# On the Radiative Decay of Orthoquarkonium Via Two Intermediate Gluons: $1^{-( }(Q \bar{Q}) \rightarrow \gamma+1^{++}(q \bar{q})$ 

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#### Abstract

We investigate the decay of a heavy ${ }^{3} S_{1}(Q \bar{Q})$ vector meson into a real photon and two off-shell intermediate gluons which create a ${ }^{3} P_{1}(q \bar{q})$ axial vector meson. As an application we suggest to look for the decay $J / \psi \rightarrow \gamma+D(1285)$ and $\gamma \rightarrow$ $\gamma_{1}+P_{c} / \chi_{1++} \rightarrow \gamma_{1}+\gamma_{2}+J / \psi$.


The transitions $e^{+} e^{-m} \rightarrow \gamma^{*} \rightarrow J / \psi \rightarrow \gamma$ (direct) + meson are an "ideal laboratory" to study the properties of $\mathrm{C}=$ + and $I=0$ mesons. Also, since the $J / \psi$ is polarized due to its formation from $e^{+} e^{-}$the polar angular distribution of the $\gamma$ (or meson) yields already some information on the spin of the meson:
$J_{\text {meson }} \geqq 1$ if $W\left(\Theta \gamma e^{ \pm}\right) \neq 1+\cos ^{2} \Theta \gamma e^{ \pm}$.
Up to now already several mesons have been clearly identified in the radiative $J / \psi$ decays:
$J^{P}=0^{-}: \eta, \eta^{\prime}, \mathscr{F}(1440) ; \quad J^{P}=2^{+}: f, \Theta(1640)[1]$
with branching ratios $B(J / \psi \rightarrow \gamma+$ meson $)$ on the order of $10^{-3}$. With present or future high statistics experiments it should be possible to identify meson states occurring with branching ratios of the order $10^{-4}$.

In lowest order QCD these Zweig forbidden transitions are mediated by two gluon exchange

where the coupling of the gluons is either to "light quarkonia' or 'gluonia'.

In this paper we present the results for the transition rate and helicity structure of
${ }^{3} S_{1}(Q \bar{Q}) \rightarrow \gamma+{ }^{3} P_{1}(q \bar{q})$
$1^{--}(Q \bar{Q}) \rightarrow \gamma+1^{++}(q \bar{q})$
by using (1) as the transition mechanism. The Lan-dau-Pomeranchuk-Yang theorem ${ }^{2}$ tells us that the transition is mediated only if at least one of the gluons is off-shell. Thus a $1^{++}$state does not occur in the $1^{--} \rightarrow \gamma+g_{1}+g_{2}$ Born term: $k_{1}^{2}=k_{2}^{2}=0$. But the Feynman rules in (1) yield a nonvanishing dispersive part stemming from the box integration. That the off-shell part of the loop contribution can be appreciable or even dominating has already been demonstrated in the transition ${ }^{3} S_{1}(Q \bar{Q}) \rightarrow \gamma+{ }^{1} S_{0}(q \bar{q})$ using again (1) as dynamical input ${ }^{3}$.

The calculation of the decay amplitude (1) for a $1^{++}$meson in the final state is performed by contracting the (off-shell) Ore-Powell matrix element $\mathscr{M}_{1}$


$$
\begin{align*}
& M_{1}=\frac{i 8 \sqrt{M_{0}} \cdot \psi(0)}{\left(k_{1}+k_{2}, k_{3}\right)\left(k_{2}+k_{3}, k_{1}\right)\left(k_{3}+k_{1}, k_{2}\right)} \\
& \cdot\left\{( \varepsilon _ { 1 } ^ { * } \varepsilon _ { 2 } ^ { * } ) \left[-\left(k_{1} k_{3}\right)\left(\varepsilon_{\varepsilon}^{*} k_{2}\right)\left(\varepsilon_{v} k_{1}\right)-\left(k_{1} k_{3}\right)\left(k_{2} k_{3}\right)\left(\varepsilon_{\varepsilon} \varepsilon_{3}^{*}\right)\right.\right. \\
& \left.-\left(k_{2} k_{3}\right)\left(\varepsilon_{3}^{*} k_{1}\right)\left(\varepsilon_{6} k_{2}\right)\right]+\left(\varepsilon_{v} \varepsilon_{3}^{*}\right)\left[\left(k_{3} k_{1}\right)\left(\varepsilon_{1}^{*} k_{2}\right)\left(\varepsilon_{2}^{*} \varepsilon_{3}\right)\right. \\
& \left.-\left(k_{1} k_{2}\right)\left(\varepsilon_{1}^{*} k_{3}\right)\left(\varepsilon_{2}^{*} k_{3}\right)+\left(k_{3} k_{2}\right)\left(\varepsilon_{2}^{*} k_{1}\right)\left(\varepsilon_{1}^{*} k_{3}\right)\right] \\
& +1 \leftrightarrow 3+2 \leftrightarrow 3\} \tag{3}
\end{align*}
$$

with the $1^{++}$decay element

$\mathscr{M}_{2}^{*}=\frac{-i 2 \sqrt{6 M_{A}} \cdot \psi_{p}^{2}(0)}{M_{A}^{3} \cdot\left(k_{1} k_{2}\right)^{2}}$

- $\left\{\left(k_{1} k_{2}\right)\left(k_{1}^{2}-k_{2}^{2}\right) \in\left(\varepsilon_{A}^{*}, \varepsilon_{1}, \varepsilon_{2}, P_{A}\right)+\left[2\left(k_{1} k_{2}\right)\left(k_{2} \varepsilon_{2}\right)\right.\right.$
$\left.\left.-\left(k_{1}^{2}+k_{2}^{2}\right)\left(k_{1} \varepsilon_{2}\right)\right] \in\left(\varepsilon_{A}^{*}, \varepsilon_{1}^{\prime}, P_{A}, k_{1}-k_{2}\right)+1 \leftrightarrow 2\right\}$
leading to the transition amplitude
$\mathscr{T}=\frac{-1}{(2 \pi)^{4}} \int d^{4} k_{1} d^{4} k_{2} \delta\left(P_{A}-k_{1}-k_{2}\right) \frac{\mathscr{M}_{1} \cdot \mathscr{M}_{2}^{*}}{k_{1}^{2} k_{2}^{2}}$
A straightforward evaluation of (5) leads to a form involving at most three parameter Feynman integrals. The results agree with an alternative evaluation using covariant helicity projectors which reduce the number of necessary Feynman parameter integrations to two. The two independent helicity amplitudes labelled by the helicity of the $1^{++}$particle

$$
\begin{equation*}
1^{--}(1,0) \rightarrow \gamma(1)+1^{++}(0,1) \equiv\left(\mathscr{H}_{0}, \mathscr{H}_{1}\right) \tag{6}
\end{equation*}
$$

are given by

$$
\begin{align*}
& \binom{\mathscr{H}_{0}}{\mathscr{H}_{1}}=\mathscr{N}\binom{\mathscr{H}_{0}(x)}{\mathscr{H}_{1}(x)}=\mathscr{N} \quad \frac{\sqrt{1-x} A_{1}(x)}{\frac{2-x}{2} A_{1}(x)-\frac{x}{2} A_{2}(x)} \\
& x=1-M_{A}^{2} / M_{v}^{2} \\
& \mathscr{N}=8 \sqrt{M_{v}} \psi(0) \cdot \frac{2 \sqrt{6 M_{A}}}{M_{A}^{3}} \psi_{p}^{\prime}(0) \cdot \frac{16 \pi^{2}}{(2 \pi)^{4} M_{v}} \\
& \cdot e_{Q}(4 \pi)^{\frac{5}{2}} \sqrt{\alpha} \cdot \alpha_{s}\left(M_{v}\right) \cdot \alpha_{s}\left(M_{A}\right) \cdot \frac{2}{3} \cdot \frac{1}{2} \tag{7}
\end{align*}
$$

and where

$$
\begin{aligned}
& A_{1}(x)=\frac{2(1-x)^{2}}{x^{3}} \log (1-x)-\frac{2(1-x)}{x^{2}} \log (1-x) \\
& +\frac{2(1-x)}{x^{2}} \log 2 x+\frac{3 x-2}{x(1-2 x)} \log 2 x \\
& +\frac{1}{x^{3}}\left[L i_{2}(1)-L i_{2}(1-2 x)\right] \\
& +\frac{6(1-x)}{x^{2}}\left[L i_{2}(1-2 x)-L i_{2}(1-x)+\log 2 \log (1-x)\right] \\
& A_{2}(x)=\frac{-8(1-x)^{2}}{x^{3}} \log (1-x)+\frac{2(1-x)}{x^{2}} \\
& -\frac{7(1-x)}{x^{2}} \log 2 x-\left(1-\frac{5}{x}+\frac{3}{x^{2}}\right) \cdot \frac{\log 2 x}{1-2 x}
\end{aligned}
$$

$+\frac{1}{x^{3}}\left[L i_{2}(1)-L i_{2}(1-2 x)\right]+(1-x) \cdot\left(\frac{12}{x^{3}}-\frac{10}{x^{2}}\right)$
$\cdot\left[L i_{2}(1-2 x)-L i_{2}(1-x)+\log 2 \log (1-x)\right]$
$\left(L i_{2}(0)=0, L i_{2}(1)=\frac{\pi^{2}}{6}\right)$
The limiting behaviour of $A_{1,2}$ is given by
$\lim _{x \rightarrow 1} A_{1}=\lim _{x \rightarrow 1} A_{2}=\frac{\pi^{2}}{4}-\log 2 ;$
$\lim _{x \rightarrow 0} A_{1}=\lim _{x \rightarrow 0} A_{2}=-\frac{2}{3} \log 2 x+\frac{13}{18}$
$A_{1}\left(\frac{1}{2}\right)=1+\frac{\pi^{2}}{3}-6 \log ^{2} 2$;
$A_{2}\left(\frac{1}{2}\right)=7-\pi^{2}+16 \log 2-14 \log ^{2} 2$.
We observe that in the limit $M_{A} \rightarrow 0$ the amplitude $\mathscr{H}_{0} \propto M_{A}^{-3 / 2}, \mathscr{H}_{1} \propto M_{A}^{-1 / 2}$, i.e. the longitudinal contri bution dominates in this limit as expected. The ratio $H_{1} / H_{0}$ is shown in Fig. 1. The angular distribution of the $\gamma$ or $1^{++}$with respect to the electron-positron beam in $e^{+} e^{-} \rightarrow Q \bar{Q} \rightarrow \gamma+1^{++}$is $W(\Theta) \propto 1$ $+\alpha(x) \cos ^{2} \Theta$ where $\alpha(x)$ is given by $\left(1-2 H_{1}^{2} / H_{0}^{2}\right) /(1$ $+2 H_{1}^{2} / H_{0}^{2}$ ). For small $x$ it approaches the electric dipole limit. Figure 2 gives the combination $x\left(H_{o}^{2}\right.$


Fig. 1. See text


Fig. 2. See text
$+H_{1}^{2}$ ) which occurs in the rate formula
$\Gamma=\frac{1}{24 \pi} \cdot \frac{x}{M_{0}}\left(\mathscr{H}_{0}^{2}+\mathscr{H}_{1}^{2}\right)$
We have estimated the rate $J / \psi \rightarrow \gamma+D(1285)$ to be of the order of 16 eV using $\alpha_{s}\left(M_{J \psi}\right)=0.2$ and $\alpha_{s}\left(M_{D}\right)$ $=0.3$ and extracting $\frac{\left|\psi_{P}^{\prime}(0)\right|^{2}}{M_{D}^{4}}=0.25 \mathrm{MeV}$ from
$\Gamma(f \rightarrow \gamma \gamma)=3 \mathrm{keV}$.

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## References

1. D.L. Scharre: Proc. 1981 Int. Symp on Lepton and Photon Interactions at High Energies, Bomn W. Preil, ed., p. 163.
2. L.D. Landau: Dokl. Akad. Nauk SSSR 207 (1948); I.Ya. Pomeranchuk: Akad. Nauk SSSR 60, 263 (1948); CN. Yang: Phys. Rev. 77, 242 (1950)
3. A. Nishimura: Univ. of Glasgow Preprint (1979); B. Guberina, f. Kuhn: Lett. Nuovo Cimento, 32, 295 (1981); T. Munehisa: Progf. Theor. Phys. 63, 734 (1980)

Note Added in Proof. If experimentally D(1285) is not found at this level it indicates, in our opinion, a different wave function for the $1^{++}$than the ${ }^{3} P_{1}$ quark-antiquark wave function used here. Since the $D$ is in the same $S U(3)$ multiplet as the $A_{1}$ and the $A_{1}$ is known from the $\tau$-lepton decay to couple to the axial vector current $\gamma_{s} \gamma_{u}$, a relativistic wave function for the $1^{++}$of the type $\gamma_{S} \psi_{A}$ may be more realistic. However, in such a case a $1^{++}$-state would not couple to two gluons since trace $\left\{\gamma_{s} \not_{A} \phi_{1} q_{2}\right)=0$. A candidate to check our nomelativistic calculation would then be the transition $y \rightarrow \gamma_{1}+P_{2} / \chi_{1}++$ where in addition to $\gamma_{1}$, the $\gamma_{2}$ from the $P_{c} / x_{1}+\rightarrow \gamma_{2}+J /$ cascade decay can be used as a good trigger.

