PHYSICAL REVIEW D VOLUME 26, NUMBER 3 1 AUGUST 1982

Constraints on recombination functions from J/ψ decays

L. M. Jones*
Deutsches Elektronen-Synchrotron, D-2000 Hamburg 52, Germany
(Received 17 December 1981)

The $J/\psi \to 3$ gluon decay is combined with current recipes for gluon recombination into hadrons to yield rates for exclusive decays.

INTRODUCTION

The purpose of this short calculation is to test, in yet another way, the current recipe for obtaining recombination functions of gluons into baryons. In this method, the gluons are first converted into quark-antiquark pairs by a splitting function which preserves momentum, and the resulting quarks and antiquarks are then recombined by an old-fashioned or primitive recombination function. In this note, I present the results thus obtained for the exclusive processes $J/\psi \to (B,\bar{B})$ and $J/\psi \to 3$ mesons. For some of the quark recombination functions currently in use, these results are surprisingly reasonable, considering their "back-of-the-envelope" nature.

PROCEDURE

The method is very simple. The differential decay rate of the heavy vector meson in its rest frame may be written as[1,2]

$$\frac{d\Gamma}{dx_1\,dx_2} = \frac{\Gamma_{3g}}{(\pi^2-9)}\left[\frac{x_1^2(1-x_1)^2+x_2^2(1-x_2)^2+x_3^2(1-x_3)^2}{x_1^2x_2^2x_3^2}\right], \tag{2.1}$$

where x_1, x_2, and x_3 are the scaled energies of the three gluons, normalized to $x_1+x_2+x_3=2$, and the x_i are allowed to vary over their entire kinematic range (this accounts for a factor-of-6 apparent difference with some other references).

We next boost the J/ψ to infinite momentum p_V so that all the gluons are (roughly) collinear. New fractional-momentum variables $\bar{x}_i$ may be defined so that, for the four-vector momenta p_i and p_V,

$$p_i = \bar{x}_i p_V + p_{ti} ,$$

where p_{ti} is such that the four-vector scalar product $p_V \cdot p_{ti}$ is zero. If the boost to infinite momentum is performed in the direction perpendicular to the plane of the original decay, this residual p_{ti} is finite. The new fractions are related to the old x_i by $x_i = 2\bar{x}_i$. We may then write the differential decay rate in terms of these variables as

$$\frac{d\Gamma}{d\bar{x}_1\,d\bar{x}_2\,d\bar{x}_3} = \frac{8\Gamma_{3g}\delta(2\bar{x}_1+2\bar{x}_2+2\bar{x}_3-2)}{(\pi^2-9)(16)}\left[\frac{(1-2\bar{x}_1)^2}{\bar{x}_2^2\bar{x}_3^2}+\frac{(1-2\bar{x}_2)^2}{\bar{x}_1^2\bar{x}_3^2}+\frac{(1-2\bar{x}_3)^2}{\bar{x}_1^2\bar{x}_2^2}\right]. \tag{2.2}$$

Each of the gluons is assumed[3] to split with probability

$$P(y) = \frac{3}{2N_f}[y^2+(1-y)^2]$$

into each possible flavor of quark-antiquark pair—the quark carrying momentum fraction y and the antiquark carrying momentum fraction $(1-y)$. To produce a baryon at x_B and an antibaryon at x_{BB}, we then multiply by the probability that the quark triplets recombine into a baryon of momentum fraction x_B, $R_B(x_{q_1}, x_{q_2}, x_{q_3}, x_B)$. We consider three different forms for this function, which are currently in use in the recombination literature. These are as follows.

(i) The form used by Ranft,[4] and more recently by Migneron, Jones, and Lassila[5]

$$R_{\rm R}(x_{q_1},x_{q_2},x_{q_3},x_B) = R_{\rm R}\frac{x_{q_1}x_{q_2}x_{q_3}}{x_B^3} \times \delta(x_{q_1}+x_{q_2}+x_{q_3}-x_B) .$$

Ranft used a normalization $R_{\rm R}=120$, whereas Migneron *et al.* use a normalization prescription due to Teper[6] which gives an upper limit of $R_{\rm R}=27$,[7] and suggest the actual value should be smaller than this by a factor of between 2 and 4.

(ii) The form used by Eilam and Zahir[8] for the proton (x_{q_1}, x_{q_2} are up quarks; x_{q_3} is a down quark)

$$R_{\rm EZ}(x_{q_1},x_{q_2},x_{q_3},x_B)=\frac{19.9(x_{q_1}x_{q_2})^{1.65}x_{q_3}{}^{1.35}}{x_B{}^{4.65}}\,\delta(x_{q_1}+x_{q_2}+x_{q_3}-x_B)\ .$$

(iii) The form originally suggested by Hwa[9] and used by Takasugi and Tata[10]

$$R_{\rm HTT}(x_{q_1},x_{q_2},x_{q_3},x_B)=R_{\rm HTT}\left(\frac{x_{q_1}x_{q_2}x_{q_3}}{x_B{}^3}\right)^{3/2}\delta(x_{q_1}+x_{q_2}+x_{q_3}-x_B)\ .$$

In using this, Hwa suggested that the normalization parameter $R_{\rm HTT}$ be $R_{\rm HTT}=105/2\pi$; however, Takasugi and Tata have allowed the normalization to run free (they find agreement with the data for $R_{\rm HTT}=8.2$). Teper's argument applied here would give $R_{\rm HTT}\leq 140.3$.

Of course we also use the same function to combine three antiquarks into an antibaryon.

The expression for decay, gluon splitting, and recombination then becomes (up to possible corrections for hadron masses, and quark combinatorics)

$$R_{\Gamma_B}=\frac{\Gamma_{BB}}{\Gamma_{3g}}=\int d\bar{x}_1 d\bar{x}_2 d\bar{x}_3\frac{d\Gamma}{d\bar{x}_1 d\bar{x}_2 d\bar{x}_3}\times\int dy_1dy_2dy_3P(y_1)P(y_2)P(y_3)\times\int dx_BR(y_1\bar{x}_1,y_2\bar{x}_2,y_3\bar{x}_3,x_B)R((1-y_1)\bar{x}_1,(1-y_2)\bar{x}_2,(1-y_3)\bar{x}_3,1-x_B) \qquad (2.3)$$

which we evaluate for the three cases as

$$(R_{\Gamma_B})_{\rm RMJL}=\frac{R_{\rm R}{}^2 0.1055\times10^{-2}}{N_f{}^3(4)(\pi^2-9)}\ ,$$

$$(R_{\Gamma_B})_{\rm EZ}=\frac{0.6937\times10^{-2}}{N_f{}^3(4)(\pi^2-9)}\ ,$$

$$(R_{\Gamma_B})_{\rm HTT}=\frac{R_{\rm HTT}{}^2 0.2477\times10^{-4}}{N_f{}^3(4)(\pi^2-9)}\ .$$

Similarly for the three-meson final state we obtain

$$R_{\Gamma_M}=\frac{\Gamma_{3M}}{\Gamma_{3g}}=\frac{R_M{}^3(0.1671)\times10^{-1}}{N_f{}^3(4)(\pi^2-9)} \qquad (2.4)$$

using the simple recombination function

$$R_M(x_{\bar q},x_q,x_m)=R_M\frac{x_q\bar{x}_q}{x_m{}^2}\delta(x_q+x_{\bar q}-x_m)\ .$$

COMPARISON WITH DATA

Baryons

To compare with data at the J/ψ we should probably include some concession to the fact that the masses of the produced objects are in fact not much smaller than those of the decaying object. For this reason in the baryon case we include the phase-space factor

$$\frac{k}{k_0}=\left(\frac{M_V{}^2-4m_B{}^2}{M_V{}^2}\right)^{1/2}=\begin{cases}0.796 \text{ for protons}\\ 0.695 \text{ for } \Lambda\text{'s}\ .\end{cases}$$

We also multiply by the number of ways that the appropriate flavors of quarks can be picked out of the gluons: 6 for the Λ, and 3 for the proton or neutron (or, for that matter, the Ξ).

We thus finally obtain the values (using $N_f=3$)

$$(R_\Gamma)_{\rm RMJL}=R_{\rm R}{}^2 2.683\times10^{-5}\ ,$$
$$(R_\Gamma)_{\rm EZ}=1.764\times10^{-4}\ , \qquad (3.1a)$$
$$(R_\Gamma)_{\rm HTT}=(R_{\rm HTT})^2 6.298\times10^{-7}\ ,$$

for protons and

$$(R_\Gamma)_{\rm RMJL}=R_{\rm R}{}^2 4.684\times10^{-5}\ ,$$
$$(R_\Gamma)_{\rm EZ}=3.08\times10^{-4}\ , \qquad (3.1b)$$
$$(R_\Gamma)_{\rm HTT}=(R_{\rm HTT})^2 1.1\times10^{-6}\ ,$$

for Λ's, which are to be compared with the experimental rates (1980 Particle Data Group Table[11])

$$(R_\Gamma)_p\approx2.5\times10^{-3},\quad (R_\Gamma)_\Lambda\approx1.28\times10^{-3}\ .$$

These agree for the proton if $R_{\rm R}=9.8$; this is reasonable since the maximum possible value of $R_{\rm R}$ is 27,[7] and this does not allow for the fact that the quarks thus made could equally well reassemble themselves into mesons or proton resonances. In the Eilam-Zahir case, the rates disagree by a factor of 14.5, which is a disagreement of about a factor of 4 in the recombination function. It is not clear whether this should be regarded as a serious disagreement with the valon-model normalization. For the Hwa-Takasugi-Tata function, we have agreement with the rates if $R_{\rm HTT}=63.76$. This is quite a bit larger than the Takasugi-Tata number of 8.2; it is again about a factor of 4 larger than the valon-model prediction of 16.71.

For Λ's, we would need a suppression factor related to the creation of the higher mass of the strange quark in the gluon-conversion process; we conclude that this would have to be about a factor of 3.5 to have agreement with the data. Again, this is not surprising in view of other experience with strange-quark production.

It is only fair to point out that the production rate for Ξ's, which should (according to these arguments) be smaller than the production rate of Λ's by an extra strange-particle-creation factor, is measured to be[11]

$$(R_\Gamma)_\Xi=(3.72\pm 0.93)\times 10^{-3}$$

for both charge states. We have no good explanation for this; the situation will be more clear when the experimental errors are reduced.

Mesons

We find, similarly, for the production rate of three mesons from quark-antiquark pairs

$$(R_\Gamma)_M=MR_M{}^3 1.78\times 10^{-4}\ , \tag{3.2}$$

where M is some integer telling how the quark pairs needed for the particular mesons in question may be chosen from the gluons. For instance, if we take the set $\omega\pi^+\pi^-$ we find $M=3$, and (3.2) is to be compared with the experimental rate of[11,12]

$$(R_\Gamma)_M=7.91\times 10^{-3}\ .$$

This implies that the recombination function for mesons is $R_M=1.14$. Again, the maximum value this could have is $R_M=4$,[6] which does not allow for baryon production; and the valon-model prediction[9] is $R_M=1$. We thus conclude that the values are in reasonable agreement with both normalization prescriptions. Note that the data show $\omega K\bar{K}/\omega\pi^+\pi^-=0.16/0.68\approx\frac{1}{4}$,[11,12] similar to the suppression factor in the baryon case.

CONCLUSIONS

At present we view these rough calculations as a test of the semiknown recombination functions rather than as predictions for the particular modes of J/ψ decay. Also, they should be taken only as a general indication of the size of the recombination parameters given the particular prescription for gluon conversion. With this in mind, all recombination constant values are consistent with the probability arguments of Teper; and the meson constant also agrees with the valon normalization of Hwa. Values of the baryon recombination constant larger than the valon-model predictions by about a factor of 4 are necessary for agreement with experiment.

There are many reasons to distrust the general procedure, but it has some chance to be applicable at the J/ψ because the total invariant mass of the produced quark triplets is limited by the ψ mass. Exactly the same calculation might be performed at the Υ or Υ' resonances, and (except for the kinematic corrections) it would produce the same ratios of pp to total hadronic width as at the ψ. This would be less reasonable, however, because as the mass of the resonance grows the effective mass of the recombining quark triplets grows.

In principle our recombination function should have a damping factor which ensures that only quark triplets of smallish size are made into protons. The experimental rates for the higher resonances could conceivably be used to measure this damping.

The technique in its present form also does not explain the rather large number of two-body meson decays found at the ψ. The implications of these for the recombination procedure are not clear; either they present a fundamental difficulty, or they can be handled by some modification of the technique.

*Permanent address: Physics Department, University of Illinois at Urbana-Champaign, 1110 W. Green St., Urbana, Ill. 61801.

[1]K. Koller and T. Walsh, Nucl. Phys. B140, 449 (1978).
[2]J. Körner and D. H. Schiller, Report No. DESY 81-043 (unpublished).
[3]V. Chang and R. C. Hwa, Phys. Rev. Lett. 44, 439 (1980).
[4]J. Ranft, Phys. Rev. D 18, 1491 (1978).
[5]R. Migneron, L. M. Jones, and K. E. Lassila, Report No. SLAC-PUB-2850 (unpublished); and Report No. SLAC-PUB-2861 (unpublished).
[6]M. J. Teper, Rutherford Report No. RL-78-022/A, 1978 (unpublished).

[7]The argument is simply that $\int dx_B\, R(x_{q_1},x_{q_2},x_{q_3},x_B)$ gives the probability that the quarks at x_{q_1}, x_{q_2}, and x_{q_3} recombine to form a baryon, and this should always be less than or equal to 1. With $R_R=27$, the upper limit of 1 does occur. If recombination of $3q$ into baryon resonances, and of $(3q\,3\bar{q})$ into mesons, is allowed, the number should be smaller.

[8]G. Eilam and M. S. Zahir, Phys. Rev. D (to be published).
[9]R. C. Hwa, Phys. Rev. D 22, 1593 (1980).
[10]E. Takasugi and X. Tata, Phys. Rev. D 23, 2573 (1981).
[11]Particle Data Group, Rev. Mod. Phys. 52, S1 (1980).
[12]F. Vanucci et al., Phys. Rev. D 15, 1814 (1977).