Nuclear Physics B232 (1984) 349–355

MONOPOLE CATALYSIS OF PROTON DECAY

T. F. WALSH

Deutsches Elektronen-Synchrotron DESY, Hamburg, Federal Republic of Germany

P. WEISZ†

II. Institut für Theoretische Physik der Universität Hamburg, Federal Republic of Germany

Tai Tsun WU*

Deutsches Elektronen-Synchrotron DESY, Hamburg, Federal Republic of Germany

Received 25 April 1983

The Rubakov effect (monopole catalysis of proton decay at strong-interaction cross sections) is absent for SU(5) grand unified monopoles, because of the non-existence of a zero-energy fermion-monopole bound state in this theory.

1. Introduction

Two years ago, Rubakov [1] obtained the very interesting result that magnetic monopoles can catalyze proton decay at strong-interaction rates. The details of this calculation were published last year [2]. There are two main assumptions in his consideration:

(a) the fermions are massless;

(b) SU(2) monopoles [3] are used instead of GUT [4] monopoles [5, 6].

Similar results were also obtained by Callan [7].

In his work, Rubakov emphasized the possible difficulties arising from assumption (a) [2]. By contrast, Callan has argued that the Rubakov effect persists after the removal of assumptions (a) [8] and (b) [9]. The issue is vital, so we give here a different analysis of the Rubakov effect, with the fermion masses taken into account. We have used an SU(5) monopole.

The question may be raised why it is essential to consider an SU(5) monopole rather than an SU(2) monopole. Their gauge-field structures are, after all very

† Heisenberg Foundation Fellow

* On leave from Harvard University, Cambridge, Massachusetts. Work supported in part by the US Department of Energy contract no. DE-AS02-76ER03227.

similar. The reason is that the Higgs structure [10] of an SU(5) theory is much more constrained than that of an SU(2) theory. In the present consideration, the Higgs belonging to the 5 representation plays a central role. The admixture of a 45 Higgs does not change the result.

The immediate motivation for the present consideration is the realization (as can be seen clearly from sect. 5 of [2]) that the Rubakov effect relies on a delicate cancellation. This cancellation depends on the asymptotic behavior of the fermion Green function in the monopole field, which is in turn controlled by the zero-energy states. When the fermions are not massless, these zero-energy states have been investigated by Jackiw and Rebbi [11], and Kazama and Yang [12]. Accordingly, we concentrate on these zero-energy states. For an SU(2) monopole, the Higgs can be such as to give a zero-energy fermion-monopole bound state, i.e. a state with energy equal to that of the monopole without the fermion. However, it is found in sect. 3 that, for SU(5), there is no such state, and hence no Rubakov effect.

Since the presence or absence of zero-energy states seems to depend on the details of the coupling, we have no cogent argument whether the Rubakov effect is present in other grand unified theories. If it is, then these theories can be naturally classified according to the presence or absence of the effect, which provides a deep probe of the structures of the underlying group and Higgs system.

2. Dirac equation

The monopole is described [6] by a pure gauge field W_μ coupled to two Higgs systems, one Φ belonging to the 24 representation and one H to the 5:

$$L = -\tfrac{1}{2}\mathrm{Tr}\left(W_{\mu\nu}W^{\mu\nu}\right) + \mathrm{Tr}\left[\left(\mathfrak{D}_\mu\Phi\right)\left(\mathfrak{D}^\mu\Phi\right)\right] + \left(D_\mu H\right)^\dagger\left(D^\mu H\right) - V(\Phi, H), \quad (2.1)$$

where the metric is $(1, -1, -1, -1)$, and the Higgs potential V is assumed to satisfy (see (3.9) below)

$$V(\Phi, H) = V(\Phi, -H) = V(-\Phi, H). \quad (2.2)$$

The SU(2) subgroup used to construct the monopole is taken to be generated by the 3rd and 4th components, and H has only a fifth component, which is real.

The standard choice of fermions is used [4], consisting of a right-handed Ψ belonging to the 5 representation and a left-handed M to the 10. The fermions interact with the monopole, treated as an external field, i.e. the monopole fields are assumed not to be perturbed by the fermions. With the Higgs coupling

$$L_{\mathrm{H}} = GH^\alpha\bar{\Psi}^\beta M_{\alpha\beta} + \tfrac{1}{4}\tilde{G}\varepsilon^{\alpha\beta\gamma\delta\eta}H_\alpha\left(M_{\beta\gamma}\right)^{\mathrm{T}}C^\dagger M_{\delta\eta}, \quad (2.3)$$

where $\varepsilon^{12345} = 1$ and C is the charge-conjugation operator, the Dirac equations are:

$$\left(i\not\partial - g\not W - \sqrt{\tfrac{1}{2}}\,GH_5\right)\begin{pmatrix} d_3 \\ e^+ \end{pmatrix} = 0\,, \tag{2.4}$$

$$\left(i\not\partial - g\not W - \tilde{G}H_5\right)\begin{pmatrix} u^1 \\ u_2 \end{pmatrix} = 0\,, \tag{2.5}$$

where

$$\not W = \begin{pmatrix} W_{\mu 33} & W_{\mu 34} \\ W_{\mu 43} & W_{\mu 44} \end{pmatrix}\gamma^\mu\,. \tag{2.6}$$

For the monopole, W has no time components, while the space components are of the form of the Wu-Yang ansatz [3]

$$\begin{pmatrix} W_{j33} & W_{j34} \\ W_{j43} & W_{j44} \end{pmatrix} = \tfrac{1}{2}\varepsilon_{jkl}\sigma_k \hat{\boldsymbol{r}}_l g^{-1} w(r)\,, \tag{2.7}$$

where σ_k are the Pauli matrices, and $\hat{\boldsymbol{r}} = \boldsymbol{r}/r$ is the unit radial vector. If ψ is defined to be [11]

$$\psi = (d_3 e^+)\sigma_2\,, \tag{2.8}$$

then (2.4) takes the simple form

$$i\gamma^\mu \partial_\mu \psi + \gamma^j \psi \varepsilon_{jkl} \tfrac{1}{2}\sigma_k \hat{r}_l w(r) - \sqrt{\tfrac{1}{2}}\,GH_5(r)\psi = 0\,. \tag{2.9}$$

The question to be studied in sect. 3 is: is there a zero-energy bound state?

3. Zero-energy bound states

To find zero-energy bound states, set $\partial_0 = 0$. Let

$$\psi = \begin{pmatrix} \psi^+ \\ \psi^- \end{pmatrix}, \tag{3.1}$$

so that ψ^+ and ψ^- are both 2×2 matrices, then

$$\boldsymbol{\sigma}\cdot\nabla\psi^\pm - \tfrac{1}{2} i\boldsymbol{\sigma}\cdot\psi^\pm \boldsymbol{\sigma}\times\hat{\boldsymbol{r}}\, w(r) \pm m(r)\psi^\pm = 0\,, \tag{3.2}$$

where

$$m(r) = \sqrt{\tfrac{1}{2}}\,GH_5(r)\,. \tag{3.3}$$

Eq. (3.2) differs from eq. (A.5) of Jackiw and Rebbi [11] by not having a factor of σ in the last term. This difference is crucial. Since the 2×2 matrices $\psi^{\pm}(\boldsymbol{r})$ must be of the form

$$\psi^{\pm}(\boldsymbol{r}) = \phi^{\pm}(\boldsymbol{r}) + \chi_j^{\pm}(\boldsymbol{r})\sigma_j, \tag{3.4}$$

it follows from (3.2) that

$$\partial_j \chi_j^{\pm} - \hat{r}_j w(r)\chi_j^{\pm} \pm m(r)\phi^{\pm} = 0, \tag{3.5}$$

$$\partial_j \phi^{\pm} + i\varepsilon_{jkl}\partial_k \chi_l^{\pm} + \hat{r}_j w(r)\phi^{\pm} \pm m(r)\chi_j^{\pm} = 0. \tag{3.6}$$

The boundary conditions are that $\phi^{\pm}(0)$ and $\chi^{\pm}(0)$ exist and that both $\phi^{\pm}$ and $\chi^{\pm}$ approach zero exponentially rapidly at infinity.

It follows from (3.5) and (3.6) that

$$\partial_j\left(\phi^{\pm *}\chi_j^{\pm}\right) = i\varepsilon_{jkl}\chi_j^{\pm}\partial_k\chi_l^{\pm *} \mp m(r)\left[|\phi^{\pm}|^2 + |\chi_j^{\pm}|^2\right]. \tag{3.7}$$

Since an integration by parts shows that

$$\int \mathrm{d}\boldsymbol{r}\, \varepsilon_{jkl}\chi_j^{\pm}\partial_k\chi_l^{\pm *}$$

is real, it follows from (3.7) and (3.3) that

$$\int \mathrm{d}\boldsymbol{r} H_5(r)\left[|\phi^{\pm}(\boldsymbol{r})|^2 + |\chi_j^{\pm}(\boldsymbol{r})|^2\right] = 0. \tag{3.8}$$

On the other hand, since a stable monopole must minimize the total energy, the standard argument that leads to the absence of nodes in the ground-state wave function can be applied to show that $H_5(r)$ does not change sign. It then follows from (3.8) that both $\phi^{\pm}(\boldsymbol{r})$ and $\chi_j^{\pm}(\boldsymbol{r})$ vanish identically. In other words, there is no zero-energy bound state. The essential role played by the non-zero fermion mass is evident.

If the Higgs potential $V(\Phi, H)$ of (1.1) is written in the following general form

$$V(\Phi, H) = -\mu^2 \operatorname{Tr}\Phi^2 - m^2 H^{\dagger}H + a_1(\operatorname{Tr}\Phi^2)^2 + a_2 \operatorname{Tr}\Phi^4$$
$$+ a_3(H^{\dagger}H)^2 + a_4 H^{\dagger}H \operatorname{Tr}\Phi^2 + a_5 H^{\dagger}\Phi^2 H, \tag{3.9}$$

then a first-order perturbation calculation near the Prasad-Sommerfield limit [13] shows that $H_5(r)$ can be either increasing or decreasing as a function of r according to the sign of a_4.

4. Discussion

We used the exact static gauge field and Higgs fields in our analysis. This is not so for the considerations of Rubakov and Callan; notably for Rubakov's calculation of the fermion-number violating condensate with massless fermions. Both authors examine the gauge fields outside a radius $r = r_0 \sim M_X^{-1}$. A boundary condition is imposed on the fermion fields at r_0, after which $r_0 \to 0$ [2,8]. It is desirable to eliminate or to justify this approximate treatment of the core. This may be especially important in future studies of the existence or non-existence of a condensate in theories with massive fermions and a zero energy state [11]. We consider the issue briefly.

The fluctuating gauge field with fixed time component $W_{0,ab} = 0$ $(a, b = 3, 4)$ is a gauge transform of the space components of the static field $W_{j,ab}$ in eq. (2.7). The gauge parameter $\lambda(r, t)$ satisfies $\lambda(0, t) = 0$ because of continuity [8]. We will use a different gauge, where the static $W_{j,ab}$ is fixed and

$$W_{0,ab} = -\frac{\Lambda}{g}(\boldsymbol{\sigma}\cdot\hat{\boldsymbol{r}})_{ab}, \tag{4.1}$$

with $\Lambda(0, t) \equiv \dot{\lambda}(0, t) = 0$. We can now consider Λ a dynamical variable. This is because the part of the action involving the gauge fields alone plus the coupling to fermions involves Λ as

$$\int \mathrm{d}^4x\, L_{\mathrm{WF}} = \frac{2\pi}{g}\int \mathrm{d}t\,\mathrm{d}r\, r^2\left[(\Lambda')^2 + 2(\Lambda)^2\frac{F^2}{r^2}\right] + \int \mathrm{d}t\,\mathrm{d}r\,\Lambda\Sigma' - \int \mathrm{d}t\,\mathrm{d}r\, w(r)S,$$

$$\Sigma(r, t) = \int_0^r \mathrm{d}r\, 4\pi r^2 \mathrm{Tr}\left[\bar{\psi}_0\gamma^0\left(\frac{\boldsymbol{\sigma}\cdot\hat{\boldsymbol{r}}}{2}\right)\psi_0\right],$$

$$S(r, t) = 4\pi r^2 \tfrac{1}{2}\varepsilon_{ikl}\hat{r}_l \mathrm{Tr}\left[\bar{\psi}_0\gamma^i\sigma_k\psi_0\right], \tag{4.2}$$

where the trace is taken with respect to the 3,4 indices and we set $F(r) = 1 - rw(r)$ (it vanishes rapidly for $r \gg M_X^{-1}$). The fermion matrices are defined by

$$\psi_0 = \begin{bmatrix}\psi_0^+\\ \psi_0^-\end{bmatrix}, \qquad \psi_0^{\pm} = \int\frac{\mathrm{d}\Omega}{4\pi}\phi^{\pm}(\boldsymbol{r}) + \sigma_j\hat{r}_j\int\frac{\mathrm{d}\Omega}{4\pi}\hat{r}_j\chi_j(\boldsymbol{r}), \tag{4.3}$$

without any further approximation, the $r \to 0$ behavior of Σ' and S is determined from $\psi_0(r, t)$, which obeys a Dirac equation with gauge fields (4.1), (2.7). Then we find that Σ' and S vanish as r^3 when $r \to 0$.

It is now possible to integrate out Λ exactly. The general solution is involved and it is not in general possible to write down an explicit expression. However, in the

Prasad-Sommerfield [13] limit $F(r)=Cr/\sinh(Cr)$ and we find

$$\int \mathrm{d}^4x\, L_{\mathrm{WF}} = \int \mathrm{d}t\, \frac{g^2}{8\pi}\left[-\int_0^\infty \mathrm{d}r\, \frac{\Sigma^2}{r^2} + \frac{1}{C}\int_0^\infty \mathrm{d}r\, \frac{\Sigma}{r^2}\left(\coth Cr + \frac{Cr}{\sinh^2(Cr)}\right)\right.$$
$$\left. \times \int_0^r \mathrm{d}s\, \frac{\Sigma}{s^2}\left(1-\frac{C^2s^2}{\sinh^2 Cs}\right)\right] + \int \mathrm{d}t\,\mathrm{d}r\, w(r) S. \tag{4.4}$$

The approximation now consists in taking the formal limit $r_0 = 1/C \to 0$ and setting $w(r)=r^{-1}$. The first term in (4.4) (that containing Σ^2/r^2) is then the interaction term in sect. 5 of ref. [2]. However, Σ and S now correspond to a different problem from the original one. In particular, Σ' and S are now in general of order unity as $r \to 0$. In order to make contact with the previous case, we have to impose a non-trivial boundary condition $\Sigma'(0,t)=S(0,t)=0$ (e.g. with both vanishing linearly as $r\to 0$). Then the term involving S in (4.4) is well defined. There are two Σ dependent terms in (4.4). Provided that Σ is bounded, the integrand of the second term vanishes rapidly for $r \gg r_0 = C^{-1}$. It approaches a constant times the integrand of the first term for $r\to 0$. There appears to be no evident difficulty with the $r_0 \to 0$ limit, once one accepts the necessity of a non-trivial boundary condition on Σ' and S for the new problem with $r_0 = 0$.

One of us (TTW) thanks Dr. A.F. Bais, Professor Gerard 't Hooft and Professor Kurt Symanzik for helpful discussions, and Professor Hans Joos, Professor Erich Lohrmann, Professor Paul Söding, and Professor Volker Soergel for their kind hospitality at DESY. T.W. and P.W. wish to thank Dr. J. Stehr and Prof. R. Peccei for their comments.

References

[1] V.A. Rubakov, ZhETF Pisma 33 (1981) 658; JETP Lett. 33 (1981) 644
[2] V.A. Rubakov, Nucl. Phys. B203 (1982) 311
[3] Tai Tsun Wu and Chen Ning Yang, in Properties of matter under unusual conditions, eds. H. Mark and S. Fernbach (Interscience, New York, 1969) p. 349;
G. 't Hooft, Nucl. Phys. B79 (1974) 276;
A.M. Polyakov, ZhETF Pisma 20 (1974) 430; JETP Lett. 20 (1974) 194
[4] H. Georgi and S. L. Glashow, Phys. Rev. Lett. 32 (1974) 438;
H. Georgi, H.R. Quinn, and S. Weinberg, Phys. Rev. Lett. 33 (1974) 451;
H. Georgi, Particles and fields-1974 (APS/DPF Williamsburg) ed. C.E. Carlson (A.I.P., New York 1975) p. 575;
M.S. Chanowitz, J. Ellis, and M.K. Gaillard, Nucl. Phys. B128 (1977) 506
[5] Z. Horvath and L. Palla, Phys. Lett. B69 (1977) 197;
M. Daniel, G. Lazarides, and Q. Shafi, Nucl. Phys. B170[FS1] (1980) 156;
D.M. Scott, Nucl. Phys. B171 (1980) 95, 109
[6] C.P. Dokos and T.N. Tomaras, Phys. Rev. D21 (1980) 2940
[7] C.G. Callan, Jr., Phys. Rev. D25 (1982) 2141

[8] C.G. Callan, Jr., Phys. Rev. D26 (1982) 2058
[9] C.G. Callan, Jr., Nucl. Phys. B212 (1983) 391
[10] P.W. Higgs, Phys. Lett. 12 (1964) 132; Phys. Rev. Lett. 13 (1964) 508;
F. Englert and R. Brout, Phys. Rev. Lett. 13 (1964) 321;
G.S. Guralnik, C.R. Hagen, and T.W.B. Kibble, Phys. Rev. Lett. 13 (1964) 585
[11] R. Jackiw and C. Rebbi, Phys. Rev. D13 (1976) 3398
[12] Yoichi Kazama and Chen Ning Yang, Phys. Rev. D15 (1977) 2300
[13] M.K. Prasad and C.M. Sommerfield, Phys. Rev. Lett. 35 (1975) 760