

SU(3) LATTICE MONTE CARLO CALCULATION OF THE GLUEBALL MASS SPECTRUM

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We have calculated the glueball masses of various spins and parities in SU(3) gauge theory. Our first results give $m_{\mathbf{M}}(0^{++}) = (3.6 \pm 0.2)\Lambda_{\text{mom}}$, $m_{\mathbf{E}}(0^{++}) = (4.3 \pm 0.3)\Lambda_{\text{mom}}$, $m(0^{-+}) = (7.2_{-0.9}^{+1.6})\Lambda_{\text{mom}}$, $m_{\mathbf{M}}(2^{++}) = (8.1 \pm 1.1)\Lambda_{\text{mom}}$ and $m_{\mathbf{E}}(2^{++}) = (8.3_{-1.0}^{+1.6})\Lambda_{\text{mom}}$ as well as information on the glueball wave functions.

In this letter we extend our recent calculation of the glueball mass spectrum [1] to the gauge group SU(3) (i.e., QCD).

We follow the calculational procedure outlined in our previous letter [1]. That is, we perform a variational calculation by choosing some a priori reasonable class of wave functions $\{\phi\}$ and vary ϕ within that class to maximize the expectation value

$$\frac{\Gamma_a}{\Gamma_0} = \frac{\langle \phi_{\mathbf{P}=0}(t+a)\phi_{\mathbf{P}=0}(t) \rangle}{\langle \phi_{\mathbf{P}=0}(t)\phi_{\mathbf{P}=0}(t) \rangle} = \frac{\langle \phi_{\mathbf{P}=0}(t)e^{-Ha}\phi_{\mathbf{P}=0}(t) \rangle}{\langle \phi_{\mathbf{P}=0}(t)\phi_{\mathbf{P}=0}(t) \rangle} \\ = \frac{\sum_{n=0}^{\infty} e^{-m_n a} |\langle n | \phi_{\mathbf{P}=0}(t) | \Omega \rangle|^2}{\sum_{n=0}^{\infty} |\langle n | \phi_{\mathbf{P}=0}(t) | \Omega \rangle|^2}, \quad (1)$$

where a is the lattice spacing and t the euclidean time. The resulting wave function is then used to calculate

$$\frac{\Gamma_{2a}}{\Gamma_a} = \frac{\langle \phi_{\mathbf{P}=0}(t+2a)\phi_{\mathbf{P}=0}(t) \rangle}{\langle \phi_{\mathbf{P}=0}(t+a)\phi_{\mathbf{P}=0}(t) \rangle} = \frac{\langle \phi_{\mathbf{P}=0}(t)e^{-H2a}\phi_{\mathbf{P}=0}(t) \rangle}{\langle \phi_{\mathbf{P}=0}(t)e^{-Ha}\phi_{\mathbf{P}=0}(t) \rangle} \\ = \frac{\sum_{n=0}^{\infty} e^{-m_n 2a} |\langle n | \phi_{\mathbf{P}=0}(t) | \Omega \rangle|^2}{\sum_{n=0}^{\infty} e^{-m_n a} |\langle n | \phi_{\mathbf{P}=0}(t) | \Omega \rangle|^2}, \quad (2)$$

the logarithm of which will give us the lowest-lying

glueball mass ($m = m_0$):

$$m = -a^{-1} \ln(\Gamma_{2a}/\Gamma_a). \quad (3)$$

The idea behind this procedure is that (i) increasing the theoretical accuracy involves searching for a larger signal and (ii) the higher mass states in (2) are double suppressed by the choice of the wave function *and* the exponential damping factor

It follows that (1) provides an *upper* bound on the mass:

$$-a^{-1} \ln(\Gamma_a/\Gamma_0) \geq m. \quad (4)$$

The *pure* variational calculations of the glueball mass(es) [2,3] assume that this upper bound will be close to the actual mass(es). How much this is actually so for the rather small lattices and the limited class of wave functions considered can be checked [1] and will be discussed further along with the results.

We work on a $4^3 \cdot 8$ lattice with 8 lattice points in the time direction. From our previous experience [1], where we have considered 4^4 , $4^3 \cdot 8$, 6^4 and 8^4 lattices, we infer that the $4^3 \cdot 8$ lattice is large enough for our purposes provided that the coupling parameter β is chosen appropriately. Working on a "rectangular" lattice has the further advantage that it lowers the physical temperature at nearly the same computer

cost and, hence, better approximates zero temperature QCD.

An appropriate value of β is one for which

$$a < \text{glueball size} < 2a. \quad (5)$$

To two loops the lattice spacing is given by ($\beta = 6/g^2$)

$$a = (83.5/\Lambda_{\text{mom}})(\frac{8}{33}\pi^2\beta)^{51/121} \exp(-\frac{4}{33}\pi^2\beta), \quad (6)$$

where the overall scale is set by the string tension $\dagger 1$ [we "measure" $\Lambda_{\text{mom}} = (0.48 \pm 0.05)\sqrt{K}$ at $\beta = 5.7$].

$$\Lambda_{\text{mom}} \approx 0.5\sqrt{K}, \quad (7)$$

which we believe is $\sqrt{K} \approx 400$ MeV, corresponding to unit Regge slope. Assuming the the glueball size to be of the order of 0.5 fermi (which proves to be correct in the actual calculation), this constrains β to the region $5 \lesssim \beta \lesssim 6$. We also require that β lies in the continuum region. Leaning on the results of ref. [4] we infer that $5 \lesssim \beta$ is a suitable value.

The occurrence of a sharp peak in the specific heat at $\dagger 2$ $\beta = 5.45 \pm 0.1$ and a rapid increase in the correlation length near this peak, which has been attributed to nearby complex β plane singularities [6], cuts the range of admissible values of β further down to $5.6 \lesssim \beta$. We have taken "data" at $\beta = 5.7$ and $\beta = 5.9$ which we have checked lie in the scaling region. Incidentally, the authors in ref. [3] have taken most of their "data" below and at the peak in the specific heat which casts some doubt on their results.

We have constructed states ϕ with $J^{PC} = 0^{++}, 0^{-+}, 1^{--}, 1^{+-}$ and 2^{++} which we expect to be low-lying. Except for 0^{-+} they can be composed of planar loops with purely space-like links ($0^{++}, 1^{+-}, 2^{++}$) or space- and time-like links ($0^{++}, 1^{--}, 2^{++}$) as shown schematically in fig. 1. To project onto zero momentum states the sum is to be taken over all spacial lattice points. The loops that we have considered in our variational calculation are shown in fig. 2. We have confined ourselves to loops that extend only one lattice spacing in time (\mathcal{O}_4 and \mathcal{O}_5 , cf. fig. 2). That is to re-

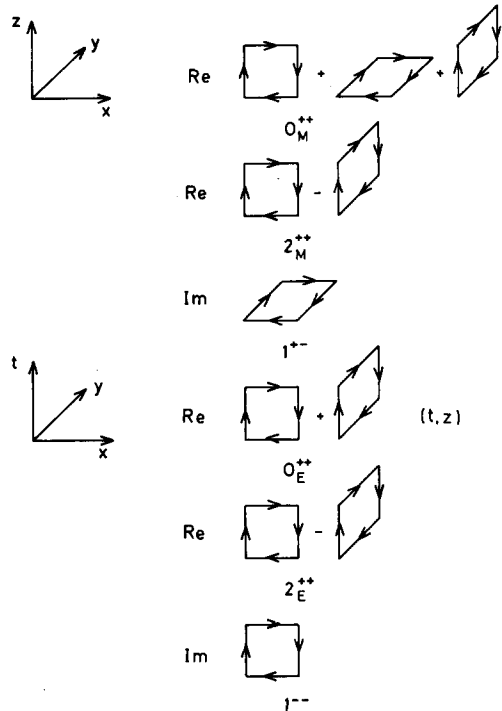


Fig. 1. $0^{++}, 2^{++}, 1^{+-}$ and 1^{--} wave functions composed of planar loops. The loops are understood to be summed over all space-like translations. The subscripts M and E stand for magnetic and electric, respectively.

duce the amount of time-like overlap in Γ_a and to avoid time-like overlaps in Γ_{2a} at all which, otherwise, could lead us into conflict with physical positivity [7]. Note that for 0^{++} and 2^{++} two types of different operators arise: magnetic ($\sim B^2$) with exclusively space-like extensions and electric ($\sim E^2$) extending also in time

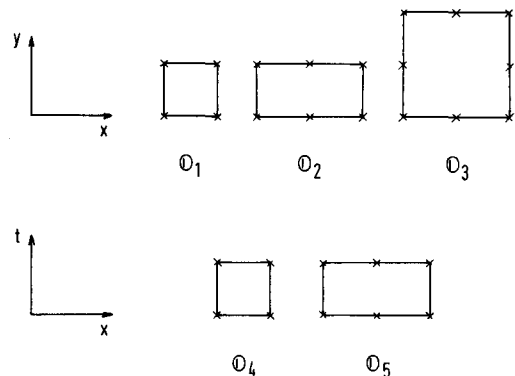


Fig. 2. Loops considered in our calculation (x, y stand for any space direction).

$\dagger 1$ We haven taken the mean value of the string tensions "measured" by the authors in ref. [4].

$\dagger 2$ This has been obtained by a straight-line interpolation $1/g^2 \sim N$, N being the number of colours, of the SU(2), SU(4), SU(5) and SU(6) peak/phase transition values reported in ref. [5]. Note that they fall very nicely on a straight line.

(cf. fig. 1). This doubling of glueball states has also been found in the bag model [8]. The most natural trial wave function for 0^{-+} is $F\tilde{F}$, the topological charge density, which is of the type $\mathbf{E} \cdot \mathbf{B}$. We have taken $\sum_{i,j,k=\pm 1}^{\pm 3} \tilde{\epsilon}_{0ijk} F^{0i} F^{jk}$ instead which extends only one lattice spacing in the time direction in contrast to $F\tilde{F}$ (which does two) but else has the same quantum numbers and topology. The reason is to secure physical positivity [7] as mentioned before. There are various ways of implementing this operator on the lattice. We use the definition employed in ref. [9].

We have performed several independent lines of iterations of the lattice configuration using the heat-bath method [10]. Before taking any "data" we have heated up the lattice starting from cold ($\beta = \infty$) and hot ($\beta = 0$) starts. So far we have collected ≥ 24000 "events" at $\beta = 5.7$ and ≥ 7000 "events" at $\beta = 5.9$. Our first results for SU(3) glueball masses and wave functions are summarized below. A more detailed presentation will be given elsewhere.

0^{++} . As 0^{++} has vacuum quantum numbers, it is necessary to replace ϕ by $\phi - \langle \phi \rangle$ in (1) and (2) to ensure that the vacuum state does not appear as intermediate state.

We first treat the magnetic and electric 0^{++} glueball states independently, i.e., not accounting for any mixing between them.

(a) Magnetic. After maximization we obtain at $\beta = 5.7$

$$\Gamma_a/\Gamma_0 = 0.22 \pm 0.01, \quad (8)$$

which, according to (4), (6), corresponds to the upper bound

$$m_M(0^{++}) \leq (5.52 \pm 0.18)\Lambda_{\text{mom}} \quad (9)$$

(where the subscript M stands for magnetic). The accompanying best wave function is (cf. fig. 2)

$$\phi_M(0^{++}) \approx 0.33 O_1 + 0.40 O_2 + 0.85 O_3, \quad (10)$$

where the operators on the right-hand side are understood to be summed over all orientations in accord with fig. 1 and over the spacial lattice for any fixed t . For ease of writing we have dropped the subscript $P = 0$.

In the next step we compute Γ_{2a}/Γ_a now using the wave function (10). We find

$$\Gamma_{2a}/\Gamma_a = 0.37 \pm 0.02, \quad (11)$$

which gives the mass

$$m_M(0^{++}) = (3.6 \pm 0.2)\Lambda_{\text{mom}}. \quad (12)$$

We observe that the straightforward variational calculation would have yielded a considerably higher 0^{++} glueball mass (9) which disqualifies the pure variational approach. This is not surprising as the class of possible wave functions is rather limited for such a small lattice.

(b) Electric. Our best wave function is ($\beta = 5.7$; cf. fig. 2)

$$\phi_E(0^{++}) \approx 0.42 O_4 + 0.9 O_5 \quad (13)$$

(where the subscript E stands for electric) which corresponds to

$$\Gamma_a/\Gamma_0 = 0.16 \pm 0.01 \quad (14)$$

(note that Γ_a is positive in line with physical positivity [7]). This results in the upper bound

$$m_E(0^{++}) \lesssim (6.68 \pm 0.22)\Lambda_{\text{mom}}. \quad (15)$$

Using the optimized wave function (13), we then obtain

$$\Gamma_{2a}/\Gamma_a = 0.31 \pm 0.02, \quad (16)$$

which gives the mass

$$m_E(0^{++}) = (4.3 \pm 0.3)\Lambda_{\text{mom}}. \quad (17)$$

Note that also in this case the pure variational calculation cannot be trusted.

(c) Mixing. In reality the magnetic and electric glueball operators will mix. Accordingly, we write

$$\phi = \cos \Theta \phi_M + \sin \Theta \phi_E. \quad (18)$$

The physical eigenstates and mass eigenvalues of the transfer matrix are then obtained by searching for two orthogonal wave functions $\phi_{1,2}$ that maximize and minimize, respectively, Γ_{2a}/Γ_a . We have attempted this. But, due to the relatively large errors, we have no reliable numbers yet and even cannot tell with certainty that there are two states (or just one), not to mention the additional mixing with quark-antiquark states. We postpone the details to a forthcoming publication.

At $\beta = 5.9$ our best wave functions have a rather small projection onto the lowest lying state which results in a large error of Γ_{2a}/Γ_a .

2^{++} . We now go through the same analysis for 2^{++} .

(a) Magnetic. At $\beta = 5.7$ the best wave function is (cf. fig. 2)

$$\phi_M(2^{++}) \approx 0.18 O_1 + 0.46 O_2 + 0.87 O_3 \quad (19)$$

with

$$\Gamma_a/\Gamma_0 = 0.058 \pm 0.002, \quad (20)$$

which gives the upper bound

$$m_M(2^{++}) \leq (10.3 \pm 0.2)\Lambda_{\text{mom}}. \quad (21)$$

Using (19) we then obtain

$$\Gamma_{2a}/\Gamma_a = 0.11 \pm 0.03, \quad (22)$$

which gives

$$m_M(2^{++}) = (8.0 \pm 1.2)\Lambda_{\text{mom}}. \quad (23)$$

Our results at $\beta = 5.9$ (with $\sim 1/4$ the statistics) agree very well with that. The optimal wave function is

$$\phi_M(2^{++}) \approx 0.14 O_1 + 0.34 O_2 + 0.93 O_3 \quad (24)$$

and gives only a slightly higher bound on m while

$$\Gamma_{2a}/\Gamma_a = 0.16 \pm 0.03, \quad (25)$$

which results in

$$m_M(2^{++}) = (8.4 \pm 1.0)\Lambda_{\text{mom}}. \quad (26)$$

This is to say that Γ_{2a}/Γ_a follows nicely the weak coupling renormalization group trajectory.

By taking the statistical average of (23) and (26) we finally obtain the mass

$$m_M(2^{++}) = (8.1 \pm 1.1)\Lambda_{\text{mom}}. \quad (27)$$

(b) Electric. At $\beta = 5.7$ the best wave function is (cf. fig. 2)

$$\phi_E(2^{++}) \approx O_5 \quad (28)$$

corresponding to

$$\Gamma_a/\Gamma_0 = 0.022 \pm 0.002 \quad (29)$$

(note that Γ_a is positive in line with physical positivity [7]) which leads to the bound

$$m_E(2^{++}) \leq (13.9 \pm 1.0)\Lambda_{\text{mom}}. \quad (30)$$

From (28) we then obtain

$$\Gamma_{2a}/\Gamma_a = 0.11 \pm 0.06 \quad (31)$$

and

$$m_E(2^{++}) = (8.0^{+2.9}_{-1.6})\Lambda_{\text{mom}}. \quad (32)$$

At $\beta = 5.9$ (with roughly the same statistics) we find the optimal wave function to be the same and

$$\Gamma_{2a}/\Gamma_a = 0.15 \pm 0.04, \quad (33)$$

which leads us to

$$m_E(2^{++}) = (8.7^{+1.4}_{-0.9})\Lambda_{\text{mom}} \quad (34)$$

in good agreement with scaling.

The statistical average of (32) and (34) gives finally the mass

$$m_E(2^{++}) = (8.3^{+1.6}_{-1.0})\Lambda_{\text{mom}}, \quad (35)$$

(c) Mixing. Our data do not allow any conclusions yet as to the mixing between the magnetic and electric 2^{++} glueball operators.

0^{-+} . In this case physical positivity [7] demands that the (naive) correlation functions Γ_a , Γ_{2a} , etc. are negative because the proper positive metric involves reflecting the time in one of the $\Sigma_{i,j,k=\pm 1}^{\pm 3} \tilde{\epsilon}_{0ijk} F^{0iFjk}$. We have found that Γ_a (and Γ_{2a} , Γ_{3a} , though these have no time-like overlaps) is (are) indeed negative.

Since the calculation of $\Sigma_{i,j,k=\pm 1}^{\pm 3} \tilde{\epsilon}_{0ijk} F^{0iFjk}$ on the lattice is very slow and the time required to obtain a reasonable signal/error ratio for $\mathbf{P} = \mathbf{0}$ wave functions is outside the present bounds of possibility, we have used momentum smeared wave functions here {as already before in our previous letter [1] on SU(2)}. That is, instead of summing the wave functions individually over all spacial lattice points to project onto zero momentum states, only nearest neighbour correlations are taken into account. We have calculated 0^{++} and 2^{++} energies in exactly the same way and, by comparing with the 0^{++} glueball mass stated above (2^{++} is less suited because of the relatively large errors), corrected for the effect of momentum smearing. We found consistently $\langle \mathbf{P}^2 \rangle \approx 4/a^2$.

All our "measurements" are at $\beta = 5.7$. From $|\Gamma_a/\Gamma_0|$ we obtain the mass upper bound

$$m(0^{-+}) \leq (8.46 \pm 0.20)\Lambda_{\text{mom}}, \quad (36)$$

while Γ_{2a}/Γ_a yields the mass estimate

$$m(0^{-+}) = (7.2^{+1.6}_{-0.9})\Lambda_{\text{mom}}. \quad (37)$$

The proximity of (36) and (37), which is a reflection

of $\Gamma_{2a}/\Gamma_a \approx |\Gamma_a/\Gamma_0|$, suggests that $\sum_{i,j,k=\pm 1}^{\pm 3} \tilde{\epsilon}_{0ijk} \times F^{0iFjk}$ is a good 0^{-+} wave function.

I^{+-}, I^{--} . We have no useful bounds and numbers yet in this case.

So far we have expressed the glueball masses in units of Λ_{mom} . To make contact to experiment [11] we may use (7) now to fix the absolute scale (but note the obvious caveats). If we do so, we obtain the mass spectrum

$$\begin{aligned} m_{\text{M}}(0^{++}) &= (720 \pm 40) \text{ MeV} , \\ m_{\text{E}}(0^{++}) &= (850 \pm 50) \text{ MeV} , \\ m(0^{-+}) &= (1430^{+320}_{-180}) \text{ MeV} , \\ m_{\text{M}}(2^{++}) &= (1620 \pm 220) \text{ MeV} , \\ m_{\text{E}}(2^{++}) &= (1670^{+320}_{-200}) \text{ MeV} \end{aligned} \quad (38)$$

[where we have ignored the uncertainties in (7)].

To summarize, we have reported first results of a calculation of the low-lying glueball masses based on altogether $\gtrsim 31\,000$ Monte Carlo "events". Our aim for the future is to reduce the errors and to make a more systematic survey of the low-lying glueball spectrum, in particular: to look for "oddballs".

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