QCD SELF-ENERGY CONTRIBUTION TO THE MASS-SHIFTS IN THE ^3P_J-STATES

M. KRAMMER and H. SCHNEIDER
Deutsches Elektronen-Synchrotron DESY, Hamburg, Fed. Rep. Germany

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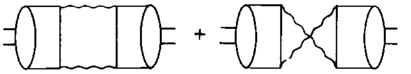
We calculate the self-energy contribution to the mass shift of the ^3P_J-states via the two-gluon intermediate state. The effect consists of an overall shift for all states, which depends on the potential energy, and individual displacements. The corrections are typically of the order of a few MeV.

Nonrelativistic potential models together with QCD describe in a reasonable way the masses and various decay modes of quarkonium states [1]. The hyperfine $^3S_1-^1S_0$ splitting and the splitting of the $^3P_{J=0,1,2}$ states arise from spin–spin, spin–orbit and tensor forces of the Breit–Fermi-hamiltonian. They are of the order of 50–100 MeV.

In the next step contributions of order α_s^2 have to be included. Among them one particular contribution arises from the self-interaction of the various $C = +$ states via a two-gluon exchange. We calculate this contribution for the ^3P_J, $J^{PC} = 0^{++}, 1^{++}, 2^{++}$ states which, in the case of the P_c/χ states of charmonium is of the order of 5–10 MeV for the experimentally measured decay-width of 16.3 ± 3.6 MeV for χ_0 [2].

The self-energy contribution is given by the two following diagrams

 (1)

The coupling of two gluons to the various ^3P_J states can be obtained by the bound state formalism as described e.g. in ref. [3]

$$A^J_{\mu_1\mu_2}\epsilon_1^{\mu_1}\epsilon_2^{\mu_2} = 8\sqrt{2}(3/4\pi M^3)^{1/2}R'_P(0)(k_1\cdot k_2 - i\epsilon)^{-2}a_J\,,$$

$$a_0 = 6^{-1/2}\{[(\epsilon_1\cdot\epsilon_2)(k_1\cdot k_2) - (\epsilon_1\cdot k_2)(\epsilon_2\cdot k_1)](M^2 + k_1\cdot k_2)$$
$$+ (\epsilon_1\cdot k_2)(\epsilon_2\cdot k_2)k_1^2 + (\epsilon_1\cdot k_1)(\epsilon_2\cdot k_1)k_2^2 - (\epsilon_1\cdot\epsilon_2)k_1^2k_2^2 - (\epsilon_1\cdot k_1)(\epsilon_2\cdot k_2)k_1\cdot k_2\}\,,$$

$$a_1 = \tfrac{1}{2}M\{[k_1^2\epsilon(\epsilon^*, \epsilon_1, \epsilon_2, k_2) + (\epsilon_1\cdot k_1)\epsilon(\epsilon^*, \epsilon_2, k_1, k_2)] + [1 \leftrightarrow 2]\}\,,$$

$$a_2 = (M^2/\sqrt{2})[(k_1\cdot k_2)\epsilon_1^a\epsilon_2^b + k_2^a k_1^b(\epsilon_1\cdot\epsilon_2) - k_1^a\epsilon_2^b(\epsilon_1\cdot k_2) - k_2^a\epsilon_1^b(\epsilon_2\cdot k_1)]e^{*ab}\,. \quad (2)$$

Here $k_{1,2}$, $\epsilon_{1,2}$ denote the momenta and polarization vectors of the two gluons. e_a (e_{ab}) stands for the polarization vector (tensor) of the spin 1 (2) bound state of momentum $P = k_1 + k_2$ and mass M and $R'_P(0)$ is the derivative of the radial P-wave function at the origin. In the formula we neglected the binding energy in the term $(k_1k_2 - i\epsilon)^{-2}$. This approximation has to be removed at a later step in the calculation.

Combining (1) and (2) the complex mass shift is given by:

$$\delta M_J^2 = 2M_J(\Delta M_J - \tfrac{1}{2}\mathrm{i}\Gamma_J) = (2J+1)^{-1}\tfrac{2}{3}(4\pi\alpha_s)^2\,[8\sqrt{2}(3/4\pi M^3)^{1/2}R'_{\mathrm{P}}(0)]^2\,\tfrac{1}{2}\mathrm{i}$$

$$\times\int\frac{\mathrm{d}^4k}{(2\pi)^4}\,\frac{1}{(k_1^2+\mathrm{i}\epsilon)(k_2^2+\mathrm{i}\epsilon)(k_1\cdot k_2-\mathrm{i}\epsilon)^4}\sum_{\text{polarisations}}(a_J^*a_J)\,,\qquad k_{1,2}=\tfrac{1}{2}P+k\,. \tag{3}$$

When evaluating (3) it proved convenient to use the Cauchy integral formula for the k_0 integration. Since the dependence on the angular variables is trivial in this case, one is then left with just one integration over $|\boldsymbol{k}|\equiv k$:

$$\int \mathrm{d}k_0 k^2\,\mathrm{d}k\,\mathrm{d}\Omega\,\frac{F(k_0,k)}{(k_1^2+\mathrm{i}\epsilon)(k_2^2+\mathrm{i}\epsilon)(k_1\cdot k_2-\mathrm{i}\epsilon)^4}$$

$$=8\pi^2\mathrm{i}\int k^2\mathrm{d}k\left[-\frac{F(k_0=-\tfrac{1}{2}M-k,k)}{4M^5k^5(k+\tfrac{1}{2}M)}+\frac{F(k_0=\tfrac{1}{2}M-k,k)}{4M^5k^5}\,[\mathbf{P}(1/(k-\tfrac{1}{2}M))+\mathrm{i}\pi\delta(k-\tfrac{1}{2}M)]\right.$$

$$\left.+\frac{1}{3!}\frac{\mathrm{d}^3}{\mathrm{d}k_0^3}\left(\frac{F(k_0,k)}{[(k_0^2-k^2+\tfrac{1}{4}M^2)^2-M^2k_0^2]\,[k_0-(k^2+\tfrac{1}{4}M^2)^{1/2}]^4}\right)_{k_0=-(k^2+M^2/4)^{1/2}}\right]. \tag{4}$$

In our case the integrals coming from the first two terms diverge logarithmically for $k=0$. We introduce a cut-off to make them finite. Physically this cut-off is determined by the binding energy of $\mathrm{Q\bar{Q}}$, which was neglected so far. The term $k_1k_2-\mathrm{i}\epsilon$ actually reads $k_1k_2+(m_{\mathrm{Q}}^2-\tfrac{1}{4}M^2-\langle q^2\rangle)-\mathrm{i}\epsilon$ with m_{Q} being the effective quark mass and $2q$ the relative momentum between Q and $\bar{\mathrm{Q}}$. In a nonrelativistic approximation one has

$$M=2m_{\mathrm{Q}}+E_{\mathrm{kin}}+E_{\mathrm{pot}}\,,\qquad \langle q^2\rangle=-\langle \boldsymbol{q}^2\rangle=-m_{\mathrm{Q}}E_{\mathrm{kin}}\,. \tag{5}$$

The term neglected in $k_1k_2-\mathrm{i}\epsilon$ is thus given by

$$m_{\mathrm{Q}}^2-\tfrac{1}{4}M^2-\langle q^2\rangle\simeq-\tfrac{1}{2}ME_{\mathrm{pot}}\,, \tag{6}$$

which leads to a cut-off of order $|E_{\mathrm{pot}}|$.

Our result for the complex mass shift is

$$\delta M_J=(\Delta M-\tfrac{1}{2}\mathrm{i}\Gamma)_J=\tfrac{8}{3}(\alpha_s^2/\pi)[R'_{\mathrm{P}}(0)]^2M^{-4}\begin{Bmatrix}8\ln(M^2/4E_{\mathrm{pot}}^2)+36\ln(2)+\tfrac{58}{3}-18\,\mathrm{i}\pi\\ 8\ln(M^2/4E_{\mathrm{pot}}^2)\qquad\qquad\qquad\tfrac{52}{3}-0\\ 8\ln(M^2/4E_{\mathrm{pot}}^2)+\tfrac{48}{5}\ln(2)+\tfrac{128}{15}-\tfrac{24}{5}\mathrm{i}\pi\end{Bmatrix}\ \text{for}\ \begin{Bmatrix}J=0\\ J=1\\ J=2\end{Bmatrix}. \tag{7}$$

The missing imaginary part for $J=1$ is a direct consequence of "Yang's"-theorem. The logarithmic part of the mass shift, which is common for all states can be determined experimentally from the decay width of $^3\mathrm{P}_1$ [4]:

$$\Gamma(^3\mathrm{P}_1\to \mathrm{g}q\bar{q})=(\tfrac{1}{3}n\alpha_s)\,\tfrac{64}{3}(\alpha_s^2/\pi)[R'_{\mathrm{P}}(0)]^2M^{-4}\ln[M^2/4(E_{\mathrm{kin}}+E_{\mathrm{pot}})^2]\,. \tag{8}$$

The different arguments in the logarithms of (7) and (8) come from neglecting $\langle q^2\rangle=-m_{\mathrm{Q}}E_{\mathrm{kin}}$ in deriving (8).

Given an experimental width around 1 MeV [2], $n=3$ for the number of flavors and an $\alpha_s=0.2$ (0.5) we get for the common mass shift a value of 5 (2) MeV. The remaining individual mass shifts can be obtained from the annihilation width of the $^3\mathrm{P}_0$ state, $\Gamma^{0^{++}}=(16.3\pm3.6)$ MeV [2]. The total mass shifts are thus given by

$$\Delta M_J=5\ (2)\ \mathrm{MeV}+\begin{Bmatrix}6.3\\ 2.5\\ 2.1\end{Bmatrix}\mathrm{MeV}\ \ \text{for}\ \begin{Bmatrix}J=0\\ J=1\\ J=2\end{Bmatrix}. \tag{9}$$

These corrections are too small and in addition do not have the right signs to correct the experimentally wrong level-splitting as obtained from the Breit–Fermi-hamiltonian

$$R = (M_{2^{++}} - M_{1^{++}})/(M_{1^{++}} - M_{0^{++}}) > 0.80 \quad \text{for Breit–Fermi}, \\ = 0.48 \quad \text{experiment}, \\ = 0.47 \quad \text{experiment corrected}. \tag{10}$$

The last entry comes from subtracting our theoretical mass shifts from the experimental values.

Finally we should mention that our corrections lead to a shift of the center of gravity of the P_c/χ states with respect to the $^1P_1$ state of 5(2) + 2.7 MeV.

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