to \nu_\mu\bar{\nu}_e;\ \bar{D}^0 \to \nu_e\bar{\nu}_\mu;\ D^0 \to \mu^- e^+;\ \bar{D}^0 \to \mu^+ e^-;\ K^0_{L,S} \to \bar{\nu}_\mu\nu_e, \nu_\mu\bar{\nu}_e;\ K^0_{L,S} \to \mu^\mp e^\pm;\ K^0_{L,S} \to \pi^0\mu^\mp e^\pm;\ K^+ \to \pi^+\nu_e\bar{\nu}_\mu;\ K^+ \to \pi^+ e^-\mu^+;\ K^- \to \pi^-\bar{\nu}_e\nu_\mu;\ K^- \to \pi^- e^+\mu^-; \quad (6)$$

Baryon $(B_A) \to$ Baryon $(B_B) + \mu e \dots$
I have calculated the branching ratios of these decays. For the results see Table 1. Here I give 3 examples to illustrate the calculations.

Table 1. The Branching Ratio Values

	Set 1	Set 2
$K_L \to \mu^+e^- + \mu^-e^+$	input[a](1.6×10^{-9})	input[b](1.6×10^{-9})
$K_S \to \mu^+e^- + \mu^-e^+$	2.7×10^{-12}	2.7×10^{-12}
$K_L \to \nu_\mu\bar{\nu}_e + \nu_e\bar{\nu}_\mu$	1.6×10^{-9}	1.6×10^{-9}
$K_S \to \nu_\mu\bar{\nu}_e + \nu_e\bar{\nu}_\mu$	2.7×10^{-12}	2.7×10^{-12}
$D^0 \to \mu^-e^+$	1.2×10^{-13}	1.2×10^{-13}
$\bar{D}^0 \to \mu^+e^-$	1.2×10^{-13}	1.2×10^{-13}
$K^+ \to \pi^+\mu^+e^-$	input[a](4.8×10^{-9})	1.7×10^{-13}
$K^0_L \to \pi^0\mu^+e^- + \pi^0\mu^-e^+$	2.0×10^{-8}	7.2×10^{-13}
$\Xi^- \to \Sigma^-\mu^-e^+$	2.4×10^{-14}	1.8×10^{-18}
$\Xi^0 \to \Lambda^0\mu^-e^+$	2.3×10^{-11}	0.9×10^{-15}
$\Xi^0 \to \Sigma^0\mu^-e^+$	1.5×10^{-14}	2.6×10^{-18}
$\Lambda^0 \to n\mu^-e^+$	7.2×10^{-12}	3.2×10^{-16}
$\Sigma^+ \to p\mu^-e^+$	2.2×10^{-11}	9.4×10^{-16}
$\Sigma^0 \to n\mu^-e^+$	0.8×10^{-22}	3.5×10^{-25}

[a] Set $\frac{M_P}{g} = 468$ TeV and $\frac{M_V}{g} = 36$ TeV

[b] Set $\frac{M_P}{g} = \frac{M_V}{g} = 468$ TeV

(*i*) For $K_L^0 \to \mu^\pm e^\mp$, the effective $\mathscr{H}$ is

$$\frac{g^2}{M_P^2}\bar{s}\gamma_5 d\bar{e}\gamma_5\mu + \frac{g^2}{M_V^2}\bar{s}\gamma_\mu d\bar{e}\gamma^\mu\mu + \text{h.c.} \tag{7}$$

From

$$\langle 0|\bar{s}\gamma_\mu\gamma_5 u|K^+\rangle = if_k k_\mu e^{-ikx}\frac{1}{\sqrt{(2\pi)^3 2k_0}}$$

and flavor SU_2 symmetry we have

$$\langle 0|\bar{s}\gamma_\mu\gamma_5 d|K_L^0\rangle = i\frac{f_k}{\sqrt{2}}k_\mu e^{-ikx}\frac{1}{\sqrt{(2\pi)^3 2^k_0}} \tag{8}$$

Furthermore, assuming d and $\bar{s}$ satisfy the naive free quark equation of motion, we can obtain

$$\langle 0|\bar{s}\gamma_5 d|K_L^0\rangle = -\frac{m_K^2}{m_d+m_s}\frac{f_K}{\sqrt{2}}e^{-ikx}\frac{1}{\sqrt{(2\pi)^3 2k_0}} \tag{9}$$

which can also be derived with help of the current algebra and K-pole dominance hypothesis. Using (7) and (9) we get

$$\frac{\Gamma(K_L^0\to\mu^+e^-)}{\Gamma(K^+\to\mu^+\nu_\mu)} = \left\{\frac{\frac{g^2}{M_P^2}m_K^2}{\frac{G}{\sqrt{2}}\sin\theta_c 2m_\mu(m_s+m_d)}\right\}^2 \tag{10}$$

$$\Gamma(K_L^0\to\mu^+e^-) = \Gamma(K_L^0\to\mu^-e^+)$$

From the experimental limit [3] and (10) we have

$$\frac{M_P}{g} \gtrsim 468\ \text{TeV} \tag{11}$$

Similar calculations lead to the values in Table 1 for D^0, $\bar{D}^0$, K_s^0 etc. where we have used (11) and assumed $f_D = f_K$.

(*ii*) For the $K^+\to\pi^+\mu^+e^-$ only the vector meson exchange part in (7) contributes. Using flavor SU_3 symmetry we obtain

$$\frac{\Gamma(K^+\to\pi^+\mu^+e^-)}{\Gamma(K^+\to\pi^0\mu^+\nu_\mu)} = \left\{\frac{\frac{g^2}{M_V^2}}{\frac{G}{\sqrt{2}}\sin\theta c}\right\}^2 \tag{12}$$

From the experimental limit [4] we obtain

$$\frac{M_V}{g} \gtrsim 36\ \text{TeV} \tag{13}$$

(*iii*) For $B_A\to B_B\mu e$ we assume K pole dominance and flavor SU_3 symmetry which leads to

$$\langle B_B|\bar{d}\gamma_5 s|B_A\rangle = \frac{m_B+m_A}{m_s+m_d}\bar{u}_B\gamma_5 u_A(aG^d+bG^f) \tag{14}$$

$$\langle B_B|\bar{d}\gamma_\mu s|B_A\rangle = b\bar{u}_B\gamma_\mu u_A \tag{15}$$

where G^d and G^f term correspond to the two different types of SU_3 couplings. For the values of a and b for

Table 2. The a and b Coefficients

	a	b
$\Xi^-\to\Sigma^-$	1	1
$\Xi^0\to\Lambda^0$	$-1/\sqrt{6}$	$\sqrt{3}/\sqrt{2}$
$\Xi^0\to\Sigma^0$	$-1/\sqrt{2}$	$-1/\sqrt{2}$
$\Xi^0\to n^0$	$-1/\sqrt{6}$	$-\sqrt{3}/\sqrt{2}$
$\Sigma^+\to p$	1	-1
$\Sigma^0\to n$	$-1/\sqrt{2}$	$1/\sqrt{2}$

various processes, see Table 2. Using (16) and (17) we have

$$\begin{aligned}\Gamma = \frac{\Sigma^3\Delta^5}{16\pi^3 m_A^3}&\left\{\frac{g^4}{M_P^4}(aG^d+bG^f)^2\left(\frac{\Delta}{m_s+m_d}\right)^2\right.\\ &\cdot\left[\left(1-\frac{m_\mu^2}{\Delta^2}\right)\left(\frac{4}{35}-\frac{22m_\mu^2}{35\Delta^2}-\frac{16m_\mu^4}{105\Delta^4}\right)\right.\\ &\left.+\frac{m_\mu^4}{\Delta^4}\left(ln\frac{1+\sqrt{1-m_\mu^2/\Delta^2}}{1-\sqrt{1-m_\mu^2/\Delta^2}}-2\sqrt{1-\frac{m_\mu^2}{\Delta^2}}\right)\right]\\ &+\frac{g^4}{M_V^4}\left[\left(1-\frac{m_\mu^2}{\Delta^2}\right)^{3/2}\left(1-\frac{7m_\mu^2}{2\Delta^2}\right)\right.\\ &\left.\left.+\frac{15}{4}\frac{m_\mu^4}{\Delta^4}\left(ln\frac{1+\sqrt{1-m_\mu^2/\Delta^2}}{1-\sqrt{1-m_\mu^2/\Delta^2}}-2\sqrt{1-\frac{m_\mu^2}{\Delta^2}}\right)\right]\right\}\end{aligned} \tag{16}$$

$$\Delta = m_A - m_B,\quad \Sigma = m_A + m_B$$

where I have omitted the higher order terms of Δ^2/Σ^2. The values of G^d and G^f can be evaluated from ordinary weak decays $n\to pe\nu$ and $\Sigma^-\to ne\nu$. At last we obtain the branching ratios of various rare baryon processes, which are also tabulated in Table 1. For the first column we have used $\frac{M_P}{g} = 468$ TeV, $\frac{M_V}{g} = 36$ TeV. But if we notice that in the ordinary hadron spectrum vector mesons are heavier than pseudoscalar mesons, then it seems reasonable to expect $M_V \gtrsim M_P$. When we take $\frac{M_V}{g} = \frac{M_P}{g} = 468$ TeV, the Branching ratios of $B_A\to B_B\mu e$ will become much smaller. For the results of these assumptions see the second column of Table 1. At present all these values are too low to be tested experimentally.

The flavor changing neutral current processes have also been discussed in [5, 6]. In [5] the effective $\mathscr{H}$ only contains a vector part which is different from (4). They obtain the branching ratio of $K_L\to\mu e$ by assuming that only g_L appears in $\mathscr{H}$. In contrast in our paper the pseudoscalar term in (4) naturally leads to this process. If we take their scale value instead of our equation (11), we would get a much larger branching ratio for $K_L\to\mu e$, namely 1.5×10^{-5}. In [6] they have not written down an effective $\mathscr{H}$. We find our results are also different from their estimates. Especially for $\Sigma \to p\mu e$, (16) in this paper is completely different from their equation (10). If we take their scale value and only calculate the vector part we have the branching ratio of $\Sigma \to p\mu e$ is 6.7×10^{-15}. This is much smaller than their value.

In summary if our arguments in this paper are correct, from all the rare processes only $K_L \to \mu e$ can give a significant lower bound on the composite scale. With the present experimental possibilities the search for other rare processes discussed in this paper cannot give more information about the compositeness of quarks and leptons.

Acknowledgements. I would like to express my gratitude to Prof. E. Lohrmann, Prof. H. Joos and colleagues in the Theory Group of DESY for their hospitality. I am grateful to A. Ali, S. Rai Choudhury, K. Ishikawa, G. Schierholz and T.T. Wu for helpful discussions.

References

1. J.C. Pati, A. Salam: Phys. Rev. **D10**, 275 (1974) J.C. Pati: Phys. Lett. **98B**, 40 (1981); H. Terazawa, Y. Chikashige, K. Akama: Phys. Rev. **D15**, 480 (1977); H. Terazawa: INS-Rep. 401 (1981); S.L. Glashow: HUTP-77/A005 (1977); H.Y. Zhu: Talk given at Huangshan conference on particle physics in China (1977), (unpublished); Y. Neeman: Phys. Lett. **82B**, 69 (1979); H. Harari: Phys. Lett. **86B**, 83 (1979) SLAC-239 (1980); H. Harari, N. Siebeg: Phys. Lett. **98B**, 269 (1981); M.A. Shupe: Phys. Lett. **86B**, 87 (1979); R. Casalbuoni, R. Gatto: Phys. Lett. **93B**, 47 (1980); UGVA-DPT 1981/06-272; H. Fritzsch, G. Mandelbaum, MPI-PAE/PTh 22/81; S.M. Phillips: Phys. Lett. **84B**, 133 (1979); M. Peskin: A Talk at 1981 International Symposium on Lepton and Photon Interactions at High Energies; G. 't Hooft: Proceedings of the Cargese Summer Institute (1979); R. Barbieri, L. Maiani, R. Petronzio: Phys. Lett. **96B**, 63 (1980); T. Banks, S. Yankilowicz, A. Schwimmer: Phys. Lett. **96B**, 67 (1980); I. Bars, S. Yankilowicz: YTP-81-04; D.D. Wu: HUTP 81/A001, 81/A015; S. Weinberg: Texas Preprint (1981); C.H. Albright, B. Schrempp, F. Schrempp: DESY 81-038
2. G. Feinberg, J. Sucher: Phys. Rev. **D20**, 1717 (1979); B. Chiu, Y.B. Dai: Texas preprint (1981)
3. A.R. Clark et al.: Phys. Rev. Lett. **26**, 1667 (1971)
4. A. Diammant-Berger et al.: Phys. Lett. **62B**, 485 (1976)
5. R.N. Cahn, H. Harari: Nucl. Phys. **B176**, 135 (1980)
6. G.L. Kahn, R. Thun: Phys. Lett. **94B**, 513 (1980)