

Charmed-quark fragmentation into charmed hadrons

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We show that in a model where fragmentation functions are created from jet calculus followed by recombination, the fragmentation function of charmed quarks into  $D$  mesons will fail to peak at low  $x$  at currently accessible values of  $Q^2$ . Likewise, the fragmentation of a  $c$  quark into a  $\Lambda_c$  is much less peaked toward small  $x$  than the production of protons by  $u$  quarks. The model predicts a magnitude for the fragmentation function much smaller than is experimentally observed.

GENERAL METHOD

We use the Konishi-Ukawa-Veneziano (KUV) jet calculus<sup>1</sup> to compute the distribution of partons in the jet, and a form of the recombination model to make these into mesons<sup>2,3</sup> or baryons.<sup>3-5</sup> As shown in our previous papers,<sup>2-5</sup> this approach has many appealing features, including approximate agreement with experiment for the production of non-

charmed mesons and baryons.

At present energies we allow only three flavors to participate fully in the jet evolution; i.e., gluons are not allowed to split into  $c\bar{c}$  pairs at any stage in the evolution or recombination. Hence each charmed-quark jet will contain only one charmed hadron, and no other jets will contain charmed particles. The "quark propagators" can be obtained for this situation; we list them here for completeness:

$$\begin{aligned}
 D_{cc}(n,y) &= \exp[A_q^{qq}(n)y], \\
 D_{ic}(n,y) &= \frac{A_q^{qq}(n)A_g^{q\bar{q}}(n)}{\lambda_1(n)-\lambda_2(n)} \left[ \frac{e^{\lambda_1(n)y} - e^{\lambda_0(n)y}}{\lambda_1(n)-\lambda_0(n)} - \frac{e^{\lambda_2(n)y} - e^{\lambda_0(n)y}}{\lambda_2(n)-\lambda_0(n)} \right], \\
 D_{gc}(n,y) &= \frac{A_q^{qq}(n)}{\lambda_1(n)-\lambda_2(n)} \left[ e^{\lambda_1(n)y} - e^{\lambda_2(n)y} \right],
 \end{aligned}
 \tag{1}$$

where  $\lambda_0(n) = A_q^{qq}(n)$ , and  $\lambda_1(n)$  and  $\lambda_2(n)$  are the other two eigenvalues of the Altarelli-Parisi equations. The propagator  $D_{gc}$  is the same as  $D_{gi}$ , and  $D_{ic} = D_{ij}$ . As a practical matter, for  $Q^2$  less than 1000 GeV<sup>2</sup>,  $D_{cc}$  and  $D_{ii}$  are almost the same for  $x$  greater than 0.2. Hence we can use the propagator  $D_{uu}$  instead of  $D_{cc}$  if we wish.

In this paper we wish to stress one important feature of this jet-calculus model which strongly influences the energy spectrum of produced heavy particles. Consider the KUV formula for the two-parton distribution:

$$\begin{aligned}
 D_{a_1 a_2; i}(x_1, x_2; Q^2) &= \sum_{b_1 b_2 j} \int_{Y_0}^Y dy \int_0^1 dx dz dw_1 dw_2 D_{a_1 b_1}(w_1, y - Y_0) D_{a_2 b_2}(w_2, y - Y_0) \\
 &\quad \times \hat{P}_{j \rightarrow b_1 b_2}(z) D_{ji}(x, Y - y) \delta(x_1 - zxw_1) \delta(x_2 - x(1-z)w_2),
 \end{aligned}
 \tag{2}$$

where

$$Y = \frac{1}{2\pi b} \ln \left[ 1 + \alpha_0 b \ln \frac{Q^2}{\Lambda^2} \right],$$

$$12\pi b = 11N_c - 2N_f,$$

with

$$\alpha_s = \frac{1}{b \ln Q^2 / \Lambda^2}, \quad \Lambda'^2 = \Lambda^2 e^{-1/b\alpha_0}.$$

We use  $\alpha_0 = 10$ .

The integration variable  $y$  represents the position of the splitting vertex  $\hat{P}$ , as shown in Fig. 1(a). If

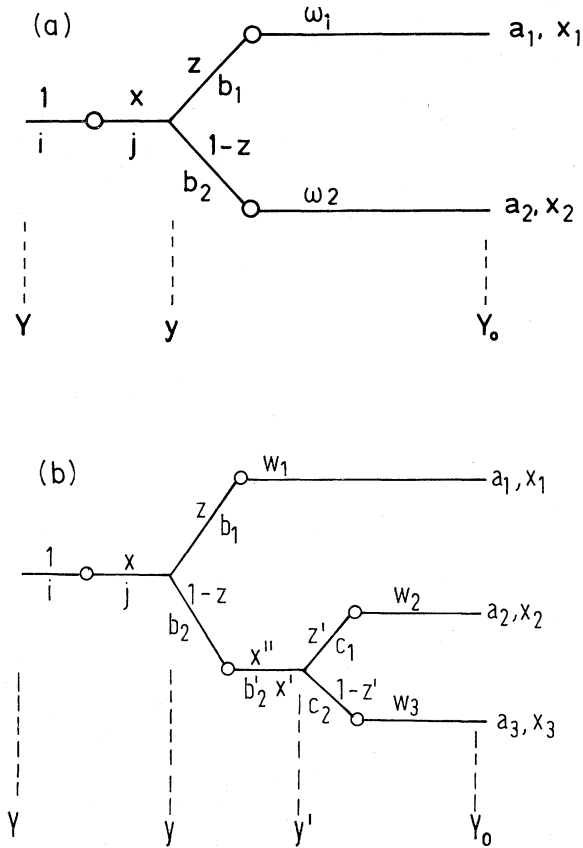


FIG. 1. (a) Pictorial representation of Eq. (1). The lines with circles represent QCD “propagators” calculated in the leading-logarithm approximation. The variable of integration  $y$  is determined by the mass of parton  $j$  just before the vertex. (b) Pictorial representation of the jet-calculus expression for the three-parton inclusive cross section.

the partons  $a_1$  and  $a_2$  are to be recombined into a hadron of mass  $M^2$ , the mass just before the splitting must always be greater than  $M^2$ . Hence the lower limit  $Y_0$  in the KUV formula must obey  $Y_0 > y(M^2)$ . Therefore, when the particle produced is quite heavy, the region of integration is much smaller than for the creation of light particles like pions, at present  $Q^2$ . Physically speaking, this means much less QCD radiation occurs and the original heavy quark tends to have most of the jet momentum. Hence charmed particles should be concentrated near large  $x$  to a greater extent than even the “leading” particles with normal quantum numbers.

In addition, many pions and other light particles are produced at small  $x$  by splitting of the produced gluons into  $q\bar{q}$  pairs and the subsequent recombination of these quarks. By forbidding splitting into  $c\bar{c}$

pairs, we “turn off” this mechanism for small  $x$  charmed particles. This is an approximation to more exact inclusion of the quark masses in the jet calculus, but it seems physically reasonable.

For the production of baryons, we must compute the three-parton distribution depicted schematically in Fig. 1(b). The explicit formula is given by Eq. (2) of Ref. 4. Again we allow only three flavors to participate in the jet evolution.

PARAMETERS AND RESULTS

In our calculations for charged pions, we computed parton sets  $q\bar{q}$ ,  $qg$ , and  $gg$ . All final gluons were then converted into  $u\bar{u}$ ,  $d\bar{d}$ , and  $s\bar{s}$  pairs by a splitting function<sup>6</sup> which conserves momentum, and all  $u\bar{d}$  and  $d\bar{u}$  pairs at  $x_a$  and  $x_b$  were recombined into mesons at  $x$  using the recombination function

$$R \frac{x_a x_b}{x^2} \delta(x_a + x_b - x), \tag{3}$$

where  $R$  is a parameter whose maximum value is 4 by unitarity arguments.<sup>7</sup> In comparisons with the data on pion and kaon production we find  $R=1$  to give agreement.<sup>3</sup>

We use the same procedure and recombination function to study  $D$  and  $D^*$  production as were used for the pions and kaons. Only  $gc$  and  $qc$  pairs need to be computed in the jet calculus. Due to the large mass of the  $D$ , we consider the two possibilities  $Q_0^2=4 \text{ GeV}^2$  and  $Q_0^2=8 \text{ GeV}^2$ . In Fig. 2(a) we show the energy dependence of the fragmentation function  $D_c^D$ , as shown it rises from a shape peaked at large  $x$  to a shape peaked at small  $x$  as  $Q^2$  increases. For very large  $Q^2$ , we would have to include charmed quarks in the QCD evolution; the resulting  $g \rightarrow c\bar{c}$  vertices would result in further peaking at low  $x$ . For comparison we show in Fig. 2(b) the range of parametrizations preferred by the European Muon Collaboration<sup>8</sup> (EMC) in fitting their charm production.

Data for  $D$  and  $D^*$  production in  $e^+e^-$  reactions are now available at several energies.<sup>9-11</sup> The values from Fig. 2 are far too small to fit this data. Even if we use the maximum  $R=4$ , and make a kinematic transformation to accommodate the fact that at present  $Q^2$  the variable used by the experimentalists ( $x_E=2E/W$ ) differs from ours ( $x_p=p/p_{\text{max}}$ ), our predictions still lie beneath the data. This is shown in Fig. 3.

As discussed in Ref. 2, the recombination term is not the whole story. There is also the possibility that there is an “intrinsic” fragmentation function present at  $Q^2=Q_0^2$  which evolves according to the Altarelli-Parisi equations as  $Q^2$  increases. This

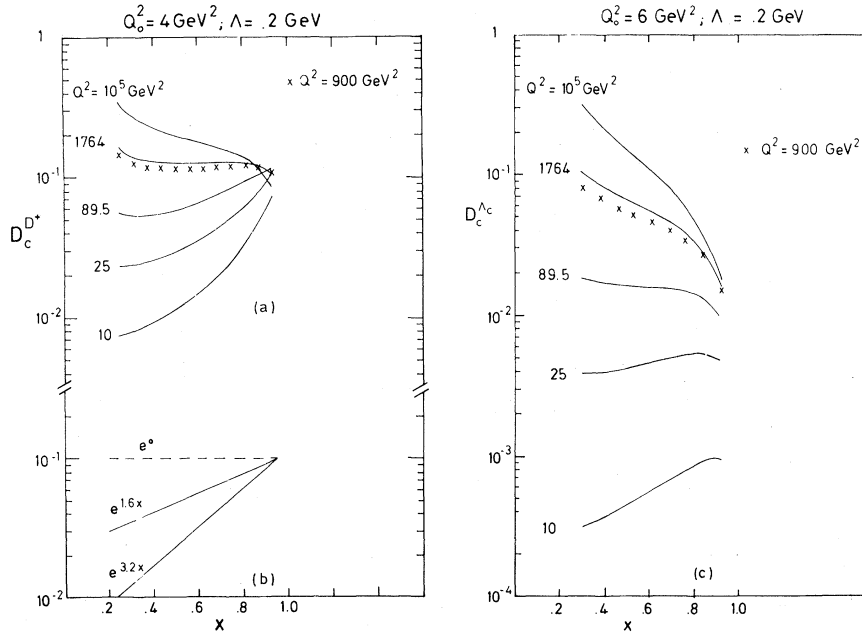


FIG. 2. (a) The fragmentation function  $D_c^D$  as a function of  $Q^2$  for  $Q_0^2 = 4 \text{ GeV}^2$ . Note the change in shape from a peak at large  $x$  at low values of  $Q^2$  to a peak at small  $x$  at large  $Q^2$ . The recombination parameter  $R$  was set to 1 in computing these graphs, since that value yields agreement with pion and kaon data. (b) For comparison with (a) we show the range of parametrizations preferred by the EMC (Ref. 8) to fit their data with  $1 < Q^2 < 100 \text{ GeV}^2$ . In preparing this graph, these functional forms have been normalized to have roughly the same size at large  $x$  as our calculations in (a). (c) The fragmentation function of charmed quarks into  $\Lambda_c$ , as a function of  $Q^2$  for  $Q_0^2 = 6 \text{ GeV}^2$  and  $\Lambda = 0.2 \text{ GeV}$ .

difference between the data and the “maximum” recombination term, as shown in Fig. 3, thus provides a lower limit for this piece. Because the recombination contribution vanishes as  $Q^2$  approaches  $Q_0^2$ , the intrinsic fragmentation is especially important in the low  $Q^2$  region.

To compute proton yields, we calculated the  $uud$  distributions in the same way and recombined them using the recombination function

$$R_B \frac{x_a x_b x_c}{x^3} \delta(x_a + x_b + x_c - x),$$

with  $R_B = \frac{27}{4}$  yielding rough agreement with the data (see Refs. 4 and 5 for details about the general features of production of noncharmed baryons as predicted in this model).

To compute charmed baryons, we simply modify our proton calculation in a manner analogous to that mentioned for the  $D$ : there are fewer terms and  $Q_0^2$  is larger. Using Eq. (4) with  $R_B = \frac{27}{4}$ , we obtain the fragmentation functions shown in Fig. 2(c).

In Fig. 4(a) we show the ratio of fragmentation functions into  $\Lambda_c$  and  $D$ , when both have been computed using a value  $Q_0^2 = 6 \text{ GeV}^2$ . Note that the ratio decreases near  $x = 1$ ; as can be seen by comparison with Fig. 4(b), the behavior is more like  $1 - x$  than like the  $(1 - x)^2$  we might expect from counting

rule arguments. This behavior is unlike that for noncharmed baryons and mesons (see Ref. 3) where the  $Q_0^2$  is much larger for the baryons than the mesons, and one obtains a decrease in the baryon/meson ratio for  $x = 1$  only for very large  $Q^2$ .

## SUMMARY AND CONCLUSIONS

The fragmentation function of charmed quarks into  $D$  mesons predicted in our model is peaked at large  $x$  for small  $Q^2$  and flattens out as  $Q^2$  grows. It is consistent with the shape required by the EMC collaboration to fit their data; and the results have many features in common with those hypothesized by Suzuki<sup>12</sup> and Bjorken<sup>13</sup> using momentum arguments. Similar peaking near large  $x$  was produced by Kartvelishvili *et al.*<sup>14</sup> in a model using Regge arguments for heavy quark trajectories. Their formula, however, lacks  $Q^2$  dependence. The work of Peterson *et al.*<sup>15</sup> shows that the expected hard spectrum is only slightly softened by  $b$  decays; we therefore feel justified in neglecting this source of charmed mesons.

Within the constraints of the recombination model, however, the size of the recombined fragmentation function cannot be made large enough to fit currently available data. This implies the need for an additional intrinsic fragmentation function.

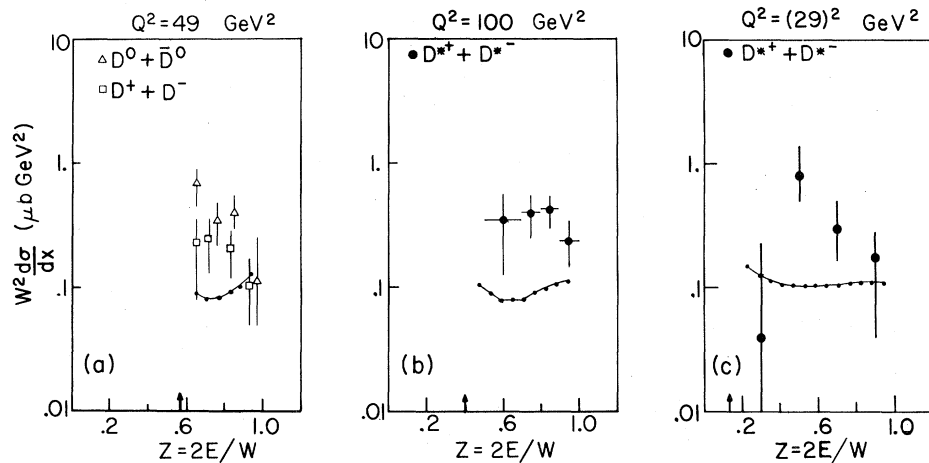


FIG. 3. Comparison of recombination-model predictions with data using maximum allowed size (four times Fig. 2) and a transformation from the momentum-based  $x_p$  of Fig. 2 to the energy-based  $x_E$  used in the data. Data are from Ref. 9 [Fig. 3(a)], Ref. 10 [Fig. 3(b)], and Ref. 11 [Fig. 3(c)].

Due to the fact that charmed baryons and mesons have similar masses, the ratio of baryons to mesons predicted from recombination will tend to decrease near  $x=1$ ; this is simply because for a given energy it is harder to make three partons perturbatively and

have them carry all the momentum than it is to make two.

Although the recombination model has proved useful in the past, its quantitative failure here probably indicates that the approach cannot be pushed much farther. The charm fragmentation functions are intuitively most like the jet-calculus description. The discrepancy between the calculations and experiment seen in Fig. 3 is thus an indictment of the recombination technique used.

We are currently computing distributions for normal quarks using a modified jet calculus<sup>16</sup> which explicitly keeps track of the color of the partons being considered. The mechanism for hadronization can then be studied in a more general framework than that of the recombination model. It is hoped that this will also lead to a better description for the  $c \rightarrow D$  fragmentation functions.

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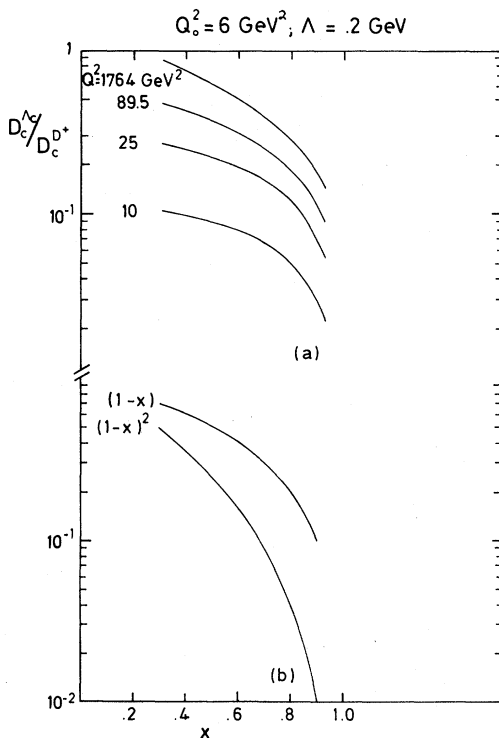


FIG. 4. (a) Ratio of  $\Lambda_c$  production to  $D$  production if both are calculated using  $Q_0^2=6 \text{ GeV}^2$  and the normalizations appropriate to pion and proton production are used. (b) Two shapes commonly postulated for the baryon/meson ratio.

- <sup>1</sup>K. Konishi, A. Ukawa, and G. Veneziano, Phys. Lett. **78B**, 243 (1978); Nucl. Phys. **B157**, 45 (1979).
- <sup>2</sup>L. M. Jones, K. E. Lassila, U. Sukhatme, and D. E. Wilen, Phys. Rev. D **23**, 717 (1981).
- <sup>3</sup>L. M. Jones and R. Migneron, Z. Phys. C (to be published).
- <sup>4</sup>R. Migneron, L. M. Jones, and K. E. Lassila, Phys. Lett. **114B**, 189 (1982).
- <sup>5</sup>R. Migneron, L. M. Jones, and K. E. Lassila, Phys. Rev. D **26**, 2235 (1982).
- <sup>6</sup>V. Chang and R. C. Hwa, Phys. Rev. Lett. **44**, 439 (1980); **44**, 1163(E) (1980).
- <sup>7</sup>M. J. Teper, Rutherford Report No. RL-78-022/A, 1978 (unpublished).
- <sup>8</sup>C. Gossling, talk given at the XVII Rencontre de Moriond, 1982 (unpublished).
- <sup>9</sup>P. A. Rapidis *et al.*, Phys. Lett. **84B**, 507 (1979).
- <sup>10</sup>C. Bebek *et al.*, Phys. Rev. Lett. **49**, 610 (1982).
- <sup>11</sup>J. M. Yelton *et al.*, Phys. Rev. Lett. **49**, 430 (1982).
- <sup>12</sup>M. Suzuki, Phys. Lett. **68B**, 164 (1977).
- <sup>13</sup>J. D. Bjorken, Phys. Rev. D **17**, 171 (1978).
- <sup>14</sup>V. G. Kartvelishvili, A. K. Likhoded, and S. R. Slavospitsky, in *Proceedings of the II International Symposium of Hadron Structure and Multiparticle Production, Kazimierz, Poland, 1979*, edited by Z. Ajduk (Institute of Theoretical Physics, Warsaw, 1979), p. 315; V. G. Kartvelishvili, A. K. Likhoded, and V. A. Petrov, Phys. Lett. **78B**, 615 (1978).
- <sup>15</sup>C. Peterson, D. Schlatter, I. Schmitt, and P. M. Zerwas, Phys. Rev. D **27**, 105 (1983).
- <sup>16</sup>B. Crespi and L. M. Jones, Trento Report No. UTF-81 (unpublished).