AN $O(\alpha_s)^2$ CALCULATION OF ENERGY–ENERGY CORRELATION IN e^+e^- ANNIHILATION AND COMPARISON WITH EXPERIMENTAL DATA

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We report, in the framework of quantum chromodynamics, an order α_s^2 calculation for the energy–energy correlation and the related asymmetry distribution in e^+e^- annihilation. The normalized distributions are found to be stable and the correction to normalization moderate. The data from PETRA and PEP are analyzed and the QCD scale parameter Λ is determined. Effects due to the quark and gluon fragmentation are also discussed.

1. e^+e^- annihilation at the storage rings PETRA and PEP provide hadronic states at very high energy. There is firm consensus that these final hadronic states arise due to the production of quarks and gluons which are highly inelastic (far off-shell) and one could quantitatively calculate the effective interaction between them. Quantum chromodynamics, QCD, provides the framework and the calculations are done using perturbation theory in the effective coupling constant $\alpha_s(Q^2)$, which is assumed to be small.

2. A crucial task in e^+e^- annihilation today is to measure the effective coupling constant $\alpha_s(Q^2)$ and hence determine the QCD scale parameter, Λ [1]. It is generally known that a theoretically meaningful determination of Λ demands a calculation at the next to leading order of the quantity being analysed. The list of such calculations is impressive and growing. However, only in very few circumstances are these perturbation theory results, obtained at the parton level, directly applicable to the experimental situation. The quarks and gluons have to be exorcised from the colorless world! Very often these effects are large and therefore turn the endeavor of determining $\alpha_s(Q^2)$ into a rather model dependent enterprise.

This is an unpleasant aspect of confinement: the perturbation theory calculations very often need an algorithm in order to compare them with the experiment [1,2]. The point is that one could calculate varying rates and distribution for final states in configurations which differ substantially from each other at the *parton level,* but can be made to look alike *after* fragmentation. A closer look indicates that this variation mainly comes from relatively soft and collinear configurations, where one is apt to distrust perturbation theory. Thus, a reliable measurement of $\alpha_s(Q^2)$ demands not only calculation of a finite quantity, in the mathematical sense of a distribution, but also a mathematically well defined procedure which tempers the rapid variation in the soft regions in an *unambiguous* way.

3. Some time ago Basham, Brown, Ellis and Love [3] (BBEL) advocated the measurement of energy–energy correlation in e^+e^- annihilation as a possible test of perturbative QCD. Experimentally, one measures the energy weighted correlation defined as

$$\frac{d^2\sigma}{d\Omega\, d\Omega'} = \sum_{N=2}^{\infty} \prod_{a=1}^{N} E_a^{-1}\, d^3p_a \frac{d^N\sigma}{E_1^{-1} d^3p_1 \dots E_N^{-1} d^3p_N}$$

$$\times\left(\sum_{b,c=1}^{N} (E_b E_c/W^2)\delta(\Omega_b - \Omega)\delta(\Omega_c - \Omega')\right),$$

$$W = E_{\mathrm{cm}}, \qquad (1)$$

where the sum runs over all the particles with appropriate statistical weight factor S_N. Finally one sums over all the events. Division by the e^+e^- flux then provides the normalization. One then calculates the same quantity for quarks and gluons in perturbative QCD. The collinear configurations can be removed by looking at angles χ (between quarks and gluons) such that $\chi \neq 0°$, 180° and the rapid variation of the QCD matrix elements is tempered in the soft region due to energy weighting. One expects to get not only finite results but results which are also expected to be stable against higher order corrections. We have, therefore, calculated eq. (1) in order α_s^2 in QCD.

In the lowest non-trivial order the energy–energy correlation was obtained by BBEL [3]. The result can be expressed as

$$(1/\sigma_0)\, d\sigma_{\mathrm{corr}}/d\cos\chi\, d\cos\theta = [3\alpha_s(Q^2)/\pi]$$

$$\times\ [A(\xi) + \cos^2\theta\, B(\xi)],$$

$$\xi = \tfrac{1}{2}(1 - \cos\chi), \qquad (2)$$

where χ is the angle between the two detectors centered at Ω and Ω' in (1) and θ is the polar angle of one of them with respect to the beam axis. The functions $A(\xi)$ and $B(\xi)$ are given in ref. [3]. A related quantity is the cross section asymmetric under the exchange $\chi \to \pi - \chi$. In lowest order perturbation theory, the distribution (2) is asymmetric.

$$(1/\sigma_0)\, d\sigma_{\mathrm{asym}}/d\cos\chi \equiv [\alpha_s(Q^2)/\pi]\, \mathcal{A}(\cos\chi), \qquad (3)$$

where

$$\mathcal{A}(\cos\chi) = F(\cos(\pi - \chi)) - F(\cos\chi),$$

$$F(\cos\chi) = \tfrac{2}{3}[3A(\cos\chi) + B(\cos\chi)].$$

In order α_s^2, the cross sections $\sigma_{\mathrm{corr}}(\chi)$ and $\sigma_{\mathrm{asym}}(\chi)$ for $\chi \neq 0°$, 180° receive additional contributions from the following processes:

(i) $e^+e^- \to q\bar{q}GG,\ q\bar{q}q\bar{q}$,
(ii) Virtual corrections to $e^+e^- \to q\bar{q}G$.

The calculations for the process (i) have been in the literature for quite some time [4]. The calculation of matrix elements in (ii) [which therefore by definition also includes the divergent parts in (i)] have been obtained by Ellis et al. [5] and by Fabricius et al. [1]. ERT present their result in a form particularly useful to us. Recently this part has also been checked by Gottschalk [2] and others [6]. We have made use of these calculations and obtained the cross sections

$$(1/\sigma)\, d\sigma_{\mathrm{corr}}/d\cos\chi \quad \text{and} \quad (1/\sigma)\, d\sigma_{\mathrm{asym}}/d\cos\chi,$$

The calculations reported here were done numerically using Monte Carlo techniques. Since we make use of the virtual corrections from the work of ERT, all results are specific to the so-called $\overline{\mathrm{MS}}$ regularization scheme [7]. The finite $O(\alpha_s)^2$ correction for $e^+e^- \to q\bar{q}G$ was obtained using a prescription originally due to Kunszt [8] and recently employed by one of us [9,1] and Gottschalk [2]. One defines

$$d\sigma_3 = d\sigma_{\mathrm{vir}} + \int_0^{y_{\min}} dy_{ij}\, \frac{d\phi}{\pi}\, [d\sigma_4(y_{ij}^{-1}\ \text{piece})], \qquad (4)$$

with $y_{ij} = (p_i + p_j)^2/s$, as a finite three-parton (massless) cross section, identified through the cut on an (scaled) invariant mass resolution parameter, $y_{\min}$. The integral in (4) is done in two steps:

$$d\sigma_4 = d\sigma_4^{\mathrm{s}} + [d\sigma_4 - d\sigma_4^{\mathrm{s}}]. \qquad (5)$$

The singular piece $d\sigma_4^{\mathrm{s}}$ is obtained by integrating analytically [see eq. (3.17) of ERT] and the rest numerically. The integral for the square bracket was done in the region $y_{\min} > y_{ij} > y_0$ with $y_0 = 10^{-7}$ and $y_{\min} = 10^{-3}$. It has been checked by one of us (A.A.) [1] and independently by Gottschalk [2] that these limits reproduce the ERT calculations for the thrust [10] and the Fox–Wolfram C-distribution [11]. We have in addition checked that the energy–energy correlations depend on the $y_{\min}$ cut but the cross sections become almost independent of $y_{\min}$ for $y_{\min} \geqslant 10^{-3}$ [‡1]. Of course, one could have worked directly with the ERT calculations, but with an eye on the fragmentation procedure, we prefer the method of a $y_{\min}$ cut-off. We emphasize that the four-jet kinematics for $y_{ij} > y_{\min}$ ($i \neq j, i, j = 1, \ldots 4$) is treated *exactly* without any recombination [‡2]. It is important to realize that we are directly calculating $O(\alpha_s)^2$ corrections to the *BBEL cross sections* themselves.

The order α_s^2 result for the energy–energy correlation can be expressed as

$$(1/\sigma_0)\, d\sigma_{\rm corr}^{(2)}/d\cos\chi = [\alpha_s(Q^2)/\pi]$$

$$\times\ \{F(\cos\chi) + [\alpha_s(Q^2)/\pi]\ G(\cos\chi)\}\ , \qquad (6)$$

and the asymmetry cross section is given by

$$(1/\sigma_0)\, d\sigma_{\rm asym}^{(1)}/d\cos\chi = [\alpha_s(Q^2)/\pi]$$

$$\times\ \{\mathcal{A}(\cos\chi) + [\alpha_s(Q^2)/\pi]\ \mathcal{B}(\cos\chi)\}. \qquad (7)$$

We work with massless quarks, hence the functions $F(\chi)$ $\mathcal{A}(\chi)$, $G(\chi)$ and $\mathcal{B}(\chi)$ are all scale-invariant, with $G(\chi)$ and $\mathcal{B}(\chi)$ specific to the $\overline{\rm MS}$ subtraction scheme. The $\overline{\rm MS}$ effective coupling constant, $\alpha_s(Q^2)$ is given by the two term Callan–Symanzik β-function:

$$\alpha_s(Q^2) = 2\pi/[b_0 \ln(Q^2/\Lambda^2) + (b_1/b_0)\ln\ln(Q^2/\Lambda^2)]\ , \qquad (8)$$

with

$$b_0 = (33 - 2n_f)/6\ , \quad b_1 = (153 - 19n_f)/6\ ,$$

n_f being the number of quark flavours (assumed 5 for all numerical estimates) and Λ, the QCD scale parameter in the $\overline{\rm MS}$ scheme. The results for these functions are calculated for the region $|\cos\chi| < 0.9$, which liberates us from doing the two-loop $O(\alpha_s)^2$ virtual corrections to $e^+e^- \to q\bar{q}$. The accuracy of the Monte Carlo calculations was checked against known results. In particular the thrust distribution from our Monte Carlo was compared with an accurate numerical calculation of the same quantity in the small $y_{\min}$ region, using the subroutine REGINA [12]. In particular, the limit $[d\sigma_4 - d\sigma_4^s]_{y_{\min}\to 0}$ exists [13] for $T < 1$, as expected from the KLN theorem.

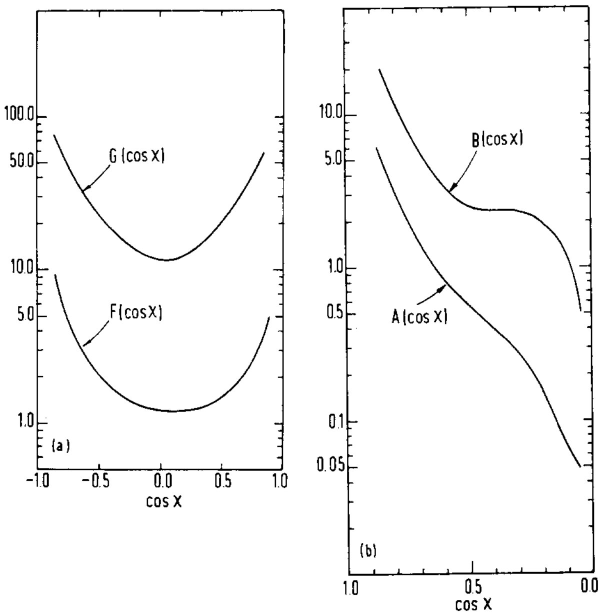

Fig.1. (a) The scalar function $F(\cos\chi)$ and $G(\cos\chi)$ for the energy–energy correlation as defined in eq. (6). (b) The scalar functions $\mathcal{A}(\cos\chi)$ and $\mathcal{B}(\cos\chi)$ for the asymmetry distribution as defined in eq. (7).

We have done a Legendre polynomial fit to the scalar function $G(\cos\chi)$ to do interpolation and the resulting function is shown in fig. 1 together with the $O(\alpha_s)$ BBEL function $F(\cos\chi)$. The scalar asymmetry functions $\mathcal{A}(\cos\chi)$ and $\mathcal{B}(\cos\chi)$ are also presented in the same figure. The $O(\alpha_s)$ and $O(\alpha_s)^2$ functions have shapes very similar to each other, showing that the normalized distributions are very stable.

The corrections to the normalization need a definition of the $O(\alpha_s)$ and $O(\alpha_s)^2$ cross sections. We would like to express the results as follows. In $O(\alpha_s)$ we define

$$(1/\sigma_0)\, d\sigma_{\rm corr}^{(1)}/d\cos\chi = [\alpha_s^{(1)}(Q^2)/\pi]\ F(\cos\chi)\ , \qquad (9)$$

where $\alpha_s^{(1)}$ is the lowest order expression obtained by

‡1 One could also use $y_{\min}$ to define finite $q\bar{q}G$ states. Then, it is easy to check that a value $y_{\min} \simeq 10^{-3}$ is needed to reproduce the BBEL formula for $e^+e^- \to q\bar{q}G$ in the region $20° < \chi < 160°$. The energy–energy correlation is a counter example to the large $y_{\min}$ (~0.05) algorithm advocated by Gottschalk. Here a small $y_{\min}$ is essential to reproduce the $O(\alpha_s)$ parton level theoretical calculations (see fig. 2) as well as the corrections in $O(\alpha_s)^2$ are small. This is not surprising, since we always have an angular (δ) cut-off but the need of an energy (ϵ) cut-off is bypassed due to energy weighting.

‡2 In the calculations reported by Fabricius et al. [1], Ali [1] and Gottschalk [2], one combines partons (quarks and gluons) in the final states $q\bar{q}gg$, $q\bar{q}q\bar{q}$ if they happen to lie in a region defined by ϵ, δ or $y_{\min}$. In general, there are various ways to combine the soft partons, each yielding a different result for distributions in perturbation theory [1,2]. We would like to avoid doing this.

setting $b_1 = 0$ in eq. (8) [‡3]. The correction to the integrated cross section can then be expressed as (for $|\cos\chi| < \chi_c$)

$$\sigma^{(2)}_{\mathrm{corr}}(\chi_c)/\sigma^{(1)}_{\mathrm{corr}}(\chi_c) = C_1 [1 + K_{\mathrm{corr}}(\chi_c)\alpha_s(Q^2)/\pi] \ , \quad (10)$$

where

$$K_{\mathrm{corr}}(\chi_c) = \frac{\int G(\cos\chi)\, \mathrm{d}\cos\chi}{\int F(\cos\chi)\, \mathrm{d}\cos\chi} \ ,$$

$$|\cos\chi| < \chi_c \ ,$$

and C_1 is given by the ratio of $\alpha_s(Q^2)$ in the lowest and second order

$$C_1 = \alpha_s(Q^2)/\alpha_s^{(1)}(Q^2) \ . \quad (11)$$

For the same value of Λ, $\alpha_s < \alpha_s^{(1)}$ and hence $C_1 < 1$. The value of the scale-invariant ratio $K(\chi_c)$ depends somewhat on $\cos\chi$ and is given in table 1 for the various values of $\cos\chi$. Typically, the value is 11, which should be compared with the corresponding K factors in thrust and c-distributions, which are typically 20 [14]. Thus, the $O(\alpha_s)^2$ corrections in the energy–energy correlations are much smaller than in the thrust or c-distributions. To have a more direct feeling for the $O(\alpha_s)^2$ corrections we note that

$$\frac{\sigma^{(2)}_{\mathrm{corr}}(|\cos\chi| \leqslant 0.85)}{\sigma^{(1)}_{\mathrm{corr}}(|\cos\chi| \leqslant 0.85)} \simeq 1.29 \ ,$$

for $\sqrt{s} = 35$ GeV, $\Lambda_{\overline{\mathrm{MS}}} = 0.1$ GeV.

The $O(\alpha_s)^2$ corrections to the asymmetry cross sections $\sigma_{\mathrm{asym}}(\cos\chi)$ are even smaller. Following the notation of eq. (10) we now have

$$\sigma^{(2)}_{\mathrm{asym}}(\chi_c)/\sigma^{(1)}_{\mathrm{asym}}(\chi) = C_1 [1 + K_{\mathrm{asym}}(\chi_c)\alpha_s(Q^2)/\pi] \ , \quad (12)$$

where

$$K_{\mathrm{asym}}(\chi_c) = \frac{\int \mathcal{B}(\cos\chi)\, \mathrm{d}\cos\chi}{\int \mathcal{A}(\cos\chi)\, \mathrm{d}\cos\chi} \ ,$$

$$|\cos\chi| < \chi_c \ ,$$

the values of $K_{\mathrm{asym}}(\chi_c)$ are also given in table 2. Again, to quote a number

$$\frac{\sigma^{(2)}_{\mathrm{asym}}(|\cos\chi| \leqslant 0.85)}{\sigma^{(1)}_{\mathrm{asym}}(|\cos\chi| \leqslant 0.85)} \simeq 1.04 \ . \quad (13)$$

The correction factors for the asymmetric cross section in the central region are somewhat larger as can be seen in table 2.

The reason for the smallness of (13) with respect to (12) can be traced to the presence of a four-jet component which reduces the asymmetric cross section since the processes in (i) are more symmetric as compared to the three-jet process $e^+e^- \to q\bar{q}G$.

These results are indeed very encouraging for a quantitative determination of the QCD scale parameter in the continuum e^+e^- region but before we attempt that we would like to discuss the problem of quark and gluon fragmentation.

4. It is generally recognized that the fragmentation effects on $\mathrm{d}\sigma_{\mathrm{corr}}$ are significant, more or less of the same order of magnitude as in the thrust distribution. On the other hand, similar effects on $\mathrm{d}\sigma_{\mathrm{asym}}$ are often assumed to be small in the large angle region. This reasoning is partly based on the use of the BBEL fragmentation correction [3], which does not take into account processes involving the emission of large

‡3 This is the conventional definition used by experimentalists while extracting "$\alpha_s(Q^2)$" using $O(\alpha_s)$ calculations.

Table 1
The K-factor for the energy–energy correlation for various values of $|\cos\chi| < \chi_c$. Typical Monte Carlo errors are ±4%.

χ_c	$K_{\mathrm{corr}}(\chi_c)$
0.85	11.6
0.75	12.0
0.65	11.9
0.55	11.65
0.45	11.3
0.35	10.85

Table 2
The K-factor for the asymmetric cross section for various values of $\cos\chi_c$. Typical Monte Carlo errors are ±8%.

$\cos\chi_c$	$K_{\mathrm{asym}}(\cos\chi_c)$
−0.85	4.46
−0.75	5.05
−0.65	5.61
−0.55	6.54
−0.45	8.47

angle hadron from a parton and is therefore in our opinion inadequate [‡4]. This view is also based on the deductions obtained from the use of event generators (13) incorporating fragmentation of quarks and gluons in a cascade manner. Having expressed our skepticism about the BBEL fragmentation correction we would like to make clear that the problem of incorporating the fragmentation effects in energy correlations in the framework of cascade models needs special care. The remarks here pertain to the hadronization of the $O(\alpha_s)$ $q\bar{q}G$ final state. The conventional method is to employ an invariant mass (or thrust) cut-off to define a $q\bar{q}G$ state as opposed to the $q\bar{q}$ state. The QCD predictions deduced from these generators are trustworthy if one is comparing theoretical prediction with data in kinematic regions, which are far from the cut-off e.g. large p_T or small T distributions. However, events generated with such a built-in invariant mass cut-off will not reproduce the perturbation theory result for energy correlations, unless one goes to the limit $y_{min} = 10^{-3}$, where the cross section $\sigma(e^+e^- \to q\bar{q}G)_{y>y_{min}=10^{-3}}$ is ridiculously large [‡6]. To describe this mismatch between energy correlations obtained via invariant mass (or thrust) cut-off generators and the $O(\alpha_s)$ BBEL formula, we plot in fig. 2 the parton level distribution $d\sigma_{corr}$ in $O(\alpha_s)$ for some "standard" T_c values. The approach of $d\sigma_{corr}(T_c)$ to $d\sigma_{corr}$(BBEL) is very slow; the discomforting feature of this is the dependence of the height of the plateau in $d\sigma_{corr}$ on the T_c (or invariant mass) values. That this effect becomes pronounced if one is studying the (unweighted) angular distribution of partons and hadrons with the help of $T_c(y_{min})$ event generators is obvious [‡7].

Continuing our discussion of the $O(\alpha_s)$ energy flow, we have estimated the fragmentation effects in $d\sigma_{corr}(\cos\chi)$ and $d\sigma_{asym}(\cos\chi)$ from the $q\bar{q}$ states using the Field–Feynman [16] ansatz for quark jets, which gives a reasonable description of the hadronic final states in e^+e^- annihilation in the low energy continuum region [‡8]. As to the fragmentation function for the $q\bar{q}G$ final state, we use a thrust cut-off $T_c =$ = 0.99, which corresponds to an invariant mass cut-off of $\simeq 3$ GeV at $\sqrt{s} = 35$ GeV. The quarks and gluons are then fragmented according to the model of Ali et al. [15]. We consider $m_{jet} = 3$ GeV as a reasonable compromise since the shape and normalization of the events generated this way at the parton level are reasonably close to the ones given by the $O(\alpha_s)$ BBEL formula (see the $T_c = 0.99$ curve in fig. 2). At the same

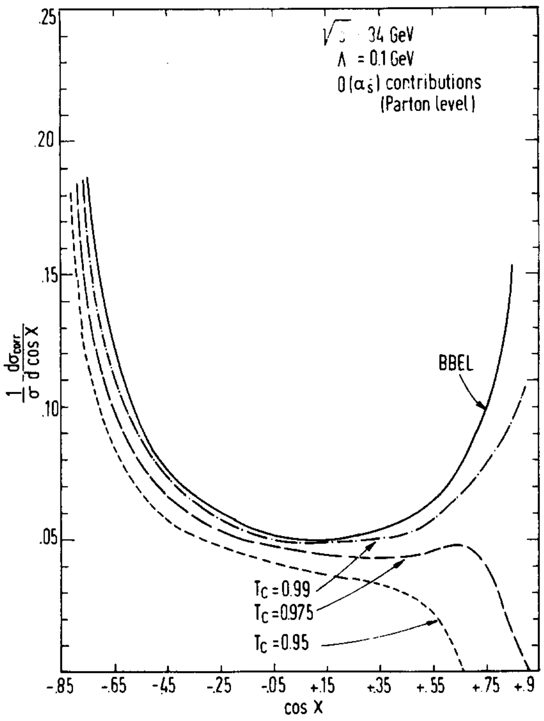

Fig. 2. The $O(\alpha_s)$ cross section $(1/\sigma)\, d\sigma(T_c)/d\cos\chi$ for $e^+e^- \to q\bar{q}G$ obtained for the various thrust cut-off, T_c. Note that there is a one to one correspondence between T_c and the scaled invariant mass cut-off, $y_{min} = m^2_{jet}/s$.

‡4 Large angle hadron emission in a jet produces a contribution to the asymmetry because of energy–momentum conservation, as a little reflection wil show.

‡5 The three commonly used event generators are given by Ali et al., Hoyer et al. and the so called Lund Monte Carlo described by Sjöstrand [15].

‡6 The energy weighted cross section for $20° < \chi < 160°$ however, does not rise as fast with decreasing y_{min}.

‡7 This circumstance has rather amusing implications for the inferences drawn about the quark and gluon fragmentation properties, based on a comparison of data with the hadron profile from cascade event generators having a different y_{min} (T_c) cut-off.

‡8 The most important parameter for energy correlations in the Field–Feynman fragmentation ansatz is the intrinsic-p_T of the hadrons, which is assumed to be distributed like a gaussian $d\sigma/dK_T^2 \sim \exp(-K_T^2/2\sigma_q^2)$. We use a value $\sigma_q = 0.3$ GeV, which is consistent with the observed hadron p_T-distribution at PETRA energies.

time partons have enough energy ($\geqslant$3 GeV) to fragment as independent quanta. With this method we get a reasonable description of the observed energy-correlation measured by the CELLO [17] and PLUTO [18] collaboration in the region 27 GeV $< \sqrt{s} <$ 34 GeV for the angular region $20° < \chi < 160°$. The asymmetric cross section is well described for all angles $\chi > 6°$. A detailed comparison and simple parametrizations of $d\sigma_{corr}^{non\text{-}pert}$ will be presented elsewhere [19].

5. We would now like to make a comparison of the QCD results with the data from the CELLO, PLUTO and MARK II collaboration. Since we primarily interested in the determination of $\alpha_s(Q^2)$ we shall concentrate on the asymmetric cross section $d\sigma_{asym}(\cos\chi)$. In fig. 3 we show the data from the CELLO collaboration which has been corrected for acceptance, photon radiation and detector efficiencies. We compare the data with the parton level perturbation theory results including the $O(\alpha_s)^2$ corrections in the region $\chi > 30°$. The shape of the $O(\alpha_s)^2$ distribution is in remarkable agreement with the data. Fitting only parton level perturbation theory [i.e. eq. (7)] we get

$$\Lambda_{\overline{MS}} = (114^{+62}_{-47})\ \mathrm{MeV}\,, \qquad (14)$$

which corresponds to $\alpha_s(Q^2) = 0.126 \pm 0.01$ at $Q^2 = 1156\ \mathrm{GeV}^2$. A similar fit to the PLUTO data (fig. 3b) yields

$$\Lambda_{\overline{MS}} = (61^{+69}_{-50})\ \mathrm{MeV}\,, \qquad (15)$$

which corresponds to $\alpha_s(Q^2) = 0.116 \pm 0.02$ at $Q^2 = 900\ \mathrm{GeV}^2$. We have also determined the QCD scale parameter using the recent MARK II data [20]. Using again the asymmetric cross section for $\chi > 30°$, we get $\alpha_s(Q^2) = 0.124 \pm 0.015$ at $Q^2 = 841\ \mathrm{GeV}^2$ giving

$$\Lambda_{\overline{MS}} = (95^{+75}_{-55})\ \mathrm{MeV}\,. \qquad (16)$$

For $\chi < 30°$, the data is dominated by the $q\bar{q}$ (and the soft $q\bar{q}G$) hadronic component and is well described by the fragmentation functions mentioned in the previous section. We remark that the fragmentation component from the $q\bar{q}$ final states in $d\sigma_{asym}$ falls off exponentially at large χ, but the $q\bar{q}G$ fragmentation contribution is non-negligible for all values of χ (~20% of the perturbative component). Including fragmentation effects in our previous analysis the distribution is also well described now for all

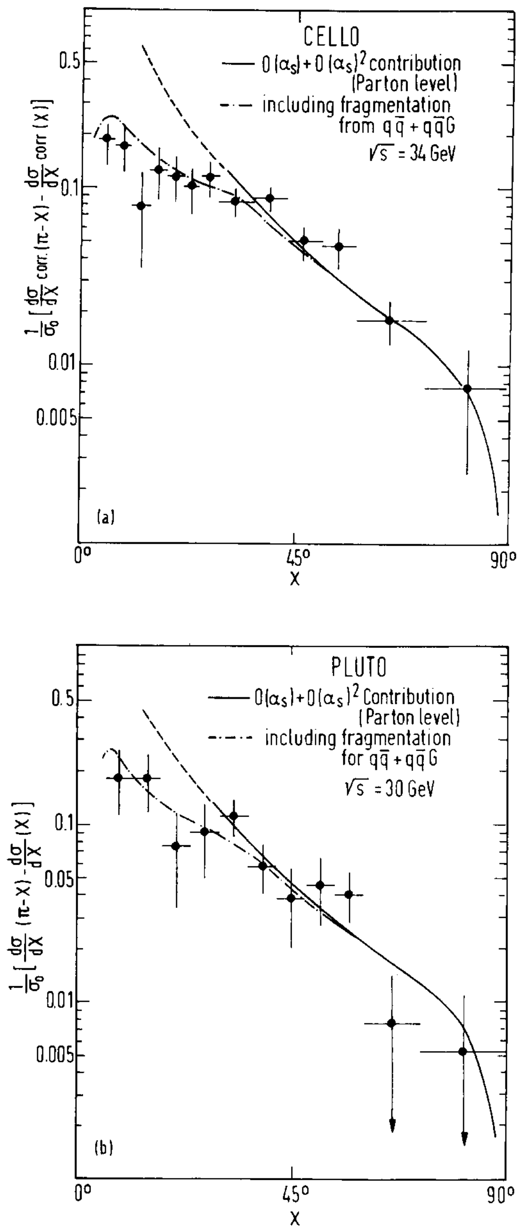

Fig. 3. (a) Comparison of the measured asymmetry distribution with the QCD calculation. The solid curve is the parton level perturbation theory result including $O(\alpha_s) + O(\alpha_s)^2$ processes, the dashed–dotted curve is the result of including in addition fragmentation effects from the $q\bar{q}$ and $q\bar{q}g$ states. The data is from the CELLO collaboration. (b) Same as in (a) with the data from the PLUTO collaboration.

angles $\chi > 6°$. However, $\alpha_s(Q^2)$ is normalized upwards by ~20%. A fit [‡9] to the CELLO data in the region $7° < \chi < 90°$ gives $[\alpha_s(Q^2) = 0.148 \pm 0.01$ at $Q^2 = 1156\ \mathrm{GeV}^2]$

$$\Lambda_{\overline{MS}} = (278^{+108}_{-90})\ \mathrm{MeV} \quad (\chi^2/\mathrm{dof} = 35/21)\,. \tag{17}$$

The fit corresponding to $\Lambda_{\overline{MS}} = 100$ MeV is marginally worse $\chi^2/\mathrm{dof} = 41/21$. A similar exercise with the PLUTO data gives $[\alpha_s(Q^2) = 0.142 \pm 0.17]$ yielding

$$\Lambda_{\overline{MS}} = (168^{+146}_{-103})\ \mathrm{MeV}\,. \tag{18}$$

Again, the fit to $\Lambda_{\overline{MS}} = 100$ MeV is also acceptable having a $\chi^2/\mathrm{dof} = 9/11$. The errors quoted (14)–(16) are statistical, whereas the errors quoted in (17) and (18) also reflect the systematic uncertainty due to fragmentation.

In conclusion, we find the $O(\alpha_s)^2$ corrections to the energy-correlation moderate and to the asymmetric cross section small. The shape of parton level asymmetric cross sections is in remarkable agreement with data for $\chi > 30°$. The uncertainty in the determination of $\alpha_s(Q^2)$ is about ±15% and should become less important at higher energies.

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‡9 The fit now includes perturbation theory contribution at the parton level including $O(\alpha_s) + O(\alpha_s)^2$ terms [i.e. eq. (7)] *and* the fragmentation component from $q\bar{q} + q\bar{q}G$ as described in section 5.

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for earlier analysis see: R. Hollebeek, Proc. Intern. Symp. on Photon–lepton interactions (Bonn, 1981) ed. W. Pfeil, p. 1.