# FOUR FAMILIES OF COMPOSITE QUARKS AND LEPTONS

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Four families of composite quarks and leptons, two standard and two non-standard, are found in a unique solution $SU(3)_{HC} \times SU(6)_L \times SU(6)_R$ of a restricted 't Hooft anomaly-matching program. Testable predictions emerge, such as prohibition of $\mu \to e\gamma$, zero charge asymmetry in $e^+e^- \to \tau^+\tau^-$ in contrast to $e^+e^- \to \mu^+\mu^-$, and a rich new hadron spectrum masses around $M_W$. A minimal set of spectator fermions contains color-singlet objects with fractional quark-like charges.

The motivation for considering quarks and leptons as composite fermions essentially comes from two sources entirely theoretical in nature. First of all, one is faced with the proliferation of quarks and leptons and their family and generation pattern. Secondly, the concept of dynamical composite Higgs scalars strongly suggests composite quarks and leptons in terms of the same elementary fermionic constituents. In a theory of composite quarks and leptons a "natural" mechanism [1] has to keep the quark and lepton masses very small compared to the "compositeness scale", $\Lambda_{HC} \gtrsim 1$ TeV. 't Hooft [1] required that the *chiral* hyperflavor symmetry $G_{HF}$, present on the level of massless elementary fermions (preons), "survive" the strong binding provided by some confining hypercolor gauge symmetry $G_{HC}$. Then $G_{HF}$ in turn naturally protects certain composite hypercolor singlet fermions from acquiring a mass. These massless composite fermions are then suitable candidates for quarks and leptons. The postulate of unbroken chiral $G_{HF}$ leads to 't Hooft's anomaly-matching equations [1] which strongly constrain the nature of $G_{HF}$ as well as the representations of the massless composite fermions with respect to $G_{HF}$.

In a recent publication [2] we imposed, in addition, a canonical ground-state criterion for massless composite fermions. The anomaly equations combined with the ground-state criterion turned out to be so restrictive that no solution exists in the simplest framework of an $SU(3)_{HC}$ hypercolor gauge theory with $N$ left-handed and $N$ right-handed massless hypercolor triplet preons:

$$G_{HC} \times G_{HF} = SU(3)_{HC} \times SU(N)_L \times SU(N)_R \times U(1)_{L+R}, \quad (1)$$

with preon content

$$P_L = (3; N, 1)_1, \quad P_R = (3; 1, N)_1. \quad (2)$$

Surprisingly a *unique* solution is found [2,3], however, if not the full $G_{HF}$, but only the *full chiral part* $SU(N)_L \times SU(N)_R$ is postulated to escape spontaneous symmetry breaking due to strong hypercolor binding – at the expense of spontaneous breakdown of the $U(1)_{L+R}$ of preon number conservation. This unique solution is simple and appealing:

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$$G_{HC} \times G_{HF} = SU(3)_{HC} \times SU(6) \times SU(6)', \tag{3}$$

with massless composite fermions

$$\psi = PP'^{+}P'^{+} = (\mathbf{1}; \square, \begin{smallmatrix}\square\\\square\end{smallmatrix}) = (\mathbf{1}; 6, 15),$$

$$\psi' = P'P^{+}P^{+} = (\mathbf{1}; \overline{\begin{smallmatrix}\square\\\square\end{smallmatrix}}, \overline{\square}) = (\mathbf{1}; \overline{15}, \overline{6}). \tag{4}$$

In eqs. (3) and (4) and throughout most of this paper we have adopted a purely left-handed notation:

$$SU(6)_L \times SU(6)_R \to SU(6) \times SU(6)', \tag{5}$$

with two-component left-handed Weyl spinors P, P′, $\psi$, $\psi'$

$$P_L \to P, \quad \psi_L \to \psi,$$

$$P_R \to (P_R)^c = P', \quad \psi_R \to (\psi_R)^c = \psi'. \tag{6}$$

In refs. [2,3] we have presented a possible multipreon condensate which could be responsible for the required spontaneous breakdown of $U(1)_{L+R}$. We also have pointed out [3] that the remaining chiral SU(6) symmetry strongly suppresses a coupling of the broken $U(1)_{L+R}$ Goldstone boson to matter.

This paper is devoted to relating this "abstract" solution to physical reality at present energies. In particular, we look for an embedding of the standard gauge groups into the hyperflavor group

$$SU(6) \times SU(6)' \supset SU(3)_c \times [SU(2) \times U(1)]_{WS}$$

$$\supset SU(3)_c \times U(1)_{em} \tag{7}$$

such that the massless composite fermion multiplets $\psi$ and $\psi'$ contain the known quarks and leptons. In ref. [3] where the possible embeddings were discussed systematically, the regular embedding turned out to be strongly favored, but the particle identification was still not unique. In this paper we shall take as a guiding principle the *requirement of identifying at least three families* of composite quarks and leptons. This, in fact, will lead us to a unique solution.

As we shall see, this unique identification shows a lot of nontrivial structure and leads to a wealth of predictions which is quite unusual for composite models. One of these predictions will soon be crucially tested at PETRA and PEP, others will be accessible at $\bar{p}p$ collider and LEP energies and most of them are independent of the specific size of the compositeness scale $\Lambda_{HC}$!

We consider [‡1] the following (regular) embedding [3]

$$SU(6) \times SU(6)'$$

$$\supset SU(3)^{(1)} \times SU(3)^{(1)'} \times SU(3)^{(2)}$$

$$\times SU(3)^{(2)'} \times U(1)_V \tag{8a}$$

$$\supset SU(3)_c \times SU(2) \times SU(2)'$$

$$\times U(1)_V \times U(1)_{V'}, \tag{8b}$$

where $SU(3)_c$ is the diagonal vectorlike subgroup of $SU(3)^{(1)} \times SU(3)^{(1)'}$ and $SU(2) \times SU(2)' \times U(1)_{V'}$ a subgroup of $SU(3)^{(2)} \times SU(3)^{(2)'}$. The preons decompose as

$$P \to \begin{cases} C = (3;3;1,1)_{-1/3} \to C = (3;3;1,1)_{-1/3,0}, \\ T = (3;1;3,1)_{1/3} \to \begin{cases} W = (3;1;2,1)_{1/3,-1/3}, \\ S = (3;1;1,1)_{1/3,2/3}, \end{cases} \end{cases}$$

$$P' \to \begin{cases} C' = (\bar{3};\bar{3};1,1)_{1/3} \to C' = (\bar{3};\bar{3};1,1)_{1/3,0}, \\ T' = (\bar{3};1;1,\bar{3})_{-1/3} \to \begin{cases} W' = (\bar{3};1;1,2)_{-1/3,1/3}, \\ S' = (\bar{3};1;1,1)_{-1/3,-2/3}, \end{cases} \end{cases} \tag{9}$$

with the simple notation C for *c*olor, T for *t*riplet, W for *w*eak and S for *s*inglet.

Table 1 presents the decomposition of the composite fermion multiplets $\psi$ and $\psi'$ with respect to the subgroups (8a) and (8b) of $SU(6) \times SU(6)'$. In identifying quarks and leptons and the physical symmetries we have the following freedom:

(i) Either SU(2) or SU(2)′ may be associated with $SU(2)_L$.

(ii) The physical U(1) quantum numbers may involve linear combinations of the quantum numbers $V$ and $V'$ associated with $U(1)_V \times U(1)_{V'}$. In particu-

‡1 All other regular embeddings of $SU(3)_c \times SU(2) \times SU(2)' \times U(1)^2$ into $SU(6) \times SU(6)'$ lead to equivalent results, if any two linear independent combinations of the two U(1) generators are admitted. As will become clear later, the embedding (8a,b) is distinguished as follows: the generators of $U(1)_V$ and $U(1)_{V'}$ are directly associated with physical quantum numbers and all leptons emerge from a single $SU(3)^{(2)} \times SU(3)^{(2)'}$ multiplet.

Table 1
$SU(6) \times SU(6)' \supset SU(3)_c \times SU(3) \times SU(3)' \times U(1)_V \supset SU(3)_c \times SU(2) \times SU(2)' \times U(1)_V \times U(1)_{V'}$ two-step decomposition of the $SU(6) \times SU(6)'$ multiplet $\psi = (6, 15) = P[P'^+P'^+]_{AS}$ of massless composite fermions; the table for $\psi' = (\overline{15}, \overline{6}) = P'[P^+P^+]_{AS}$ is obtained analogously.

| | | | | | |
|---|---|---|---|---|---|
| $\psi = (6,15) = PP'^+P'^+$ | $\to (1;3,\bar{3})_1 = TT'^+T'^+$ | $\boxed{(1;1,2)_{1,1} = SS'^+W'^+}$ $\equiv (\ell_R^{(1)})^c$ | $\boxed{(1;2,1)_{1,-1} = WW'^+W'^+}$ $\equiv \ell_L^{(2)}$ | $\boxed{(1;2,2)_{1,0} = WW'^+S'^+}$ $\equiv (\ell_R^{(3)})^c + \ell_L^{(4)}$ | $(1;1,1)_{1,0} = SW'^+W'^+$ |
| | $\to (3;3,3)_{1/3} = TT'^+C'^+$ | $\boxed{(3;2,2)_{1/3,-2/3} = WW'^+C'^+}$ $\equiv q_L^{(3)} + q_L^{(4)}$ | $\boxed{(3;2,1)_{1/3,1/3} = WS'^+C'^+}$ $\equiv q_L^{(2)}$ | $(3;1,2)_{1/3,1/3} = SW'^+C'^+$ | $(3;1,1)_{1/3,4/3} = SS'^+C'^+$ |
| | $\to (\bar{3};3,1)_{-1/3} = TC'^+C'^+$ | $(\bar{3};2,1)_{-1/3,-1/3} = WC'^+C'^+$ | $(\bar{3};1,1)_{-1/3,2/3} = SC'^+C'^+$ | | |
| | $\to (3;1,\bar{3})_{1/3} = CT'^+T'^+$ | $(3;1,2)_{1/3,1/3} = CW'^+S'^+$ | $(3;1,1)_{1/3,-2/3} = CW'^+W'^+$ | | |
| | $\to (\bar{3};1,3)_{-1/3} = CC'^+T'^+$ | $\boxed{(\bar{3};1,2)_{-1/3,-1/3} = CC'^+W'^+}$ $\equiv (q_R^{(1)})^c$ | $(\bar{3};1,1)_{-1/3,2/3} = CC'^+S'^+$ | | |
| | $\to (6;1,3)_{-1/3} = CC'^+T'^+$ | $(6;1,2)_{-1/3,-1/3} = CC'^+W'^+$ | $(6;1,1)_{-1/3,2/3} = CC'^+S'^+$ | | |
| | $\to (8;1,1)_{-1} + (1;1,1)_{-1} = CC'^+C'^+$ | $(8;1,1)_{-1,0} = CC'^+C'^+$ | $(1;1,1)_{-1,0} = CC'^+C'^+$ | | |

lar, the most general ansatz for the generator of the electromagnetic charge U(1) is given by

$$Q = \tfrac{1}{2}\ (\text{a linear combination of } V \text{ and } V') + I_{3L} + I_{3R}\ , \qquad (10)$$

with $I_{3L,R}$ being the standard diagonal generators of $SU(2)_{L,R}$.

(iii) Left-handed quarks and leptons may appear in the decomposition of $\psi$ as well as of $\psi'$.

As may be easily shown, a *unique* assignment for (i)–(iii) is obtained, if we require the *identification of at least three families of composite quarks and leptons* with correct color and electromagnetic charge quantum numbers: $SU(2)_L$ turns out to be associated with SU(2) and $SU(2)_R$ with SU(2)′, the electric charge is given by

$$Q = \tfrac{1}{2} V' + I_{3L} + I_{3R}\ , \qquad (11)$$

and the resulting quark and lepton spectrum is presented in table 2. The physical interpretation of $U(1)_V$ will be given later.

Let us discuss the important issues contained in table 2. The first two families, labeled by superscripts (1) and (2), are absolutely standard with respect to their color, electromagnetic charge, $SU(2)_L$ and hypercharge,

$$Y = V' + 2I_{3R}\ , \qquad (12)$$

assignments. In this case $V'$ is identical to what is generally identified with $B–L$, if $L(e^-) = L(\mu^-)$ is

Table 2
Two standard and two non-standard families of quarks and leptons given in terms of their preon content and classified according to $SU(3)_c \times SU(2)_L \times SU(2)_R \times U(1)_V \times U(1)_{V'}$. The index c means charge conjugation.

| left-handed quarks and leptons | electromagnetic charge $Q$ | right-handed quarks and leptons |
|---|---|---|
| $q_L^{(1)} = C'C^+W^+ = (3;2,1)_{1/3,1/3}$ | $(\frac{2}{3}, -\frac{1}{3})$ | $q_R^{(1)} = (CC'^+W'^+)^c = (3;1,2)_{1/3,1/3}$ |
| $q_L^{(2)} = WS'^+C'^+ = (3;2,1)_{1/3,1/3}$ | $(\frac{2}{3}, -\frac{1}{3})$ | $q_R^{(2)} = (W'S^+C^+)^c = (3;1,2)_{1/3,1/3}$ |
| $\ell_L^{(1)} = S'S^+W^+ = (1;2,1)_{-1,-1}$ | $(0, -1)$ | $\ell_R^{(1)} = (SS'^+W'^+)^c = (1;1,2)_{-1,-1}$ |
| $\ell_L^{(2)} = WW'^+W'^+ = (1;2,1)_{+1,-1}$ | $(0, -1)$ | $\ell_R^{(2)} = (W'W^+W^+)^c = (1;1,2)_{+1,-1}$ |
| $(q_L^{(3)}, q_L^{(4)}) = WW'^+C'^+ = (3;2,2)_{1/3,-2/3}$ | $((\frac{2}{3}, -\frac{1}{3}), (-\frac{1}{3}, -\frac{4}{3}))$ | $(q_R^{(3)}, q_R^{(4)}) = (W'W^+C^+)^c = (3;2,2)_{1/3,-2/3}$ |
| $((\ell_R^{(3)})^c, \ell_L^{(4)}) = WW'^+S'^+ = (1;2,2)_{1,0}$ | $((1,0), (0,-1))$ | $((\ell_L^{(3)})^c, \ell_R^{(4)}) = (W'W^+S^+)^c = (1;2,2)_{1,0}$ |

assumed. These two standard families are naturally identified with the first two generations of physical quarks and leptons. However, it is important to notice that at this level one still has the choice of identifying (u, d) with either $q^{(1)}$ or $q^{(2)}$ and, in principle, independently identifying $(\nu_e, e)$ with either $\ell^{(1)}$ or $\ell^{(2)}$.

The third and fourth family of quarks and leptons, labeled by superscripts (3) and (4), respectively, have standard color and electromagnetic charge assignments, except for the fourth family quark charges which are –1/3 and –4/3. Both families are *nonstandard* with respect to $SU(2)_L \times U(1)_Y$ in that the right-handed quarks and leptons also appear in $SU(2)_L$ doublets with the same hypercharges as the left-handed ones. The quantum number $V'$ correspondingly has lost its interpretation as $B-L$. The bottom quark b naturally fits into $q^{(3)}$. (An embedding of b into $q^{(4)}$ and paired with an exotic quark charge of –4/3 is probably already ruled out by the observed decay rate [4] for "b → c".) The $\tau$ lepton may be an element of either $\ell^{(3)}$ or $\ell^{(4)}$.

Can one survive with such a non-standard (t, b, $\nu_\tau$, $\tau$) family? So far, it seems the answer is "yes".

(1) The nature of the weak couplings of $q^{(3)}$ and $q^{(4)}$ will be V instead of V–A; this will be open to direct test above the top threshold. The V–A part of the b quark will mix with the d and s quarks.

(2) *The $\tau$ lepton necessarily has a V type coupling to the neutral weak current* resulting in the prediction of *zero charge asymmetry* [‡2] in $e^+e^- \to \tau^+\tau^-$. It is interesting that recent data [5] for the $\tau^+\tau^-$ charge asymmetry are indeed compatible with zero whereas, in contrast, the measured asymmetry in $e^+e^- \to \mu^+\mu^-$ is rather large. Within half a year higher precision data should provide a crucial test.

(3) As concerns the charged current, the measured $\tau$ coupling favors a V–A structure [6]. If, as usual, the right-handed tau neutrino $N_R$, is assumed to have a large Majorana mass ($\gtrsim$several hundred GeV), the right-handed $SU(2)_L$ doublet $(N_R, \tau_R)$ strongly decouples from the charged current at PETRA/PEP energies and an *effective V–A coupling* to $(\nu_L, \tau_L)$ is obtained [7].

We next identify the $U(1)_V$ characterized by the first subindex in tables 1 and 2. With the normalization chosen it is 1/3 for all four quark families, coinciding with their baryon number $B$. For the leptons it comes out non-zero and alternating

$$V = -1, +1, -1, +1 \quad \text{for } \ell^{(1)}, \ell^{(2)}, \ell^{(3)}, \ell^{(4)}, \tag{13}$$

respectively. It is somewhat unusual, albeit very beneficial as we shall see, to identify $V$ with a generalized baryon – lepton quantum number

$$V \equiv B - \sum_{i=1}^{4} L^{(i)}. \tag{14}$$

‡2 In lowest order and absence of mixing.

Eq. (13) then requires that the lepton numbers ($L^{(i)}$) of the four lepton families alternate in sign

$$L^{(1)} = L^{(3)} = +1, \quad L^{(2)} = L^{(4)} = -1. \tag{15}$$

In particular this implies

$$L(e^-) = -L(\mu^-). \tag{16}$$

Thus:

(i) $\mu \to e\gamma$ and other related rare processes such as $\mu^+ \to e^+e^-e^+$, $K_L \to e\bar{\mu}$, $K^+ \to \pi^+ e\bar{\mu}$, $\Sigma^+ \to pe\bar{\mu}$, $\mu^- S \to e^- S$ (S = sulfur) are strictly forbidden [‡3].

(ii) There is no $\mu$–e mixing.

(iii) $\mu^- Z(A) \to e^+ Z(A-2)$ and either $\tau \to e\gamma$ with $\tau$–e mixing, or $\tau \to \mu\gamma$ with $\tau$–$\mu$ mixing are allowed from the point of view of lepton number conservation. Since all four lepton families have different preon content, there will, however, be considerable dynamical suppression of these processes by inverse powers of $\Lambda_{HC}$.

The origin of the alternating lepton number rule (15) becomes transparent if the embedding (8a,b) of $SU(2) \times SU(2)' \times U(1)_{V'}$ in $SU(3) \times SU(3)'$ is considered. All four lepton families together with one additional heavy $SU(2) \times SU(2)'$ singlet lepton $\ell_h$ are contained in one $SU(3) \times SU(3)'$ composite fermion multiplet of the form (c.f. table 1)

$$\psi' \to T'T^+T^+ = (1;3,\bar{3})_{-1}$$
$$\hat{=} (\ell^{(1)}, \ell^{(2)\prime}, \ell^{(3)}, \ell^{(4)\prime}, \ell_h)_{-1},$$
$$\psi \to TT'^+T'^+ = (1;3,\bar{3})_{+1}$$
$$\hat{=} (\ell^{(1)\prime}, \ell^{(2)}, \ell^{(3)\prime}, \ell^{(4)}, \ell_h')_{+1}, \tag{17}$$

carrying a *single* generalized $B-L$ quantum number $V = \mp 1$. [Recall the correspondence $\ell^{(i)} \leftrightarrow \ell_L^{(i)}$, $\ell^{(i)\prime} \leftrightarrow (\ell_R^{(i)})^c$]. Alternating lepton numbers $L(\mu^-) = -L(e^-)$ in the framework of a unifying $SU(3) \times SU(3)'$ theory have also been obtained years ago by Weinberg [8], though with a different representation content of the leptons.

Besides the four quark–lepton families the following composite fermions are contained in $\psi$ and $\psi'$ (c.f. table 1):

(i) Three families of mirror quarks

$$q_{3M} = (3;1,2)_{1/3,1/3}, \quad q'_{3M} = (\bar{3};2,1)_{-1/3,-1/3} \tag{18}$$

and one family of color sextet quarks

$$q_6 = (\bar{6};2,1)_{1/3,1/3}, \quad q_6' = (6;1,2)_{-1/3,-1/3}. \tag{19}$$

At the level of $SU(2) \times SU(2)'$ they are protected by this symmetry from "pairing off" into massive Dirac fermions. Correspondingly, their masses will be $\lesssim O(M_W)$, the breaking scale of $SU(2)_L$, and a rich new hadronic spectroscopy is predicted to show up in $e^+e^-$ and purely hadronic reactions, such as $\bar{p}p$.

(ii) $SU(2) \times SU(2)'$ singlets, which can become massive [‡4] by "pairing off" already well above O(100 GeV): two pairs of (neutral) color singlet leptons, four pairs of color triplet quarks, one pair of color sextet quarks and one pair of (neutral) color octet leptons (!).

Next, let us mention an amusing observation. As is well known, but rarely commented on in papers on composite models, anomaly freedom with respect to the hyperflavor group $G_{HF}$ requires the presence of hypercolor singlet "spectators" along with the preons [1]. In our case of $SU(3)_{HC} \times SU(6) \times SU(6)'$, the only spectator representations which are physically acceptable are

either

$$3 \times (1;\bar{6},1) \quad \text{and} \quad 3 \times (1;1,6), \tag{20a}$$

or

$$(1;\bar{6},1) \oplus (1;\overline{15},1) \quad \text{and} \quad (1;1,6) \oplus (1;1,15). \tag{20b}$$

In a decomposition of $(1;\bar{6},1)$ and $(1;1,6)$ with respect to $SU(3)_c \times SU(2) \times SU(2)' \times U(1)_V \times U(1)_{V'}$, the only occurring $SU(2) \times SU(2)'$ non-singlets are $(1;2,1)_{-1/3,-1/3}$ and $(1;1,2)_{1/3,1/3}$ with electric charges $(1/3, -2/3)$ and $(2/3, -1/3)$ and $V$ = "$B-L$" quantum numbers $-1/3$ and $+1/3$, respectively. These spectator fermions have *quark-like charge and baryon number*, but they are *color singlets*. Thus, they represent perfect candidates for the tantalizing objects reported by Fairbank et al. [9].

Finally, let us comment about proton decay. The relevant transition is

$$q_L^{(1)} + q_L^{(2)} \to (\ell_R^{(1)})^c + (q_R^{(2)})^c. \tag{21}$$

According to table 1 it may occur via preon rearrange-

‡3 Apart from a slight lepton-number violation due to large Majorana masses for right-handed neutrinos.

‡4 Discrete symmetries may prevent the pairing off of some $SU(2) \times SU(2)'$ singlets.

ment, has $\Delta B \neq 0$, $\Delta(B - \Sigma L^{(i)}) = 0$ and necessarily involves a generation flip. Via additional weak interaction and unitarity it gives rise to proton decay. Thus it seems there is some, but not a very strong dynamical suppression of proton decay (the details depend on the identification of (u, d) and ($\nu_e$, $e^-$) among the $q^{(1,2)}$ and $\ell^{(1,2)}$ composites, respectively, and will be discussed elsewhere). Consequently, the compositeness scale $\Lambda_{HC}$ will have to be chosen uncomfortably high, unless one manages to invoke further selection rules, e.g., due to discrete (axial) symmetries [10]. On the other hand, this result should be seen in the light of recent arguments [11], according to which *any* generic composite model has to face high values of $\Lambda_{HC}$, irrespective of proton decay, because of the problem of baryon asymmetry in the universe. In fact, the present model automatically provides two (dynamical) Higgs scalars ($P^6$ and $P'^6$ responsible [2,3] for the required spontaneous breaking of the preon number U(1)), with exactly the properties needed, according to ref. [11], for a generation of sufficient baryon asymmetry. Apart from carrying preon number they are overall singlets, are very long-lived, have mass $O(\Lambda_{HC})$ and contribute only weakly to proton decay.

In concluding, let us emphasize that the choice of $\Lambda_{HC}$ does *not* affect the predictions contained in this paper, in particular the prohibition of $\mu \to e\gamma$ and related processes, vanishing charge asymmetry in $e^+e^- \to \tau^+\tau^-$, and the new hadron spectrum with masses $\lesssim O(M_W)$. They all depend on the "quantum numerology" aspect of our composite model only and not on the detailed preon dynamics around $\Lambda_{HC}$.

Irrespective of how realistic this composite model may be, we feel it is quite instructive: although much of the dynamics remains unknown in a 't Hooft type framework, the present analysis demonstrates that firm and testable predictions may emerge from it together with a wealth of non-trivial structure.

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