PREDICTION OF LOW-LYING ODDBALLS IN LATTICE QCD

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We find, in a high precision Monte Carlo calculation of the glueball mass spectrum in pure SU(3) lattice gauge theory, a low-lying oddball with quantum numbers 1^{-+}. We estimate its mass to be $m(1^{-+}) = 1.68 \pm 0.18$ GeV. We also measure the mass of the 0^{--} oddball and find $m(0^{--}) = 2.79 \pm 0.22$ GeV.

Recently, there have been several calculations of the low-lying glueball mass spectrum in both SU(2) [1–3] and SU(3) [4,5] lattice gauge theory. The experimental discovery of such states would help to confirm in a very direct way the non-abelian (self-coupling) character of the gluonic degrees of freedom in strong interactions. It is therefore interesting to note that the theoretical predictions [4] for the 0^{-+} and 2^{++} masses coincide with the masses of the two glueball candidates in J/ψ radiative decays [6] [‡1]; although at present the identification of these resonances as glueball rather than quark states is controversial.

The above situation accentuates the importance of those glueball states which have quantum numbers not accessible to a quark–antiquark pair (i.e. $J^{PC} = 0^{--}$, $(\mathrm{odd})^{-+}$, $(\mathrm{even})^{+-}$), and faces theory with the challenge of *predicting* their masses. In this letter we report on our calculation of the masses of two such oddball states, the 0^{--} and 1^{-+}, which we find to be sufficiently low-lying as to be of immediate experimental interest.

We work on a lattice with 4 lattice sites in each spatial direction and 16 in the (imaginary) time direction. Using the Wilson action [8] for SU(3) gauge fields and applying Monte Carlo techniques [9], we construct 6000 typical gauge field configurations of the vacuum of this lattice. In this calculation we have taken particular care with the random number generation because we have found that for the calculation of the masses of some states the simpler and most common (pseudo-) random number generators give incorrect results. Our procedure has been to choose the link to be updated at random using *true* random numbers [10], and then using an improved (pseudo-) random number generator [11] in the actual upgrading of the SU(3) matrix on the link. For a more detailed discussion we refer to ref. [11]. Here we only make the one further remark that we find this procedure to be particularly efficient in reducing the final error.

We now measure, within these configurations, correlation functions $\langle \phi(t')\, \phi(0) \rangle$ of zero-momentum (translationally invariant) operators $\phi(t)$. For times t' and t large enough we have that

$$\Gamma_{t'}/\Gamma_t \equiv \langle \phi(t')\, \phi(0) \rangle / \langle \phi(t)\, \phi(0) \rangle \approx \exp[-m(t'-t)] \,, \qquad (1)$$

where m is the mass of the lightest particle communicating with the operator ϕ. So to obtain an estimate of the lowest glueball mass of a particular J^{PC} we construct an operator ϕ with these quantum numbers and measure the correlation functions as in (1).

In order to minimise the computing time, the spatial lattice should be as coarse as is consistent with the requirement that the glueball size be (considerably)

‡1 For a theoretical discussion, see ref. [7].

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greater than the spatial lattice spacing, a_s, and (considerably) smaller than half the total spatial extent of the lattice. On the other hand, in order to minimise the error to signal ratio in the measurement of ratios of correlation functions as in (1), we obviously want to use t and t' as small as is consistent with (1) being accurately satisfied. To best satisfy these requirements we deviate from the traditional use of the hypercubic lattice spacings; in particular we shall choose $a_t < a_s$. This has the additional advantage of being closer to the continuum limit than a hypercubic lattice of spacing $a_t = a_s$.

The lattice spacings a_s, a_t are determined by the magnitudes chosen for the inverse spatial and temporal couplings, β_s and β_t, in the action [11]. In making an appropriate choice of β_s, β_t we can use as a guide the perturbative relationship between β_s, β_t and a_s, a_t as obtained in ref. [12]. We have used

$$\beta_s = 3.3\,, \quad \beta_t = 10.68\,, \tag{2}$$

which, according to the perturbative formulae [12], would correspond to $a_t/a_s = 0.5$ and $a_s \approx a\ (\beta = 5.8)$ where $a\ (\beta)$ is the hypercubic lattice spacing at inverse coupling β. Of course, we do not expect these low-order perturbative results to be very accurate for our relatively coarse lattice, so we have also measured the lattice spacings directly by comparing the behaviour of various correlation functions with their values on a hypercubic lattice at $\beta = 5.7$ [4]. We find

$$a_t \approx 0.68a \quad (\beta = 5.7)\,. \tag{3}$$

Because of the limited spatial extent of our lattice, we do not have very precise estimates of a_s, but we do find ourselves consistent with $a_s \approx a\ (\beta = 5.8)$. Our measurements thus suggest corrections of O(30%) to the perturbative estimate. In any case, these measurements confirm that, according to the criteria we specified earlier [4], our lattice at these values of β_s, β_t is suitable for glueball mass calculations. Moreover, on the basis of our previous experience [4], we expect that (1) will certainly be accurate for $t = 2a_t$, $t' = 3a_t$, and that for heavier glueballs $t = a_t$, $t' = 2a_t$ should suffice.

In the calculation reported on here we confine our systematic exploration of wavefunctions to those whose basic components are purely spatial loops of the form shown in fig. 1. In terms of oddball states we limit ourselves to the (expectedly low-lying) 0^{--} and 1^{-+} glueballs. It is possible that there also exists a light 2^{+-} oddball, but at the moment we cannot comment on that possibility. Taking the trace of such loops makes the operator a colour singlet, and when we talk of any loop it is to be understood that this operation has been performed. To obtain states of differing charge configuration we use the fact that the real part of a loop transforms as $C = +1$ and the imaginary part as $C = -1$. Parity inverts a spatial loops as in fig. 1; taking linear combinations of a loop and its parity inverse allows us to construct operators of $P = +1$ and $P = -1$. Rotating through multiples of $\pi/2$ around the three axes and taking appropriate linear combinations allows us to isolate operators of definite lowest spin J. Finally, to isolate operators of definite lowest spin J. Finally, by summing such operators over all sites at a given time t gives us our zero-momentum operators $\phi(t)$. Applying these rules to the operators in fig. 1, we find that a suitable 0^{--} wavefunction can be formed from $\mathrm{Im}\,(\phi_1 - \phi_2)$ and 1^{-+} from $\mathrm{Re}\,(\phi_1 - \phi_2)$. We construct a spin 2 wavefunction by adding the operator ϕ_1 in fig. 1 and the three operators obtained by rotations of $\pi/2$, π, $3\pi/2$ around the x-axis, and subtracting from this sum a similar sum with respect of the y-axis. The real part of the resulting wavefunction

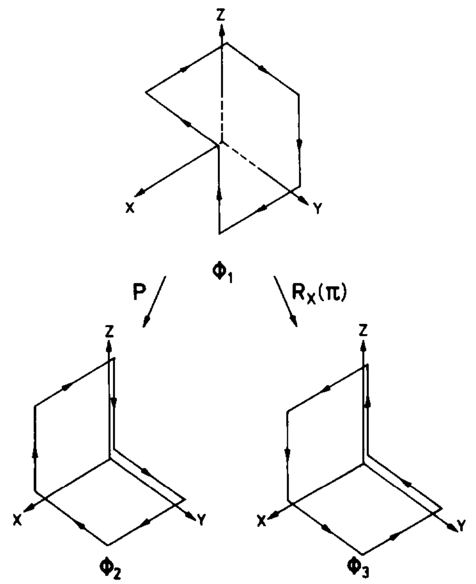

Fig. 1. The operator that forms the basic component of our oddball wavefunctions. Parity inversion takes ϕ_1 to ϕ_2, and a rotation by π about the x-axis takes ϕ_1 to ϕ_3.

gives us a 2^{++} operator and the imaginary part a 2^{--} operator. We also construct 0^{++} operators from the operators in fig. 1 as well as from planar loops; this allows us to set the scale of the lattice.

To optimise our wavefunctions (and the signal-to-noise ratio in the spirit of refs. [1,4]) we consider linear combinations of the basic 6 link operators (with 1 link on each side) and the scaled-up 12 link operators (with 2 links on each side). In fig. 2 we plot the measured ratios Γ_{2a}/Γ_a for the "optimised" wavefunctions (see below) for the 0^{--} and 1^{-+} glueballs, and the ratio Γ_{3a}/Γ_{2a} for the 0^{++}. These ratios are plotted versus the number of lattice configurations over which they have been averaged. The final values (based on 6000 iterations) provide us with best estimates of $\exp(-ma_t)$ [as in eq. (1)], and the fluctuations, visible on fig. 2, provide us with an error estimate. We find

$$\begin{aligned} 0^{++}&: \quad ma_t = 0.67 \pm 0.04\,, \\ 0^{--}&: \quad ma_t = 2.3^{+1.6}_{-0.7}\,, \\ 1^{-+}&: \quad ma_t = 1.6^{+0.32}_{-0.24}\,, \\ 2^{++}&: \quad ma_t = 1.46 \pm 0.10\,. \end{aligned} \tag{4}$$

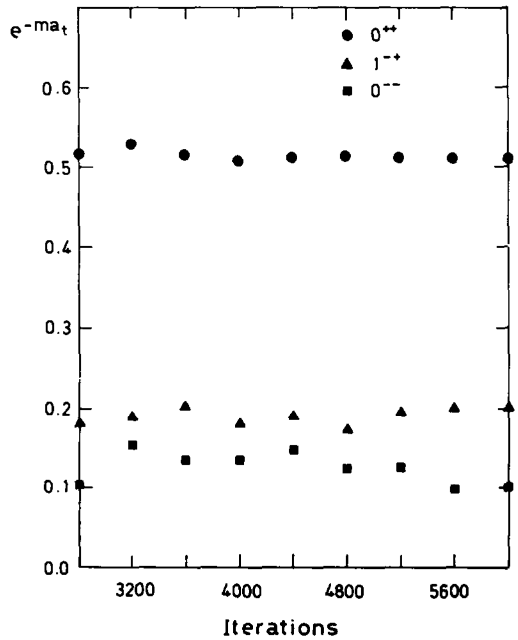

Fig. 2. Estimates of $\exp(-ma_t)$ for the 0^{++}, 0^{--}, 1^{-+} glueballs as a function of the number of iterations (lattice configurations) averaged over.

In obtaining these numbers we find that for the 1^{-+} (and also the 2^{++} not listed here) the Γ_{2a}/Γ_a ratios for different wavefunctions vary by almost a factor of two. This implies that this ratio is, for these glueballs, still somewhat sensitive to the quality of the wavefunction used, and that one should probably go to Γ_{3a}/Γ_{2a} to be confident of obtaining a good estimate of these masses (indeed all our 3 values of Γ_{3a}/Γ_{2a} for the 0^{++} are identical within small errors). In any case since the mass estimates are always upper bounds on the true mass, the best mass estimate is obtained by using the largest Γ_{2a}/Γ_a value, and this is the procedure we used in obtaining the numbers in (4), except for the 0^{--} where the errors were so large that we took the statistical average of the 6 link and 12 link operators.

To obtain Γ_{3a}/Γ_{2a} using zero momentum wavefunctions would require a great deal of computer time. Instead we shall take for our wavefunctions the elementary components described earlier, around a single lattice point, and measure the correlation functions of these localized operators. Since we now get a separate measurement of a correlation function for each lattice point, this procedure will give us numbers with much smaller errors. Since the operators now have non-zero momentum what we measure through ratios of correlation functions is not ma_t but Ea_t where we parametrize the momentum smearing as [1]

$$E^2a_t^2 = m^2a_t^2 + \delta^2\,. \tag{5}$$

This is a reliable procedure as long as

$$m^2a_t^2 \gg \delta^2\,, \tag{6}$$

which will turn out to be the case for all but the 0^{++} glueball. Taking our measured values of Γ_{3a}/Γ_{2a} for the 2^{++} glueball and comparing to the zero-momentum value we previously obtained [4] allows us to estimate δ^2:

$$\delta^2 = 0.4 \pm 0.4\,. \tag{7}$$

We see that (6) is well satisfied for the 2^{++}, but not for the 0^{++} which is why we use the 2^{++} to extract δ^2. We now use (7) together with our measured values of Γ_{3a}/Γ_{2a} for the momentum smeared wavefunctions to obtain

$$\begin{aligned} 1^{-+}&: \quad ma_t = 1.57 \pm 0.17\,, \\ 0^{--}&: \quad ma_t = 2.61 \pm 0.21\,. \end{aligned} \tag{8}$$

Now in our previous calculation [4] we estimated the 0^{++} mass to be ≈ 0.72 GeV (by calculating the string tension at that β and assuming its value to be 400 MeV). If we use this value in (8) we obtain

$$1^{-+}: \quad m = 1.68 \pm 0.18 \text{ GeV} ,$$

$$0^{--}: \quad m = 2.79 \pm 0.22 \text{ GeV} . \qquad (9)$$

We hasten to caution the reader against taking this translation into physical units too definitively; our calculation was in the pure gauge theory and the precise value of the string tension (as inferred from Regge slopes) may exhibit some variation with the inclusion of fermions. Nonetheless the mass ratios we obtain should remain very similar in the full QCD theory with fermions; and this should be particularly true for the oddballs which to lowest order do not mix with $q\bar{q}$ pairs at all. We can therefore reexpress the results of our calculation as demonstrating the approximate equality of the 1^{-+}, 0^{-+} and 2^{++} masses, with the 0^{--} about 70% heavier. To the extent that either the 0^{-+} or the 2^{++} may be ascribed to their candidate states [6,7], the masses of the 0^{--} and 1^{-+} are predicted to be as in (9).

The prediction of such relatively light oddballs [‡2] should provide encouragement for experimental searches. For example, the 0^{--} might be seen in $\chi \to \gamma + X$ or in $J/\psi \to \eta + X$, and the 1^{-+} might be seen in $J/\psi \to \gamma + X$.

‡2 Upper bounds for the 0^{--}, 1^{-+} masses are given in ref. [5]; our mass estimates are much smaller and hence are consistent with their numbers.

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