NOTE ON THE ANGULAR ANALYSIS OF THE DECAY $J/\psi \to f(\to \pi\pi) + \gamma$

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We derive the general (4-parameter) angular decay distribution for the cascade decay $J/\psi \to f(\to \pi\pi) + \gamma$ and determine its angular coefficients from the absorptive and dispersive contributions of the lowest order QCD diagram.

Following a suggestion by Krammer in 1979 [1] the PLUTO group at DESY performed an angular analysis of the cascade decay $J/\psi \to f(\to 2\pi) + \gamma$ [2]. The angular analysis was repeated by the Crystal Ball Collaboration [3] and the MARK II Collaboration [4] on a much larger data sample.

A straight-forward theoretical angular momentum analysis of the above cascade decay yields the (4-parameter) angular decay distribution:

$$\begin{aligned}
&d\sigma/d\cos\theta_\gamma\, d\phi_p\, d\cos\theta_p \propto (1+\cos^2\theta_\gamma)[\tfrac{3}{4}\sin^4\theta_p\, |H_2|^2 + \tfrac{1}{2}(3\cos^2\theta_p - 1)^2\, |H_0|^2] \\
&+ 2\sin^2\theta_\gamma\, [\tfrac{3}{4}\sin^2 2\theta_p\, |H_1|^2] + 2\sin^2\theta_\gamma \cos 2\phi_p\, [\tfrac{1}{4}\sqrt{6}\sin^2\theta_p\, (3\cos^2\theta_p - 1)\,\mathrm{Re}(H_0 H_2^*)] \\
&+ 2\sqrt{2}\sin 2\theta_\gamma \cos\phi_p\, [-\tfrac{3}{8}\sin 2\theta_p \sin^2\theta_p\, \mathrm{Re}(H_1 H_2^*) + \tfrac{1}{8}\sqrt{6}\sin 2\theta_p (3\cos^2\theta_p - 1)\,\mathrm{Re}(H_0 H_1^*)], \qquad (1)
\end{aligned}$$

where θ_γ is the polar angle between the photon and the e^+e^- beam axis. θ_p and ϕ_p are the polar and azimuthal angles of the pseudoscalar mesons in the 2^{++} rest frame, relative to the photon with $\phi_p = 0$ defined by the e^+e^- beam direction. The H_i are the 3 helicity amplitudes describing the decay $J/\psi \to f + \gamma$ and are labelled by the helicity of the f-meson.

Unfortunately, the experimental analyses [2–4] were based on a restricted (2-parameter) form of the general decay distribution (1), assuming erroneously that the 3 helicity amplitudes in (1) are relatively real. Thus the conclusion reached in refs. [3,4] that the experimental results are at variance with QCD must be considered premature. First, the experimental results were based on a restricted angular analysis and second, they were compared to an incomplete evaluation of the relevant lowest order QCD diagram in which only the absorptive part had been considered [1].

It is the purpose of this note to complete the evaluation of the lowest order QCD diagram fig. 1 by calculating also its dispersive part. We urge the experimentalists to reanalyze their data using the general distribution (1) and to compare their results with the complete lowest order QCD result which we shall present in this paper. Any light that can be shed on the production mechanism of ordinary quark–antiquark meson states in radiative ψ-decays will

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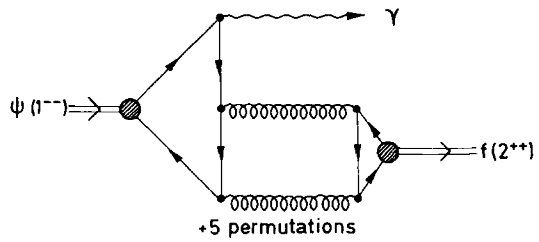

Fig. 1. Lowest order QCD diagram contributing to $\psi \to f + \gamma$.

help in the identification of the long-sought-after elusive glueball states expected to appear in these decays [4].

In the following we shall present the results of evaluating the dispersive and absorptive contributions of the lowest order QCD Feynman diagram depicted in fig. 1.

The vertices $J/\psi \to g_V + g_V + \gamma$ and $f \to g_V + g_V$ are calculated in the usual static quark approximation with non-relativistic constituent quarks as given, e.g. in ref. [5]. The vertex $J/\psi \to g_V + g_V + \gamma$ involves the radial wave function of the constituent quarks at the origin R_0, whereas the vertex $f \to g_V + g_V$ involves the derivative of the radial wave function R'_1 at the origin, since the f-meson is a ${}^3P_2$-state in the static quark approximation. The necessary ${}^3S_1$ and ${}^3P_2$ spin projections of the constituent quarks have been dealt with in a covariant fashion as described in refs. [5,6].

The loop integrands to be treated in the evaluation of fig. 1 are in general fourth degree tensors. These have been contracted to scalars by using covariant helicity projectors which directly project onto the 3 helicity amplitudes H_i of the process. By expanding the scalar integrands into partial fractions we encountered at most two-dimensional Feynman parameter integrals. The real and imaginary parts of the Feynman parameter integrals were then evaluated analytically. Details of the calculation will be reported in a forthcoming publication [7].

We shall present our results in terms of reduced helicity amplitudes $\hat{H}_i$. These are related to the helicity amplitudes by

$$H_i = -(16i/\sqrt{3})\,(\alpha/\pi)^{1/2}\,\alpha_s^2 Q_c (R_0/M_\psi^{3/2})\,(R'_1/M_f^{5/2})\,M_f \hat{H}_i\,, \qquad (2)$$

where α_s is the strong coupling constant, Q_c is the charge of the charm quark, and v is the scaled momentum of the f $(0 \leqslant v \leqslant 1)$.

$$v = (M_\psi^2 - M_f^2)/M_\psi^2 .$$

For the reduced helicity amplitudes one obtains

$$\hat{H}_0 = (4\sqrt{6}/v^3)\{(6-5v)\,v + \tfrac{2}{3}\,[(6-19v+18v^2)/v]\,(1-v)\ln(1-v)$$
$$-\tfrac{1}{3}\,[(10-12v+5v^2)/(2-v)]\,[\mathcal{L}_2(1) - \mathcal{L}_2(1-2v)] + \tfrac{2}{3}\,[(6-38v+71v^2-37v^3)/(1-2v)]\ln(2v)$$
$$-8\,[(1-v)^2/v^2(2-v)]\,[\mathcal{L}_2(1-2v) - 2\,\mathcal{L}_2(1-v) - \tfrac{1}{2}\ln^2(1-v) + \mathcal{L}_2(1) + i\pi\ln(1-v)]$$
$$+\tfrac{4}{3}\,[(6-6v-v^2)/v]\,(\ln 2 - \tfrac{1}{2}\,i\pi) - \tfrac{4}{3}\,(12-26v+13v^2)\,[\mathcal{L}_2(1-v) - \mathcal{L}_2(1-2v) - \ln 2\,\ln(1-v)]\}, \qquad (3)$$

$$\hat{H}_1 = 4\sqrt{2}\,[(1-v)^{1/2}/v^3]\,\{-\tfrac{1}{3}(38-9v)\,v-(2/v)\,(4-13v+16v^2-4v^3)\ln(1-v)$$
$$+\,2\,[v(1-v)/(2-v)]\,[\mathcal{L}_2(1)-\mathcal{L}_2(1-2v)]-[4/(1-2v)]\,(2-11v+16v^2-4v^3)\ln(2v)$$
$$+\,8\,[(1-v)\,(2-2v+v^2)/(2-v)\,v^2]\,[\mathcal{L}_2(1-2v)-2\,\mathcal{L}_2(1-v)-\tfrac{1}{2}\ln^2(1-v)+\mathcal{L}_2(1)+\mathrm{i}\pi\ln(1-v)]$$
$$-\tfrac{16}{3}\,[(3-3v+v^2)/v]\,(\ln 2-\tfrac{1}{2}\,\mathrm{i}\pi)+4(8-12v+3v^2)\,[\mathcal{L}_2(1-v)-\mathcal{L}_2(1-2v)-\ln 2\,\ln(1-v)]\}\,,$$

$$\hat{H}_2 = 4[(1-v)/v^3\,]\,\{\tfrac{16}{3}\,v+(4/v)\,(1-6v+6v^2)\ln(1-v)$$
$$-\,[2/(2-v)]\,(5-6v+2v^2)\,[\mathcal{L}_2(1)-\mathcal{L}_2(1-2v)]+4(1-6v)\ln(2v)$$
$$-\,4\,[(2-4v+6v^2-4v^3+v^4)/(2-v)\,v^2]\,[\mathcal{L}_2(1-2v)-2\,\mathcal{L}_2(1-v)-\tfrac{1}{2}\ln^2(1-v)+\mathcal{L}_2(1)+\mathrm{i}\pi\ln(1-v)]$$
$$+\,\tfrac{4}{3}\,[(6-6v+11v^2)/v]\,(\ln 2-\tfrac{1}{2}\,\mathrm{i}\pi)-16(1-v)\,[\mathcal{L}_2(1-v)-\mathcal{L}_2(1-2v)-\ln 2\,\ln(1-v)]\}. \qquad \text{(3 con'd)}$$

Note that the imaginary parts of the three helicity amplitudes (3) agree with the previous evaluation [1].

We shall first discuss the limiting behaviour of the helicity amplitudes (3) for the two limits $v \to 0$ ($M_{2^{++}} \to M_{Q\bar{Q}}$) and $v \to 1$ ($M_{Q\bar{Q}} \gg M_{2^{++}}$). For $v \to 0$ one finds

$$\hat{H}_0 \approx (16/5\sqrt{6})\,(\tfrac{10}{3}\ln v+\tfrac{4}{3}\ln 2-\tfrac{16}{9}+\mathrm{i}\pi)+\mathrm{O}(v)\,,$$

$$\hat{H}_1 \approx (16/5\sqrt{2})\,(\tfrac{10}{3}\ln v+\tfrac{4}{3}\ln 2-\tfrac{16}{9}+\mathrm{i}\pi)+\mathrm{O}(v),$$

$$\hat{H}_2 \approx \tfrac{16}{5}\,(\tfrac{10}{3}\ln v+\tfrac{4}{3}\ln 2-\tfrac{16}{9}+\mathrm{i}\pi)+\mathrm{O}(v). \qquad (4)$$

As expected on very general grounds the transition reduces to an electric dipole transition $[H_0 \approx H_1(3)^{-1/2} \approx H_2(6)^{-1/2}]$ in this (soft photon) limit which provides a nice check on our calculation.

Next we discuss the limit $v \to 1$ where the quarkonium mass is very much larger than the 2^{++}-mass. One finds

$$\hat{H}_0 \approx (16/\sqrt{6})\,[-2\ln 2\,\ln(1-v)-4\ln 2+\tfrac{3}{2}-\tfrac{5}{4}\;\mathcal{L}_2(1)+\mathrm{i}\pi]+\mathrm{O}(1-v),$$

$$\hat{H}_1 \approx (32\sqrt{2}/3)\,(1-v)^{1/2}\;[\tfrac{3}{2}\ln 2\,\ln(1-v)-\tfrac{9}{4}\ln(1-v)+\tfrac{5}{2}\ln 2-\tfrac{29}{8}-\tfrac{1}{8}\pi^2+\mathrm{i}\pi]+\mathrm{O}((1-v)^{3/2}),$$

$$\hat{H}_2 \approx -16(1-v)\,\{-\tfrac{1}{2}\ln^2(1-v)-\ln(1-v)+\tfrac{4}{3}\ln 2-\tfrac{4}{3}+\tfrac{5}{24}\pi^2+\mathrm{i}\pi\,[\ln(1-v)+\tfrac{11}{6}]\}+\mathrm{O}((1-v)^2). \qquad (5)$$

Up to logarithms the three helicity amplitudes $\hat{H}_0, \hat{H}_1$ and $\hat{H}_2$ behave as $(1-v)^0, (1-v)^{1/2}$ and $(1-v)$ in this limit, respectively. The origin of this relative $(1-v)$-power dependence is the mass dependent normalization factor $m_{2^{++}}^{-1} = (1-v)^{-1/2} M_{Q\bar{Q}}^{-1}$ of the helicity-zero spin-1 states that are used to construct the spin-2 f-meson state.

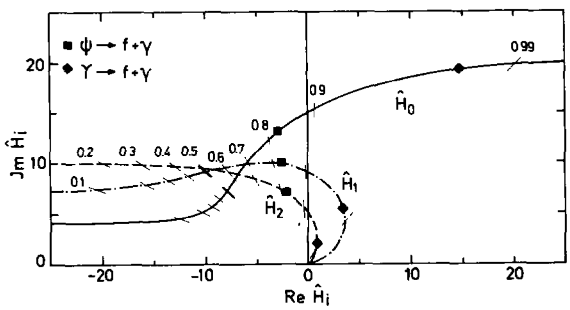

Fig. 2. Argand plot of reduced helicity amplitudes $\hat{H}_i$ as functions of the parameter $v = 1-(M_{2^{++}}/M_{1^{--}})^2$.

Table 1
Reduced helicity amplitudes and phase angles for $\psi \to f + \gamma$ and $\Upsilon \to f + \gamma$.

	$\mathrm{Re}\,\hat{H}_i$	$\mathrm{Im}\,\hat{H}_i$	$\lvert\hat{H}_i\rvert$	φ_i (deg)		$\mathrm{Re}\,\hat{H}_i$	$\mathrm{Im}\,\hat{H}_i$	$\lvert\hat{H}_i\rvert$	φ_i (deg)
$\psi \to f + \gamma\ (v = 0.832)$					$\Upsilon \to f + \gamma\ (v = 0.982)$				
$\hat{H}_0$	−2.95	13.10	13.43	102.7	$\hat{H}_0$	14.66	19.24	24.19	52.7
$\hat{H}_1$	−2.61	9.93	10.27	104.7	$\hat{H}_1$	3.44	5.48	6.47	57.9
$\hat{H}_2$	−2.12	7.04	7.36	106.7	$\hat{H}_2$	0.84	2.17	2.33	68.9

As can be seen from eqs. (4) and (5) the helicity amplitudes are dominantly real both at $v \to 0$ and $v \to 1$. In fig. 2 we have plotted the three helicity amplitudes in an Argand plot. The individual phases monotonically decrease from 180° at $v = 0$ to 0° at $v = 1$. It is quite remarkable that the three helicity amplitudes are very close in phase over the whole range of v-values. In fact the phase difference between the amplitudes never exceeds one degree for $0 \leqslant v \leqslant 0.6$, whereas somewhat larger phase differences develop for $0.6 \leqslant v \leqslant 1$.

In table 1 we list the values of the helicity amplitudes for the two cases of physical interest, i.e. $J/\psi \to f + \gamma$ ($v = 0.832$) and $\Upsilon \to f + \gamma$ ($v = 0.982$), as well as their absolute magnitudes and phases.

In the case of $J/\psi \to f + \gamma$ the real parts of the helicity amplitudes amount to approximately 25% of the imaginary parts. However, as noted above, the amplitudes are quite close in phase [‡1]. It would be quite important to experimentally confirm this general feature of the QCD model calculation. Vice versa, since the complete QCD contribution effectively only introduces a common phase and leaves relative magnitudes unchanged, the predicted angular distribution will not change much from what was obtained from the absorptive part alone [1]. It remains to be seen whether a reanalysis of the data using the general distribution (1) reduces the significance of disagreement with the QCD-result reported in refs. [3,4].

For $\Upsilon \to f + \gamma$ the helicity zero contribution $\hat{H}_0$ dominates the transition showing that the m_f/m_Υ mass ratio is small enough to actuate the spin-kinematical enhancement effect mentioned above. The distribution is thus dominantly $(1 + \cos^2\theta_\gamma)(3\cos^2\theta_p - 1)^2$ with a small correction coming from the $\mathrm{Re}(H_0 H_1^*)$ term.

Finally we consider the absolute normalization for the two decays. From the rate formula

$$\Gamma = \frac{v}{24\pi M_{Q\bar{Q}}} \sum_{i=0,1,2} |H_i|^2, \tag{6}$$

and normalizing $R(0)$ to $\psi \to \ell^+\ell^-$ and $R'(0)$ to $f \to \gamma\gamma$ we obtain:

$$\Gamma_{J/\psi \to f+\gamma} = \alpha_s^4 \cdot 17.3\ \mathrm{keV}. \tag{7}$$

In order to fit the experimental rate $\Gamma_{J/\psi \to f+\gamma} = 95$ eV one would require the effective strong coupling constant to be $\alpha_s \approx 0.27$. Presumably the scale for the running coupling constant in this process is set by the low f-mass. This would give $\Lambda \approx 96$ MeV if one uses the naive first order formula $\alpha_s(Q^2) = 12\pi/[27\ln(Q^2/\Lambda^2)]$.

Assuming that the scale for α_s is approximately equal for $J/\psi \to f + \gamma$ and $\Upsilon \to f + \gamma$ we obtain for the ratio of decay rates

$$\Gamma_{\Upsilon \to f+\gamma}/\Gamma_{J/\psi \to f+\gamma} = \tfrac{1}{4}\,[v(\Upsilon)/v(J/\psi)]\,(M_{J/\psi}^2/M_\Upsilon^2) \sum |\hat{H}_i^\Upsilon|^2 \Big/ \sum |\hat{H}_i^{J/\psi}|^2 \approx 6\%, \tag{8}$$

where we have used the experimental near equality of $R(0)^2/M_{Q\bar{Q}}^2$ for quarkonium states. Thus, since $B_{J/\psi \to f+\gamma} \approx 1.5 \times 10^{-3}$ we expect $B_{\Upsilon \to f+\gamma} \approx 10^{-4}$. Since one can expect to produce 10^5–10^6 Υ's in the next few years in e^+e^--machines, the above branching rate is high enough to allow one to experimentally study this interesting exclusive radiative Υ-decay channel.

‡1 The single tensor-meson-dominance model of ref. [8] also predicts a common phase for the three helicity amplitudes.

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