THE GLUEBALL MASS SPECTRUM IN QCD: FIRST RESULTS OF A LATTICE MONTE CARLO CALCULATION

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We perform a variational calculation of the masses of glueballs of various spins and parities in SU(2) gauge theory. The quantum vacuum we use is generated by the lattice Monte Carlo technique. Our first results, obtained on medium sized lattices give $m(0^+) = (3.6 \pm 0.35)\Lambda_{\rm mom}$, $m(0^-) = (6.0 \pm 1.0)\Lambda_{\rm mom}$, $m(2^+) = (6.5^{+1.8}_{-1.1})\Lambda_{\rm mom}$, various mass upper bounds and information on glueball wave functions.

QCD without fermions, the pure non-abelian gauge theory, will possess its own spectrum of states that are composed solely out of gluon fields. If this theory is colour confining – and all the numerical evidence now points to this [1] – then these "glueball" states will be colour singlets and should have their place in, and impact upon, the full spectrum of the chromodynamic theory with fermions. The lack of any firm experimental information on glueballs [‡1] makes the prediction of their properties a theoretical challenge with particularly high stakes.

In this letter we present the first results of a theoretical program to calculate the masses of the detailed glueball spectrum. Our calculations are on a finite sized lattice [3,1] using the Monte Carlo technique [3,1]. Previous [4] [‡2] Monte Carlo calculations attempted to calculate the lowest glueball mass, μ, using the fact that correlation functions for any operator ϕ should have the property

$$\langle\phi(r)\phi(0)\rangle \sim f(r)\, e^{-\mu r}, \quad \text{as } r \to \infty, \tag{1}$$

where $f(r)$ is an inverse power of r (and where one chooses ϕ such that $\phi|\text{vacuum}\rangle$ has a zero projection on the vacuum and a non-zero projection on the lowest glueball state). This approach proved inaccurate because for r greater than 2 lattice spacings the signal is lost in the noise of statistical fluctuations. The problem with this method is that the more accurate a measurement one wants the (exponentially) smaller is the signal one has to look for – since the theoretical accuracy of (1) is improved by going to larger r; and this is not going to work in a Monte Carlo calculation of limited statistics.

The lesson here is that one should use a method where increasing the theoretical accuracy involves searching for a *larger* signal. Such a method is available It is the standard variational method for deriving energy levels [‡3]. One chooses some à priori reasonable class of wave functions ϕ and varies within that class to minimize the energy expectation

$$\delta\{\langle\Omega|\phi^+ H\phi|\Omega\rangle/\langle\Omega|\phi^+\phi|\Omega\rangle\} = 0. \tag{2}$$

[‡1] Interesting recent candidates have been located in J/ψ radiative decays [2].

[‡2] Rough glueball mass estimates have also been obtained from the temperature dependence of QCD [5].

[‡3] This has been emphasized by Wilson [6].

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On the lattice, the operator one has available is the transfer matrix, $V = e^{-Ha}$, where a is the lattice spacing. The analogue of (2) on the lattice is

$$\delta\{\langle\Omega|\phi^+ e^{-Ha}\phi|\Omega\rangle/\langle\Omega|\phi^+\phi|\Omega\rangle\} = 0, \tag{3}$$

where now one searches for a wave function ϕ that *maximizes* the expectation value (minimizing H maximizes e^{-Ha}). In our work we shall use the lattice Monte Carlo technique to construct the vacuum $|\Omega\rangle$ and hence to calculate the matrix elements in (3).

An important added advantage of the variational method is that as well as providing estimates of energy levels it simultaneously provides an estimate of the wave functions. This will enable us to make, later on in this paper, non-trivial statements about the internal structure of glueballs of different spins and parities.

On the other hand, a serious disadvantage of the variational method is that while it is easy to obtain an estimate of an energy level, it is difficult to judge the accuracy of this estimate (although what we do know is that our estimate provides an *upper bound* on the true value). For this reason we supplement the naive variational method as embodied in eq. (3) with a calculation of the expectation value of the *square* of the transfer matrix: $\langle\Omega|\phi^+ e^{-2Ha}\phi|\Omega\rangle$. In general

$$\frac{\langle\Omega|\phi^+ e^{-Ha}\phi|\Omega\rangle}{\langle\Omega|\phi^+\phi|\Omega\rangle} \leqslant \frac{\langle\Omega|\phi^+ e^{-2Ha}\phi|\Omega\rangle}{\langle\Omega|\phi^+ e^{-Ha}\phi|\Omega\rangle}, \tag{4}$$

with equality being achieved when our trial wavefunction exactly reproduces the correct wavefunction, so that the deviation from equality will measure the extent to which our trial wavefunction and energy level estimate deviate from their correct values. Indeed (see below) for a given wavefunction the right-hand side of eq. (4) provides in principle a much better estimate of the energy level than the left-hand side; however, for reasons to do with statistical accuracy it is best to do both calculations in tandem (see below).

The analogue of this in a more familiar context – say a potential model – would be to compare different moments of the energy, for example $\int\psi^* H\psi$ and $\int\psi^* H^2\psi$. Typically the latter quantity would be much more complicated to evaluate than the former, so this test is not usually implemented. In our present context the evaluation of (3) and (4) is equally simple, so in this sense the Monte Carlo approach lends itself especially readily to such a variational treatment.

We now outline our method; details we leave to a later publication. For brevity we sketch the argument as though the lattice were of infinite extent. The set of all closed loops on the lattice forms an (over) complete set of colour singlet operators for the dynamical system represented by the lattice. So our wavefunction operator ϕ will in general be composed of such loops. By taking a suitable linear combination of such loops the operator ϕ can be made to transform as a momentum $\vec{p}$, spin J, parity P operator. The rotational discreteness of course implies that in terms of continuum angular wave functions our operator will not be a pure spin J operator: here J will label the lowest spin contributing to the operator. Now choose one axis as the imaginary time, t, axis and consider the correlation function $\langle\phi(J^P, \vec{p}; t = a)\phi(J^P, \vec{p}; t = 0)\rangle$ normalized to the equal-time expectation value $\langle\phi(t = 0)\phi(t = 0)\rangle$, where we shall suppress arguments where convenient. Expanding in a complete set of energy eigenstates labelled by $\nu = 0, 1, 2, \ldots$ etc. (and suppressed indices J, P and $\vec{p}$) we have

$$\frac{\Gamma_a}{\Gamma_0} \equiv \frac{\langle\phi(t = a)\phi(t = 0)\rangle}{\langle\phi(t = 0)\phi(t = 0)\rangle} = \frac{\langle\phi\, e^{-Ha}\phi\rangle}{\langle\phi\phi\rangle}$$

$$= \sum_\nu e^{-E_\nu a}\, \frac{|\langle\nu|\phi|\Omega\rangle|^2}{\Sigma_\nu |\langle\nu|\phi|\Omega\rangle|^2}, \tag{5}$$

where necessary (in particular for $J^P = 0^+$) we replace ϕ by $\phi - \langle\phi\rangle$ to ensure that the vacuum state does not appear in the above sum. Then the lowest energy state $|\nu = 0\rangle$ contributing to this sum will generally be a one-particle state (or a continuum formed out of lower mass glueballs of other J^P); the lowest mass particle of the particular J^P considered. Since we are interested in masses we take a zero-momentum translation-invariant operator, $\phi(J^P, \vec{p} = 0)$, so that $E_0 = m(J^P)$. From the fact that we have normalized the correlation function in (5) it follows that *any* wave function ϕ immediately gives us an *upper* bound on $m(J^P)$:

$$m(J^P) \leqslant \frac{1}{a} \ln \frac{\langle\phi(t = 0)\phi(t = 0)\rangle}{\langle\phi(t = a)\phi(t = 0)\rangle}. \tag{6}$$

The variational part of the calculation now consists of varying ϕ judiciously so as to minimize the right-hand side of (6). (Note that this means one *maximizes* the left-hand side of (5), which is the quantity directly calculated on the lattice.)

In the spirit of the variational approach, one hopes

that the resulting upper bound will be close to the actual mass itself. How much this is actually so can be checked by measuring also the correlation function over two timelike lattice spacings, giving in analogy to (5),

$$\frac{\Gamma_{2a}}{\Gamma_a} = \frac{\langle\phi(t=2a)\phi(t=0)\rangle}{\langle\phi(t=a)\phi(t=0)\rangle} = \frac{\Sigma_\nu e^{-E_\nu\cdot 2a}|\langle\nu|\phi|\Omega\rangle|^2}{\Sigma_\nu e^{-E_\nu\cdot a}|\langle\nu|\phi|\Omega\rangle|^2}. \quad (7)$$

The higher mass states are more harshly suppressed in (7) than (5), and so one expects approximate equality between (7) and (5) only when one has found a very good wavefunction for the lowest mass state. Of course, in general

$$\langle\phi(t=2a)\phi(t=0)\rangle \ll \langle\phi(t=a)\phi(t=0)\rangle \ll \langle\phi(t=0)\phi(t=0)\rangle, \quad (8)$$

whereas the statistical noise is comparable in all cases, so the errors on (7) will be much larger than on (5), and this is an added reason to use (7) in tandem with (5) rather than alone (see below).

As a brief aside at this stage note how we differ from earlier efforts [4] [‡2]: normalizing our correlation function allows us an immediate bound on the energy of the lowest excited state; taking suitable J^P combinations of loop operators allows us to obtain lowest mass states of differing J^P; taking $\vec{p} = 0$ means that the energy is the mass and does not contain a, perhaps large, momentum contribution [‡4]. Moreover, we obtain non-trivial information on the various glueball wavefunctions.

We now turn to the practical implementation of the above program. The first point to note is that the use of $\vec{p} = 0$ states, is an expensive strategy from the point of view of minimizing statistical noise, sound as it might be theoretically. This is basically because any given equal-time slice of a given generated lattice configuration gives us just one $\vec{p} = 0$ wave function, $\phi|\Omega\rangle$, so a lattice with LT spacings in the time direction gives us only LT measurements of $\langle\Omega|\phi\phi|\Omega\rangle$ for each generated configuration. Moreover, the error to signal ratio for a given measurement is roughly independent of the spatial lattice size LS, so that the computing time required to achieve a given accuracy increases rapidly with lattice size, as LS^3.

The solution to this problem lies in the observation that one loses nothing by considering states with momentum smearing Δp as long as

$$\Delta p \ll m_g, \quad (9)$$

so that $E(p) \approx m_g$ (unfortunately one does not know the dispersion relation for large p). Equivalently one considers partially translation invariant operators, e.g. finite blocks of loop operators. This cures the statistics problem. However, to implement it requires a rough idea of the mass spectrum; higher mass states allow a larger momentum smearing, which increases statistics, which nicely compensates for the lower signal coming in (5) for higher mass states. Accordingly the first part of our program has been to calculate the glueball mass spectrum using exact $\vec{p} = 0$ states on small/medium sized lattices. It is the results of this part of our program that we wish to report on in this letter.

We work on lattices of sizes 4^4, $4^3\cdot 8$ and 6^4, using operators ϕ of definite J^P and with $\vec{p} = 0$ (and using the Wilson action [3,1]). Such lattices are large enough that the mass spectrum should undergo no qualitative change in going to larger lattices. This expectation has been borne out by a comparison of our results for the 4^4 and 6^4 lattices. The present calculation is for the SU(2) group; again theoretically one expects no great change going to SU(3). SU(2) calculations are much faster so it is expedient to let SU(3) wait till the momentum smeared wavefunctions have been constructed and can be used. Most of our calculations are at one value of β ($= 4/g^2$) so as not to dissipate our statistics. An appropriate value for β is one for which

$$a \ll \text{size glueball} \ll a\cdot LS/2, \quad (10)$$

so that the effects of discreteness and of the boundary conditions are minimal. We also require that β should be large enough (g^2 small enough) that two-loop perturbation theory should be good on the size scale of the lattice spacing – so that we are indeed close to the continuum limit. For the kind of lattice sizes currently amenable to Monte Carlo treatment, such requirements could well have been found to be mutually exclusive; the surprise of the first calculations [1] has been that this is not so, nature has been kind to us. We lean on the results of ref. [1] to infer that $\beta = 2.3$ is a suitable value. For this value of β, the two-loop perturbative ex-

[‡4] The advisability of using $\vec{p} = 0$ states was stressed in ref. [7]. Note that this strong-coupling series calculation does indeed give a lower mass than obtained in (4).

pansion is good on the size scale of the lattice spacing a, and hence we can obtain the relationship between a and $\Lambda_{\rm mom}$:

$$a \approx [57.5/\Lambda_{\rm mom}({\rm GeV})]\ {\rm e}^{-3\pi^2\beta/11}(\tfrac{6}{11}\pi^2\beta)^{51/121} \approx_{\beta=2.3} 0.34\Lambda^{-1}_{\rm mom}({\rm GeV}). \quad (11)$$

At $\beta = 2.3$ on the 6^4 lattice we measure the string tension to be

$$\sqrt{K} \approx 1/2a \approx 1.47\,\Lambda_{\rm mom}({\rm GeV}). \quad (12)$$

Using the SU(3) string tension of ~400 MeV (corresponding to unit Regge slope) gives $a \approx 0.24$ fermi. Hence for this β, for our lattices of sizes 4^4, $4^3\cdot 8$, 6^4, condition (10) appears to be "satisfied" (if one adopts the optimistic interpretation of inequalities that has become standard usage in Monte Carlo work).

On these lattices, at $\beta = 2.3$, we have constructed operators with $J^P = 0^+, 0^-, 1^+, 1^-, 2^+, 2^-$ and 3^+ and have measured the quantities in (5) and (7). Since in fact the glueball size is not expected to be much bigger than the lattice spacing, only small loops need be considered. Furthermore here we neglect products of loops: basically to simplify matters, although one can also remark (somewhat disingenuously) that a product of loops looks like a state with two colour singlet objects, and here we are not so interested in continuum states. For odd parity loops one needs loops with a geometric handedness and such loops have at least 8 links. In figs. 1a–1i we show the loops that we have used as basic components of ϕ in our calculation (note all the links are spacelike in figs. 1a–1h). The loops in figs. 1d–1g are 8 link loops (not products of plaquettes, although their differences are simple plaquette products) and the arrows indicate the ordering of the matrices (there are only two such orderings because in SU(2) the overall direction around the loop is irrelevant). The loops in figs. 1f–1i have a geometric handedness and are used to construct odd-parity states (as well as even-parity states).

To construct states of definite spin J out of these basic components we construct linear combinations which have appropriate properties under rotations of $\pi/2$ and/or π about the spatial axes, x, y, z. For example the totally symmetric combination of simple plaquettes (see fig. 2a) contains states of spin–parity

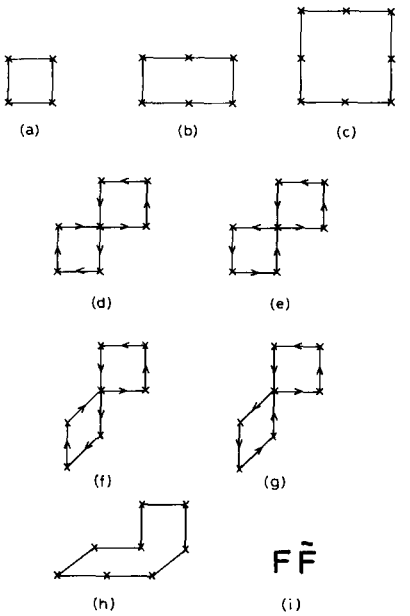


Fig. 1. Loops on the lattice (crosses represent lattice sites) that are used as the basic components for our trial wave functions: (a)–(e) loops are planar; (f)–(h) loops are three-dimensional (and purely spatial); (i) is not drawn explicitly since it is four dimensional (see text). Where necessary arrows indicate the path ordering along the loop.

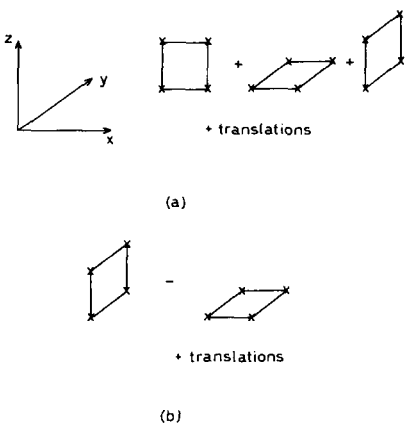


Fig. 2. (a) a sum of plaquettes with 0^+ as its lowest spin component. (b) A sum of plaquettes with 2^+ as its lowest spin component.

$$J^P = 0^+, 4^+, \ldots \tag{13}$$

and so is a suitable trial wavefunction for the 0^+ glueball. As a non-trivial example consider the 2^+ operator. Such an operator can be obtained by imposing the rotational properties

$$R_z(\pi) = R_y(\pi) = -R_y(\pi/2) = 1, \quad P = +1. \tag{14}$$

In fig. 2b we show a linear combination of plaquettes satisfying the conditions in (14) (the simplest example). This operator will contain J^P contributions

$$J^P = 2^+, 4^+, \ldots \tag{15}$$

and hence is a possible trial operator for the 2^+ glueball. We perform analogous steps for our other wave functions.

The results we present here are based on the following wave functions. We construct 0^+, 2^+ wave functions out of all the basic components shown in fig. 1. In addition we calculate arbitrary linear combinations of figs. 1a–1c and, separately, arbitrary linear combinations of figs. 1f,1g. (In subsequent results we plan to have arbitrary linear combinations of all our basic components.) The components in figs. 1f–1i are used for trial 0^-, $1^\pm$, 2^- wave functions, and figs. 1d,1e for the 3^+ wave function.

We have performed several independent series of iterations of the lattice configuration. Typically before taking any data, we first heat up the lattice with O(2000) iterations starting from an ordered (all matrices on links are unit matrices) or a disordered (all matrices totally random) start; corresponding to starting with $\beta = \infty$, $\beta = 0$, respectively. Most of our data here is from a disordered start; the purpose of taking data from both starts is to provide an estimate of some of the *systematic* errors as well as giving an additional measure of the statistical errors.

We now summarize our first results for glueball masses and wavefunctions: a detailed presentation and discussion of our results and error analysis will appear elsewhere.

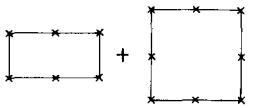
0^+: measurements of Γ_a/Γ_0 show that the best operators in fig. 1 are 1h and a linear combination of 1a –1c which consists almost equally of 1b and 1c. We obtain from Γ_a/Γ_0 for these operators an upper bound

$$m(0^+) \leqslant (4.88 \pm 0.15)\Lambda_{\text{mom}}. \tag{16}$$

Measurements on about 30 000 lattice ($4^3 \cdot 8$ and 6^4) configurations have been used. We measure Γ_{2a}/Γ_a for these wave functions to obtain the mass

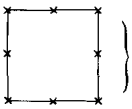
$$m(0^+) = (3.6 \pm 0.35)\Lambda_{\text{mom}}. \tag{17}$$

Our observed statistical errors on Γ_{2a}/Γ_a are consistent with the errors expected from the observed errors on the more accurately determined Γ_a/Γ_0. (All our quoted errors are statistical.) If we confine ourselves to the operators in figs. 1a–1c, we find that the best linear combination is

$\phi \approx \frac{1}{2}\Big\{$  $\Big\}$. (18)

Observe that a straightforward variational calculation would have yielded the right-hand side of (16) as our best estimate, the concurrent use of Γ_{2a}/Γ_a has enabled us to obtain (17) – a substantially different and, we believe, better estimate.

2^+: measurements of Γ_a/Γ_0 show that the best operator in fig. 1 is a linear combination of 1a–1c dominated by 1b and 1c:

$\phi \approx \Big\{0.6$  $+ 0.4$  $\Big\}$. (19)

This gives us an upper bound

$$m(2^+) \leqslant (8.53 \pm 0.27)\Lambda_{\text{mom}}, \tag{20}$$

based on some 25 000 lattice configurations. Measurements of Γ_{2a}/Γ_a for our best operators give the mass

$$m(2^+) = (6.5^{+1.8}_{-1.1})\Lambda_{\text{mom}}. \tag{21}$$

The error quoted here is the error expected on the basis of the error observed in Γ_a/Γ_0: the apparent statistical error in Γ_{2a}/Γ_a appeared much smaller.

0^-: the most natural trial operator for the 0^- glueball is $F\tilde{F}$, the topological charge density, which has an $\boldsymbol{E}\cdot\boldsymbol{B}$ structure. Unfortunately the calculation of this object on the lattice is very slow, and the time required to obtain a reasonable signal/error ratio with $\bar{p} = 0$ wavefunctions would have been prohibitive. Accordingly we have used momentum smeared wavefunctions; specifically our wavefunction consisted of a nearest neighbour sum of $F\tilde{F}$ [+5] terms (all on a 6^4 lattice). We have calculated 2^+ energies in precisely the same way, and comparing with our 2^+ masses allows

us to unravel (approximately) the effects of this momentum smearing. Our mass upper bound from Γ_a/Γ_0 turns out to be

$$m(0^-) \leqslant (8.04 \pm 0.23)\Lambda_{\rm mom} \qquad (22)$$

and our mass estimate from Γ_{2a}/Γ_a gives $m(0^-) = (6.5 \pm 1.1)\Lambda_{\rm mom}$. In this case we find $\Gamma_{2a}/\Gamma_a \approx \Gamma_a/\Gamma_0$ which suggests that $F\tilde{F}$ is a good wave function. Observe that we find Γ_a/Γ_0 for 0^- to be larger than for our best 2^+ wave functions suggesting perhaps that the 0^- is lighter than the 2^+. We have also used the operators in figs. 1f–1h as trial wave functions for the 0^-; in these cases we used $\vec{p} = 0$ operators. We find that these are far worse wave functions than $F\tilde{F}$ for the lowest lying 0^-. The qualitative difference between say fig. 1h and $F\tilde{F}$ is that the former contains no $\boldsymbol{E}$ field, it contains pieces like B^3; and it may well be that the two classes of wave functions have their largest projections on different 0^- glueballs. Nonetheless Γ_{2a}/Γ_a should be much less sensitive to the quality of the wavefunction, and indeed we obtain a mass estimate $m(0^-) = (5.4 \pm 1.2)\Lambda_{\rm mom}$ consistent with that from $F\tilde{F}$. (Γ_{2a}/Γ_a was obtained on a 6^4 lattice at $\beta = 2.5$ to give a larger signal: in fact some of our other data was also taken at $\beta = 2.5$). Averaging for all our wave functions gives finally

$$m(0^-) = (6.0 \pm 1.0)\Lambda_{\rm mom}. \qquad (23)$$

2^-: from $\vec{p} = 0$ wave functions based on figs. 1f–1h we obtain only a crude upper bound

$$m(2^-) \leqslant (11.5 \pm 0.5)\Lambda_{\rm mom}. \qquad (24)$$

We expect a $F\tilde{F}$ type operator to do better, but we have no numbers yet.

3^+: based on figs. 1d,1e we obtain a crude upper bound

$$m(3^+) \leqslant (13 \pm 1)\Lambda_{\rm mom}. \qquad (25)$$

$1^\pm$: no useful numbers as yet.

We have expressed the masses in terms of $\Lambda_{\rm mom}$; using eq. (12) the reader may reexpress the masses in terms of the string tension. Because SU(2) is not the "correct" colour group, there is no obviously correct way to express our masses in GeV units. One possibility is to set the string tension equal to the string tension which gives unit Regge slopes (but note the obvious caveats). If we were to do so we would obtain

$$m(0^+) = 1.07 \pm 0.11 \text{ GeV},$$

$$m(0^-) = 1.77 \pm 0.3 \text{ GeV},$$

$$m(2^+) = 1.92\ {}^{+0.53}_{-0.32} \text{ GeV}.$$

In summary: we have applied a modified variational approach to the Monte Carlo generated vacuum of SU(2) lattice gauge theory to obtain the glueball masses and wave functions. Our results reported here are for 4^4, $4^3 \cdot 8$ and 6^4 lattices; however, the variation in going from 4^4 to 6^4 lattices is sufficiently small that we do not expect large changes in going to larger lattices. We are now calculating on an 8^4 lattice with a wider class of momentum smeared wave functions. Having established what are good wave functions we plan to extend our calculations to the SU(3) gauge theory. Nonetheless, the current folklore is that in going from 2 to 3 colours there should be no qualitative change in mass ratios, so that the numbers presented in this paper should be a guide to the true glueball spectrum.

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Note added in proof. It is interesting to note that the mass spectrum obtained here can also be accommodated in the bag model if hyperfine interactions are included [10].

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