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# A CONFINING $SU(2)_L \times SU(2)_R$ GAUGE MODEL OF THE WEAK INTERACTIONS

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With the aim of understanding weak interactions as residual hypercolor interactions among composite quarks and leptons, we investigate a confining $SU(2)_L \times SU(2)_R$ hypercolor gauge theory with only fundamental fermions (i.e. without fundamental scalars). A scenario with two different confinement scales, $\Lambda_L \sim G_F^{-1/2} \sim 300$ GeV and $\Lambda_R > \Lambda_L$, is systematically analyzed as a function of the parameter $\xi = \Lambda_R/\Lambda_L$. The general requirement of anomaly saturation is combined with effective lagrangian techniques—a powerful procedure also of interest for more general confining product groups. Two solutions, separated by a phase boundary, emerge. Phase I for $1 \leq \xi < \xi_{crit}$: a *left–right symmetric* spectrum of massless composite quarks and leptons with a possibility for dominance of V–A effective low-energy interactions. Phase II for $\xi > \xi_{crit}$: only left-handed massless composite quarks and leptons (of the Abbott–Farhi type). Phase II results from separate anomaly saturation at the two scales $\Lambda_R$ and $\Lambda_L$, Phase I from joint anomaly saturation (matching). Dynamical insight comes from treating confinement in two steps, at $\Lambda_R$ and $\Lambda_L$ consecutively, which amounts to a complementary version of the technicolor mechanism; it necessarily entails the appearance of Goldstone bosons in the intermediate momentum range, $\Lambda_L < p < \Lambda_R$, which play the rôle of dynamical scalars being subject to $SU(2)_L$ confinement.

## 1. Introduction

Recently strong-coupling confining versions of electroweak gauge models [1–4] have stimulated the intriguing question of whether weak interactions are in fact "strong". The general idea is that a set of fundamental fields experience a strong confining gauge force at an energy scale $\Lambda \sim G_F^{-1/2} = O$ (300 GeV). Quarks and leptons emerge as bound states of these fundamental fields; they are singlets with respect to the "strong" electroweak gauge force. Weak interactions as measured at presently available energies are *residual* interactions among *composite* quarks and leptons much in the same way as the conventional strong interactions (scale $\Lambda_{QCD} \sim$ 200 MeV) among composite hadrons. Some chiral symmetry has to be present in order to prevent "naturally" the quarks and leptons from acquiring a mass of the order of $\Lambda = O(300$ GeV). (Here the analogy with QCD ceases.) Usually 't Hooft's framework [5] is adopted, where the massless fundamental fermionic fields introduce a chiral symmetry into the gauge lagrangian. This chiral symmetry is assumed to

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"survive" strong binding and to keep (via 't Hooft's anomaly matching conditions [5–7]) a set of fermionic bound states, the quarks and leptons, massless.

The confining $SU(2)_L$ model of the weak interactions, as proposed by Abbott and Farhi [3], provides a simple generic realization of these ideas. Let us briefly recapitulate the basic features of this model, since it represents a starting point for our approach. The Abbott and Farhi *gauge lagrangian* is *formally identical* with the one of the standard electroweak Glashow-Salam-Weinberg (GSW) model.

(i) It exhibits an $SU(2)_L \times U(1)$ local gauge symmetry.

(ii) It involves the same fundamental fields, i.e. $4 \times n_f$ massless left-handed fermion doublets $F$ and one complex scalar doublet $\phi$ ($n_f$ = number of families).

(iii) It has the same global $SU(4n_f) \times SU(2)$ symmetry in the limit of vanishing electromagnetic and color coupling constants ($e, g_c \to 0$) and absence of Yukawa couplings; the chiral $SU(4n_f)$ being introduced by the massless fundamental fermions, the global $SU(2)$ by the fact that $\phi$ is complex and thus appears in the potential $V(\phi)$ only in the combination $\phi\phi^\dagger$.

However, the *parameters* entering the gauge lagrangian are *different.*

(a) The scalar potential $V(\phi)$ is supposed to be such that no spontaneous symmetry breaking, i.e. no Higgs mechanism takes place. Correspondingly, the gauged $U(1)$ symmetry is identified with the electromagnetic $U(1)_{\rm e.m.}$, the global $SU(2)$ with the $SU(2)_{\rm WI}$ of *global* weak isospin.

(b) The $SU(2)_L$ gauge coupling constant is supposed to be large such that the $SU(2)_L$ gauge interactions become strong at an energy scale of order $\Lambda_L \sim G_F^{-1/2} \sim 300$ GeV. Confinement is presumed to occur; physical states are $SU(2)_L$ singlet bound states.

The physical fermions, to be identified with the left-handed quarks and leptons, are the $SU(2)_L$ singlet bound states of the fundamental fermion doublet $F$ and the scalar doublet $\phi$, each. The relevant composite $SU(2)_L$ singlet operators are

$$f_{\rm up} = F_i \phi^{*i} \text{ abbreviated by } \text{“}F\phi^\dagger\text{”},$$
$$f_{\rm down} = F_i \varepsilon^{ij} \phi_j \text{ abbreviated by } \text{“}F\phi\text{”}, \tag{1}$$

for $i, j = 1, 2$. All these composite, left-handed quarks and leptons are kept massless most elegantly by 't Hooft's anomaly matching conditions with respect to the triangle anomaly involving three chiral $SU(4n_f)$ currents. The $W^\pm, Z^\circ$ bosons are $SU(2)_L$ singlet bound states with masses of order $\Lambda_L$.

The confining Abbott and Farhi model is complementary [5, 8–10] to the standard GSW model; it leads essentially to the same effective, low-energy weak interaction lagrangian as the GSW model [3, 11, 12].

Even though Abbott and Farhi's model is very attractive, it exhibits two theoretically unsatisfactory features.

(i) It involves elementary scalar fields. At least without supersymmetry such theories are in conflict with 't Hooft's principle of "naturalness" [5]; an incredible fine-tuning of parameters with respect to a large inherent scale (grand unification scale or Planck mass) is required.

(ii) Only the left-handed quarks and leptons are composite fermions with radius $\Lambda_L^{-1}$. Their right-handed partners, in contrast, are point-like spectators.

Other existing realizations [2, 4, 13–17] of a strong-coupling confining gauge theory for the weak interactions share at least one of the problems (i) and (ii). In ref. [13], for example, problem (ii) is avoided in the framework of a confining hypercolor theory with a global $SU(2)_L \times SU(2)_R$ symmetry (for $e \to 0$). A left–right symmetric spectrum of composite quarks and leptons is obtained and parity violation arises via spontaneous symmetry breaking of the chiral SU(2). This model, however, still involves fundamental scalars along with fundamental fermions.

Problem (i) of violation of "naturalness" is shared with the GSW model. A "dynamical scalar" as a bound state of fundamental fermions is a natural answer to (i). Such a "complementary" version of the technicolor [18, 19] idea for confining models has been recently proposed in refs. [14–16]. Double confinement due to product gauge groups

$$SU(2)_L \times SU(N)_{HC} \tag{2}$$

with different confinement scales $\Lambda_L \ll \Lambda_{HC}$, $\Lambda_L \sim G_F^{-1/2} \sim 300$ GeV are considered. $SU(N)_{HC}$ confines first at $\Lambda_{HC}$. Dynamical scalars as bound states of pairs of fundamental fermions are formed. Such a scalar $\phi$ has to transform like a doublet under $SU(2)_L$. In order that it does not decouple from the physics at the scale $\Lambda_L$ (Symanzik-Appelquist-Carazzone theorem [20]) its mass should not be too large as compared to $\Lambda_L$. This is most naturally achieved, if $\phi$ is a (pseudo-)Goldstone boson [14], associated with the spontaneous breakdown of some global symmetry through $SU(N)_{HC}$ confinement. The consistent realization of a dynamical scalar in the form of a Goldstone boson is by no means trivial. E.g. in the class of models discussed by Abbott, Farhi and Schwimmer [15], the scalar is composite but not obviously a Goldstone boson. The models of refs. [14–16] live with the asymmetry of composite left-handed quarks and leptons as opposed to elementary right-handed ones. The starting point of our investigation was an attempt to find a natural joint remedy for the two shortcomings (i) and (ii) in the framework of a strong-coupling, confining gauge theory of the weak interactions. More precisely, we looked for a simple scenario which describes *left-handed as well as right-handed* quarks and leptons as massless bound states of *fundamental fermions* only, but which is potentially able to explain parity violation of low-energy residual weak interactions.

The specific model to be presented in this paper is analyzed by *combining* the general requirement of *anomaly saturation* [5–7] *with effective lagrangian techniques* in various momentum ranges. This way a considerable amount of dynamical information about the non-perturbative sector of the theory may be obtained. It is a second goal of this paper, to illustrate in detail this method of analysis which is useful for a much larger class of theories.

## 2. A scenario of the weak interactions with double confinement

In this section we outline the framework, the strategy of investigation and the main results of this paper.

We take Abbott and Farhi's confining $SU(2)_L$ model with confinement scale

$$\Lambda_L \sim G_F^{-1/2} \sim 300\,\mathrm{GeV} \tag{3}$$

as a starting point. Next we try to implement the complementary version of the technicolor idea, i.e. a double confinement mechanism. The *simplest* product gauge group leading to double confinement is

$$SU(2)_L \times SU(2)_R\,, \tag{4}$$

where $SU(2)_R$ is assumed to confine at an energy scale $\Lambda_R$ with

$$\Lambda_R > \Lambda_L\,. \tag{5}$$

A set of fundamental fermions (preons) is chosen with the following transformation properties

$$F=(2,1)\,, \qquad F'=(1,2)\,, \qquad T=(2,2)\,, \tag{6}$$

with respect to $SU(2)_L \times SU(2)_R$. Fermionic bound states, which are singlets under (4) can be built, if an $F$, an $F'$ and a $T$ type preon are combined. Clearly the choice (6) is *minimal* for obtaining *three-preon, fermionic* bound states which are $SU(2)_L \times SU(2)_R$ singlets.

The multiplicities and handedness with which the different types of preons $F$, $F'$ and $T$ appear give rise to a global chiral symmetry. Since we aim at a left–right symmetric spectrum of massless fermionic bound states, we choose the same multiplicity $M$ for $F$ and $F'$ and, for simplicity, multiplicity 1 for $T$. To be more specific, we choose $M$ left-handed preons of the type $F$, $M$ right-handed preons of the type $F'$ and one left-handed preon* of the type $T$. In a purely left-handed Weyl formulation (where each right-handed state is replaced by its left-handed charge conjugate one) we have the following classification of preons with respect to the resulting symmetry group

$$(SU(2)_L \times SU(2)_R)_{\substack{\text{gauged,}\\ \text{strong}}} \times (SU(M) \times SU(M)' \times U(1)_F \times U(1)_{F'} \times U(1)_T)_{\text{global}}\,, \tag{7}$$

$$F=(2,1/M,1)_{1,0,0}\,, \qquad F'=(1,2/1,\bar{M})_{0,-1,0}\,, \qquad T=(2,2/1,1)_{0,0,1}\,. \tag{8}$$

The set of three subindices denotes the three U(1) quantum numbers. Only one combination of the three U(1)s, an axial U(1), survives breaking through $SU(2)_L \times SU(2)_R$ instantons.

* We could have equally well started from a right-handed Weyl multiplet $T$; it will turn out to be a Majorana fermion anyway.

The global chiral symmetry group will have to contain the exactly conserved $SU(3)_c \times U(1)_{e.m.}$ as a subgroup (treating the color and electromagnetic gauge couplings, $g_c$ and $e$, as negligibly small at $\Lambda_L \sim 300$ GeV). The minimal choice for $M$ is correspondingly $M = 4$ leading to

$$SU(4) \times SU(4)' \supset SU(3)_c \times U(1)_{e.m.} \,. \tag{9}$$

The minimal model with $M = 4$, i.e. symmetry group

$$(SU(2)_L \times SU(2)_R)_{\substack{\text{gauged}\\ \text{strong}}} \times (SU(4) \times SU(4)' \times U(1)_{\text{axial}})_{\text{global}} \tag{10}$$

and preon content

$$F = (2, 1/4, 1)_{-1} \,, \qquad F' = (1, 2/1, \bar{4})_{-1} \,, \qquad T = (2, 2/1, 1)_2 \,, \tag{11}$$

will be systematically explored in this paper.

In the spirit of 't Hooft's program [5], the global chiral symmetry is assumed to survive, at least partially, the $SU(2)_L \times SU(2)_R$ confinement. The surviving chiral symmetry in turn singles out, via 't Hooft's anomaly matching conditions [5–7], a set of massless fermionic bound states, the candidates for composite quarks and leptons.

As an aside let us briefly confront this $SU(2)_L \times SU(2)_R$ double-confining gauge model with the well-known left–right symmetric extension [21, 22] of the GSW model. The latter is also an $SU(2)_L \times SU(2)_R$ gauge theory involving, however, a double Higgs mechanism. By comparing the degrees of freedom of fundamental fields we feel that our preon content is convincingly economical. Both, representations and multiplicities of the fundamental fermion fields in the spontaneously broken version are identical to those of our preon fields $F$ and $F'$. The simple additional fermion multiplet $T$ in the confining version substitutes, however, the very complicated Higgs sector [22] of the spontaneously broken version.

As formulated above, the dynamics of our model depends on two independent parameters, the confinement scales $\Lambda_L$ and $\Lambda_R$. $\Lambda_L$ is considered to be fixed, $\Lambda_L \sim G_F^{-1/2} \sim 300$ GeV. Thus it is convenient to treat the dynamics as a function of the dimensionless parameter

$$\xi = \frac{\Lambda_R}{\Lambda_L} \geqslant 1 \,. \tag{12}$$

There are two regions in $\xi$, where information on the dynamics can be obtained *by combining effective lagrangian techniques with the general requirement of anomaly saturation.*

(i) $\xi \simeq 1, (\Lambda_R \simeq \Lambda_L)$. A *one-step confinement* with respect to the non-abelian product gauge group $SU(2)_L \times SU(2)_R$ takes place. All anomalies are saturated (matched) in one-step. The appropriate language for

$$p \gtrsim \Lambda_L \simeq \Lambda_R : \quad \mathscr{L}_{\text{gauge(R),gauge(L)}} \,, \tag{13}$$

is a lagrangian which is gauged with respect to $SU(2)_L$ and $SU(2)_R$ with comparable gauge couplings, $g_L \simeq g_R$. The fundamental fields (besides the gauge fields) are the fermions $F$, $F'$ and $T$. For

$$p \ll \Lambda_L \simeq \Lambda_R: \quad \mathcal{L}_{\text{eff(R),eff(L)}}, \tag{14}$$

the system is described by a lagrangian which is effective with respect to $SU(2)_L$ and $SU(2)_R$. The fields are those composite fermions (quarks, leptons, ...) which are kept massless on account of 't Hooft's anomaly matching conditions.

(ii) $\xi \gg 1$, $(\Lambda_R \gg \Lambda_L)$. At momenta $p = 0(\Lambda_R)$ $SU(2)_R$ confines, while the $SU(2)_L$ gauge coupling is still negligibly small $g_L \approx 0$; at momenta $p = 0(\Lambda_L)$ $SU(2)_L$ eventually becomes confining as well. Parallel to this *two-step confinement* runs an anomaly saturation in two steps. One distinguishes three different regions of momenta, each having a different appropriate language:

$$p \gtrsim \Lambda_R \gg \Lambda_L: \quad \mathcal{L}_{\text{gauge(R)}}, \qquad g_L = 0, \tag{15}$$

where the lagrangian is gauged only with respect to $SU(2)_R$. The $SU(2)_R$ singlet fermions $F$ decouple and play the rôle of spectators. The fundamental fields are the $M+2$ left-handed, $SU(2)_R$ doublet fermions $F'$ and $T$ giving rise to a global chiral

$$SU(M+2) \text{ symmetry (i.e. } SU(6) \text{ for } M=4) \tag{16}$$

of $\mathcal{L}_{\text{gauge(R)}}$. The triangle anomalies involving three global $SU(M+2)$ currents have to be saturated by massless $SU(2)_R$ singlet bound states [6, 7]. In sect. 3 these are shown to be necessarily Goldstone bosons accompanying a spontaneous breakdown of the chiral $SU(M+2)$ symmetry.

For

$$\Lambda_L < p < \Lambda_R: \quad \mathcal{L}_{\text{eff(R),gauge(L)}}(g_L \neq 0), \tag{17}$$

the appropriate lagrangian is effective with respect to $SU(2)_R$ and gauged with respect to $SU(2)_L$. The relevant fields are the fundamental fermions $F$ and the Goldstone bosons appearing due to $SU(2)_R$ confinement. Clearly, the $\mathcal{L}_{\text{eff}}$ methods [18] developed for dynamical symmetry breaking, i.e. technicolor, can be adapted easily to this complementary version of technicolor, where the $SU(2)_L$ non-singlet Goldstone bosons are confined (instead of being responsible for the Higgs mechanism).

Finally, for

$$p \ll \Lambda_L: \quad \mathcal{L}_{\text{eff(R),eff(L)}}, \tag{18}$$

we again encounter the effective lagrangian involving those massless composite fermions which are necessary to match the anomalies in the second confinement step. These massless composite fermions now have two alternative interpretations: either as three preon bound states (of the type "$FF'T$") or bound states of the type "$F\phi$" of a preon $F$ and a dynamical scalar $\phi$, which has been identified as a (pseudo-) Goldstone boson.

We perform separately a two-step and a one-step confinement analysis. Combining the results we obtain the following two solutions, separated by a *phase boundary* at some *critical value* $\xi_{\text{crit}}$ of $\xi = \Lambda_{\text{R}}/\Lambda_{\text{L}}$.

(i) In the region $1 \leqslant \xi < \xi_{\text{crit}}$ we find the solution we were looking for: a *left–right symmetric* spectrum of one family of massless composite quarks and leptons. The global SU(2) symmetry of weak isospin as well as the $V-A$ nature of the residual, low-energy weak interactions among the composite quarks and leptons will emerge approximately (sect. 5) provided $\Lambda_{\text{R}}$ is sufficiently much larger than $\Lambda_{\text{L}}$ (i.e. $\xi$ sufficiently much larger than 1). These important dynamical features will be inferrred from the two-step confinement analysis in sects. 3 and 5.

(ii) In the region $\xi > \xi_{\text{crit}}$ a spectrum of *left-handed* massless composite quarks and leptons appears. This is an Abbott–Farhi [3] type, or more precisely an Abbott–Farhi–Schwimmer [15] type solution, however, with composite *Goldstone bosons* playing the rôle of the scalars in the confining $\text{SU}(2)_{\text{L}}$ gauge theory. The $V-A$ nature and global weak isospin symmetry will be exact in this solution.

In both solutions *no exotics*, i.e. no unwanted massless composite fermions with exotic quantum numbers are present.

Both solutions are interesting in their own right, both are candidates (or come close to being candidates) for the description of weak interactions. The issue of which one is the more likely solution of course depends on the size of $\xi_{\text{crit}}$, the critical value of $\xi = \Lambda_{\text{R}}/\Lambda_{\text{L}}$, over which we have no control.

## 3. Two-step confinement analysis

The idealized two-step confinement analysis is appropriate for $\Lambda_{\text{R}} \gg \Lambda_{\text{L}}$. As outlined in sect. 2, in the first step $\text{SU}(2)_{\text{R}}$ confinement for $p \sim \Lambda_{\text{R}}$ is considered with negligibly small $\text{SU}(2)_{\text{L}}$ gauge coupling, $g_{\text{L}} \approx 0$; in a second step $\text{SU}(2)_{\text{L}}$ confines at $p \sim \Lambda_{\text{L}}$. In the first step, for momenta $p \gtrsim \Lambda_{\text{R}}$, the $\text{SU}(2)_{\text{R}}$ singlet preons $F$ decouple and play the rôle of spectators. The fundamental fermions entering the $\text{SU}(2)_{\text{R}}$ gauge lagrangian are the $\text{SU}(2)_{\text{R}}$ doublet preons $F'$ and $T$.

A major rôle in the two-step analysis will play the requirement of anomaly saturation [6, 7]. Quite generally, the triangle anomaly present on the preon level of a confining gauge theory has to be accounted for in the bound state sector by the appearance of *massless* bound states. These can be *massless composite Goldstone bosons* signalling spontaneous breakdown of the chiral symmetry present on the preon level or *massless composite fermions* if the chiral symmetry survives confinement in Wigner–Weyl realization (or a combination of both in case of partial spontaneous breakdown). The appearing massless composite fermions are then subject to 't Hooft's anomaly matching [5] conditions.

In our two-step analysis the crucial question is whether the anomalies introduced in the first step by the preons $T$ and $F'$ and those introduced in the second step by the preons $F$ (for $g_L \neq 0$) are *saturated separately* or *jointly*. Corresponding to these two distinct possibilities we shall find two different solutions.

Let us now go through the two steps of the two-step analysis in detail. *In the first step*

$$\text{for } p \gg \Lambda_R \quad \text{and} \quad g_L \approx 0\,, \tag{19}$$

there are $M+2$ left-handed preons transforming as doublets under $SU(2)_R$, 2 contained in $T$ and $M$ contained in $F'$. Correspondingly, the $SU(2)_R$ gauge lagrangian has a

$$\text{global } SU(M+2) \text{ symmetry}^{\star}\,, \tag{20}$$

which of course contains $SU(2)_L$ as a subgroup. The $M+2$ preon doublets, let us call them $\psi$, transform according to

$$\psi = \begin{pmatrix} T \\ F' \end{pmatrix} = (2, \overline{M+2}) = (2, \overline{\Box})\,, \tag{21}$$

with respect to

$$SU(2)_{R,\text{gauged}} \times SU(M+2)_{\text{global}}\,. \tag{22}$$

As mentioned already in sect. 2 there are non-vanishing triangle anomalies involving three global $SU(M+2)$ currents in the preon sector of the $SU(2)_R$ gauge theory. These anomalies have to be accounted for in the bound state sector by massless $SU(2)_R$ singlet bound states (bosonic or fermionic [6, 7]). *All* $SU(2)_R$ singlet bound states necessarily involve an even number of preons and hence *necessarily are bosons*. Thus the $SU(M+2)^3$ anomaly has to be accounted for by composite Goldstone bosons accompanying a *necessary* spontaneous breakdown of the global $SU(M+2)$ symmetry.

Next, since there are no fundamental scalars available, a condensate of preons must form, when $SU(2)_R$ forces become strong, in order to effect the required spontaneous breakdown of the $SU(M+2)$ symmetry. Given the single preon multiplet $\psi$ there is a unique candidate for a two-preon condensate: $\langle\psi\psi\rangle \neq 0$. Using the "most attractive channel" rules [23], the $\psi\psi$ condensate is an $SU(2)_R$ singlet and thus automatically leaves $SU(2)_R$ unbroken, in accord with our intention to have a confining $SU(2)_R$:

$$\psi\psi = (1, \overline{M+2} \times \overline{M+2}) = \left(1, \overline{\begin{smallmatrix}\Box\\\Box\end{smallmatrix}}\right) + (1, \overline{\Box\Box})\,, \tag{23}$$

with respect to $SU(2)_R \times SU(M+2)$. The Fermi principle forbids the formation of a $(1, \overline{\Box\Box})$ scalar condensate leaving a *uniquely determined two-preon condensate*

$$\psi\psi = \left(1, \overline{\begin{smallmatrix}\Box\\\Box\end{smallmatrix}}\right) \text{ with respect to } SU(2)_R \times SU(M+2)\,. \tag{24}$$

According to the standard analysis of classical potentials [24] (restricted to fourth-order polynomials), a scalar in the antisymmetrical tensor representation leads to

$^{\star}$ The $U(M+2)$ symmetry is broken by $SU(2)_R$ instantons to the $SU(M+2)$ symmetry.

two possible stability channels*

$$\mathrm{SU}(M+2) \to \mathrm{Sp}(M+2)\,, \tag{25a}$$

$$\mathrm{SU}(M+2) \to \mathrm{SU}(2) \times \mathrm{SU}(M)'\,. \tag{25b}$$

(Here we assumed for simplicity of presentation $M$ to be even.) The SU($M$) group in eq. (25b) is distinguished by a prime from the SU($M$) associated with the preons $F$.

The Goldstone bosons accompanying the symmetry breaking (25) transform like the broken generators of SU($M+2$), i.e. like

$$\chi(\tfrac{1}{2}M(M+3)) \text{ with respect to Sp } (M+2)\,, \tag{26a}$$

$$\chi(2, M) + \chi^{\star}(2, \bar{M}) + \varphi(1,1) \text{ with respect to } \mathrm{SU}(2) \times \mathrm{SU}(M)'\,, \tag{26b}$$

respectively.

Next we consider *step 2* of our confinement analysis for $\Lambda_L \leq p < \Lambda_R$. The $\mathrm{SU}(2)_L$ gauge coupling $g_L$ becomes appreciable in this momentum range. A lagrangian which is gauged with respect to $\mathrm{SU}(2)_L$ and effective with respect to $\mathrm{SU}(2)_R$ becomes the appropriate language. The participating fields are the $\mathrm{SU}(2)_L$ gauge fields, the massless $\mathrm{SU}(2)_L$ doublet fermions $F$ and finally the massless composite Goldstone bosons, surviving as the only (leading) manifestations of $\mathrm{SU}(2)_R$ confinement in the momentum range $p \ll \Lambda_R$. The original global SU($M+2$) symmetry is realized *non-linearly* in terms of these Goldstone bosons alone (see e.g. refs. [25, 26]). It is explicitly broken by the $\mathrm{SU}(2)_L$ gauge interactions (which also give rise to radiative masses [18] for the Goldstone bosons).

The explicit form of the lagrangian $\mathscr{L}_{\mathrm{eff(R),gauge(L)}}$ (for $p/\Lambda_R = \mathrm{O}(g_L)$) can be obtained from a straightforward adaptation of Weinberg's general analysis [18] of dynamical symmetry breaking to the complementary, double-confinement scenario (see sect. 5). Of particular importance is the question of vacuum alignment [18], i.e. the question of how the gauged $\mathrm{SU}(2)_L$ is embedded in the original group SU($M+2$) with respect to the rest symmetries, Sp($M+2$) and $\mathrm{SU}(2) \times \mathrm{SU}(M)'$, respectively, which remain after spontaneous symmetry breaking. The technicolor

* channel (25a): $\langle\psi\psi\rangle = C \begin{pmatrix} 0 & 1 & & & & & \\ -1 & 0 & & & & \mathbf{0} & \\ & & 0 & 1 & & & \\ & & -1 & 0 & & & \\ & & & & \ddots & & \\ & & & & & 0 & 1 \\ & \mathbf{0} & & & & -1 & 0 \end{pmatrix}$, i.e. $\langle TT\rangle, \langle F'F'\rangle \neq 0$,

channel (25b): $\langle\psi\psi\rangle = C \begin{pmatrix} 0 & 1 & & \mathbf{0} \\ -1 & 0 & & \\ & & \mathbf{0} & \\ & & & \ddots \\ & \mathbf{0} & & & \mathbf{0} \end{pmatrix}$, i.e. $\langle TT\rangle \neq 0$ but $\langle F'F'\rangle = 0$.

type mechanism of dynamical symmetry breaking would require the $SU(2)_L$ generators to be among the broken $SU(M+2)$ generators [18, 19]. The Goldstone bosons corresponding to these generators would play the rôle of dynamical Higgs scalars, the $SU(2)_L$ would suffer "spontaneous breakdown" by a dynamical Higgs mechanism. This is *not* what we aim at. We want a *confining* $SU(2)_L$, i.e. the *complementary* version of the technicolor mechanism. This realization is possible, whenever the rest symmetry group is large enough to contain the gauged subgroup, which is the case in our setting:

$$\mathrm{Sp}(M+2) \supset \mathrm{SU}(2)_L \times \mathrm{Sp}(M)\,, \tag{27}$$

with the Goldstone bosons transforming as

$$\chi = \phi(2, M) + \mathrm{SU}(2)_L \text{ singlets}\,, \tag{28}$$

and (most conveniently)

$$\mathrm{SU}(2) \times \mathrm{SU}(M)' \equiv \mathrm{SU}(2)_L \times \mathrm{SU}(M)'\,, \tag{29}$$

with the Goldstone bosons transforming as

$$\chi(2, M) + \chi^*(2, \bar{M}) + \varphi(1, 1)\,, \tag{30}$$

respectively.

We observe that in both cases, those Goldstone bosons which are subject to $SU(2)_L$ gauge interactions are $SU(2)_L$ *doublets*! They are composite scalars which suffer $SU(2)_L$ confinement along with the fermions $F$ for $p \simeq \Lambda_L$.

Let us briefly comment on the preon content of the $\psi\psi$ condensate and the Goldstone bosons. The $\psi\psi$ condensate of course has to be a singlet with respect to the rest symmetry. In the case of $Sp(M+2)$ all $M+2$ preons $T$ and $F'$ participate in the condensate [24] and acquire a spontaneous Majorana mass. In the case of $SU(2) \times SU(M)'$ with $SU(2) \equiv SU(2)_L$ it is straightforward to see [24] that only the $T$ component of the $(M+2)$-plet $\psi$ participates in the condensate, i.e. that "Re($TT$)" acquires a non-vanishing vacuum expectation value. Thereby the preon $T$ becomes a Majorana fermion

$$T_L = T_R^c\,, \tag{31}$$

with a spontaneous Majorana mass. The $F'$ preons do not participate in the condensate and thus remain massless. This is reflected in the remaining global $SU(M)'$ symmetry. The preon content of the Goldstone bosons $\chi$ and $\varphi$ associated with the spontaneous breakdown (25b), is easily established to be

$$\begin{aligned} \chi(2, M) &= \text{“}(TF')^{+}\text{”}\,, \\ \chi^*(2, \bar{M}) &= \text{“}TF'\text{”}\,, \\ \varphi(1, 1) &= \mathrm{Im}\ \text{“}TT\text{”}\,. \end{aligned} \tag{32}$$

In the subsequent part of the analysis we shall specialize to $M = 4$ (see eqs. (25): $SU(M+2)$ becomes $SU(6)$, $Sp(M+2)$ becomes $Sp(6)$ and $SU(2)_L \times SU(M)'$

becomes $SU(2)_L \times SU(4)'$). We shall mention in the end which of our results go through for general $M$.

We now return to the question of anomaly saturation, in particular to the issue of separate versus joint anomaly saturation.

The spontaneous breakdown (25a), $SU(6) \to Sp(6)$, leads to the *anomaly free* subgroup $Sp(6)$. This means that all anomalies present on the level of the preons $T$ and $F'$ have been saturated in the $SU(2)_R$ bound state sector in terms of the associated Goldstone bosons (26a). The $SU(4)^3$ anomalies on the level of the $SU(2)_R$ singlet preons $F$ then have to be saturated *separately* in the bound state sector of the $SU(2)_L$ confining gauge theory. Thus the channel $SU(6) \to Sp(6)$ necessarily corresponds to *separate anomaly saturation at the two scales* $\Lambda_R$ *and* $\Lambda_L$.

In the case of the spontaneous breakdown (25b), $SU(6) \to SU(2)_L \times SU(4)'$, there remain the non-vanishing $SU(4)'^3$ anomalies, associated with the massless preons $F'$, which have *not* been saturated in the bound state sector of $SU(2)_R$. Obviously this channel could not be dynamically realized, if one were to insist on separate anomaly saturation. It is only accessible, if the remaining $SU(4)'^3$ anomalies are saturated *jointly* with the $SU(4)$ anomalies associated with the preons $F$. Thus, the channel $SU(6) \to SU(2)_L \times SU(4)'$ necessarily corresponds to *joint anomaly saturation* (of the anomalies of $F$ and $F'$). Joint anomaly saturation may well make sense for $\Lambda_R > \Lambda_L$, as long as $\Lambda_R$ is not too large as compared to $\Lambda_L$. The meaning of "not too large" will be specified further.

Let us now discuss both cases, $SU(6) \to Sp(6)$ with separate anomaly saturation and $SU(6) \to SU(2)_L \times SU(4)'$ with joint anomaly saturation.

In the case $SU(6) \to Sp(6) \supset SU(2)_L \times Sp(4)$ it only remains to saturate the $SU(4)^3$ anomalies associated with the preons $F$ in the bound state sector of the confining $SU(2)_L$. We aim at anomaly matching in terms of massless composite $SU(2)_L$ singlet fermions, to be identified with quarks and leptons. These will have to be bound states of the $SU(2)_L$ doublet preons $F$ and the Goldstone bosons $\phi$, transforming as $(2, 4)$ with respect to $SU(2)_L \times Sp(4)$ (cf. eq. (28)). A solution to 't Hooft's anomaly matching equations is only obtained, if a further breakdown of the global $Sp(4)$ to its maximal subgroup $SU(2) \times SU(2)$ occurs. With respect to $SU(2)_L \times (SU(2) \times SU(2))_{\text{global}}$ the Goldstone boson multiplet $\phi$ decomposes as

$$\phi \to \phi_1(2/2, 1) + \phi_2(2/1, 2) . \tag{33}$$

Both, $\phi_1$ and $\phi_2$, behave like doublets with respect to the confining $SU(2)_L$ and like doublets with respect to one of the global SU(2)s (and like singlets with respect to the other one). Comparing now our intermediate lagrangian $\mathscr{L}_{\text{eff(R),gauge(L)}}$ with the Abbott and Farhi lagrangian [3] (specialized to one family) we can *identify* the *composite Goldstone bosons* $\phi_1$, say, with their *fundamental scalar multiplet*. Its classification as a doublet with respect to a *global SU(2)* symmetry allows for anomaly matching with the habitual solution of one family of left-handed massless composite quarks and leptons. The global SU(2) is of course nothing else but the

$SU(2)_{WI}$ of weak isospin. Notice, the anomaly matching equations only allow one of the two $\phi_i$s in eq. (33) to appear as constituent of light $SU(2)_L$ singlet composite fermions.

This is the result of strict separation of anomaly saturation. It can be summarized as follows. Saturation of the anomalies due to the $T$ and $F'$ preons in the bound state sector of the confining $SU(2)_R$ theory leads to those massless composite Goldstone bosons necessary to play the rôle of the fundamental scalars in Abbott and Farhi's lagrangian. Saturation of the anomalies due to the four preons $F$ in the bound state sector of the confining $SU(2)_L$ theory then leads in the well-known way to one family of left-handed, massless composite quarks and leptons.

This result can be rather straightforwardly extended to general $M \geq 4$. For $M = 4 \cdot n_f$ one obtains $n_f$ *families* of left-handed, massless composite quarks and leptons. However, for increasing values of $M$ one has to live with an increasing number of superfluous Goldstone bosons and with an increasing degree of global symmetries in the Goldstone boson sector of the intermediate lagrangian $\mathscr{L}_{\mathrm{eff(R),gauge(L)}}$.

Next, we discuss the case $SU(6) \to SU(2)_L \times SU(4)'$ with joint saturation of the $SU(4)'^3$ anomalies due to $F'$ and the $SU(4)^3$ anomalies due to $F$. As far as anomaly saturation is concerned, we find ourselves almost in the same starting position as in the one-step confinement analysis, where besides $SU(4)'^3$ and $SU(4)^3$ anomalies also the anomalies related to the axial $U(1)_A$, the $SU(4)'^2 \times U(1)_A$, $SU(4)^2 \times U(1)_A$ and $U(1)_A{}^3$ anomalies have to be saturated. However, as we shall see in sect. 4, the requirement of anomaly matching in terms of massless composite $SU(2)_L \times SU(2)_R$ singlet fermions (quarks and leptons) will enforce a spontaneous breakdown of the $U(1)_A$ also in the one-step confinement analysis. Hence, as far as *joint saturation* of the $SU(4)'^3$ and $SU(4)^3$ anomalies is concerned, the *results of the two-step and the one-step analysis are identical.* In sect. 4 we shall find the solution to joint anomaly matching: one standard family of left-handed as well as right-handed massless composite quarks and leptons.

However, as concerns the dynamics of the residual interactions among the quarks and leptons at low momenta, $p \ll \Lambda_L$, the two-step confinement analysis will be crucial. From the one-step analysis for $\Lambda_L \simeq \Lambda_R$ one obviously would obtain left–right symmetric residual interactions and furthermore, there is no apparent origin for the global $SU(2)_{WI}$ symmetry of weak isospin. Both characteristic features of weak interactions, the V$-$A nature as well as global weak isospin symmetry, will have to arise as approximate properties for $\Lambda_R > \Lambda_L$ in the framework of the two-step confinement analysis (see sect. 5).

Let us summarize. Joint anomaly saturation which makes sense only if $\Lambda_R$ is "sufficiently close" to $\Lambda_L$ leads to a solution of one family of *left-handed and right-handed* composite quarks and leptons. Separate anomaly saturation which makes sense only for $\Lambda_R$ "sufficiently much larger" than $\Lambda_L$ has led to a solution of one family of *left-handed* composite quarks and leptons only.

The two solutions can obviously not coexist in the same phase. There have to be two phases and it is extremely suggestive that the *phase transition* happens at some *critical value* $\xi_{\text{crit}}$ of the parameter $\xi = \Lambda_R/\Lambda_L$. The vague terms, "sufficiently close" and "sufficiently much larger", used above, are then substantiated as follows: the symmetric phase with a left–right symmetric spectrum lives below $\xi_{\text{crit}}$, $1 \leq \xi < \xi_{\text{crit}}$, the asymmetric phase with a purely left-handed spectrum above $\xi_{\text{crit}}$, $\xi > \xi_{\text{crit}}$. Unfortunately, we have no control over the size of $\xi_{\text{crit}}$ (even $\xi_{\text{crit}} = 1$ cannot be excluded). The left–right symmetric solution becomes an admissible solution for the description of weak interactions as residual $SU(2)_L \times SU(2)_R$ interactions, if $\xi_{\text{crit}}$ is at least of the order of 3, say.

Throughout the one-step confinement analysis in sect. 4 we shall assume $\xi_{\text{crit}} > 1$.

## 4. One-step confinement analysis

This analysis is appropriate for $\Lambda_L \simeq \Lambda_R$. The preons $F$, $F'$ and $T$ are treated on an equal footing. They are subject to confinement due to the product gauge group $SU(2)_L \times SU(2)_R$ at $p \sim \Lambda_L \simeq \Lambda_R$. The global symmetry is $SU(4) \times SU(4)' \times U(1)_A$, where $U(1)_A$ is anomaly free with respect to $SU(2)_L \times SU(2)_R$ (see eqs. (10), (11)).

The complete list of left-handed, spin-$\frac{1}{2}$, fermionic three-preon bound states which are singlets under the confining $SU(2)_L \times SU(2)_R$ is given by

$$TFF' , \qquad T^+F^+F' , \qquad T^+FF'^+ , \qquad TF^+F'^+ . \tag{34}$$

They are the candidates for massless composite quarks and leptons. The *massless* composite fermions among (34) have to be singled out by means of 't Hooft's anomaly matching conditions. There are five different types of triangle anomalies to be matched, the $SU(4)^3$ anomaly involving three SU(4) currents the $(SU(4)')^3$ anomaly, the ${U(1)_A}^3$ anomaly and the mixed $SU(4)^2 \times U(1)_A$ and $(SU(4)')^2 \times U(1)_A$ anomalies.

It is straightforward to work out that there is *no* solution to 't Hooft's anomaly matching equations for unbroken $U(1)_A$, SU(4) and SU(4)′ symmetries. We are forced to assume that the strong $SU(2)_L \times SU(2)_R$ forces cause spontaneous breakdown of $U(1)_A$ as well as of $SU(4) \times SU(4)'$. The maximal left–right symmetric subgroup, which admits a solution is

$$\underbrace{SU(3) \times SU(3)'}_{\text{"chiral color"}} \times \underbrace{U(1) \times U(1)'}_{\text{"chiral charge"}} \subset SU(4) \times SU(4)' . \tag{35}$$

It contains $SU(3)_c \times U(1)_{\text{e.m.}}$ as the diagonal (vectorial) subgroup. We feel that we would be stretching the model too far, if we went too much into the details of these breakings. Nevertheless let us mention the following interesting features.

As became already apparent in the two-step analysis the obvious candidate for a dynamical scalar breaking the $U(1)_A$ is a "Re($TT$)" condensate. The spontaneously broken $U(1)_A$ may be interpreted as playing the rôle of an automatic Peccei–Quinn

[27] U(1) in a confining $SU(2)_L$ framework: it is axial and thus forbids mass terms: moreover it is anomalous with respect to $SU(3)_c$ if the color gauge coupling is turned on. Hence, there is an *automatic* solution of the strong $P$ and $CP$ problem. The chiral symmetry needed to eliminate the $\theta$ dependence of QCD is not ad hoc but a consequence of the fermion representations of the fundamental hypercolor theory [28]. The Goldstone boson "Im $TT$", associated with the spontaneous breakdown of $U(1)_A$, then plays the rôle of an axion with extremely small coupling.

The breakdowns (35) are not unwelcome, since they single out $SU(3)_c$ and $U(1)_{e.m.}$ from a unifying Pati–Salam type SU(4). Obvious candidates for dynamical scalars achieving the spontaneous breakings [24]

$$\begin{aligned} &SU(4) \to SU(3) \times U(1)\,, \\ &SU(4)' \to SU(3)' \times U(1)' \end{aligned} \tag{36}$$

respectively, are the condensates*

$$(TF)(TF)^+ = (1, 1/15, 1)\,, \qquad (TF')(TF')^+ = (1, 1/1, 15)\,, \tag{37}$$

respectively, classified with respect to $SU(2)_L \times SU(2)_R \times SU(4) \times SU(4)'$. They transform like the adjoint of SU(4) and SU(4)' respectively. The Goldstone bosons, associated with these breakings are *all* color triplets or antitriplets and thus eventually subject to QCD confinement, when the color coupling becomes large. (Possibly interesting consequences for the hadron spectrum and QCD phenomenology remain to be explored.)

With respect to the remaining symmetry group

$$(SU(2)_L \times SU(2)_R)_{\text{gauge}} \times (SU(3) \times SU(3)' \times U(1) \times U(1)')_{\text{global}} \tag{38}$$

the preons are classified as

$$\begin{aligned} &T = (2, 2/1, 1)_{0,0}\,, \\ &L = (2, 1/1, 1)_{-1/2,0}\,, \qquad L' = (1, 2/1, 1)_{0,1/2}\,, \\ &Q = (2, 1/3, 1)_{1/6,0}\,, \qquad Q' = (1, 2/1, \bar{3})_{0,-1/6}\,, \end{aligned} \tag{39}$$

with $F = (L, Q)$ and $F' = (L', Q')$. Notice that only the preons $Q$ and $Q'$ carry "chiral color". The set of candidates for massless composite fermions is listed in table 1. For each candidate we introduce an integer index $l_i$, $k_i$, $m_i$, $n_i$ with $i = 1, 2, 3, 4$. Remember, the index is defined as the number of left-handed minus the number of right-handed massless fermions in a given representation.

* The formation of these condensates is supported by the 'most attractive channel' rules extended to multi-preon condensates [29]. Exploiting for $\Lambda_L = \Lambda_R$ the isomorphism $SU(2)_L \times SU(2)_R \sim SO(4)$ we find the following relative strengths of attraction from the Casimir rules of ref. [29]. The condensates (37) are as likely as the $TT$ condensate, responsible for the breaking of $U(1)_A$, and "twice as likely" as e.g. an $F'F'$ or $FF$ condensate. This is, moreover, consistent with our previous result of sect. 3 that *no* $F'F'$ condensate is formed in the phase appropriate for $1 \leq \xi < \xi_{\text{crit}}$.

TABLE 1

| Index | Three preon composite fermions classified w.r.t. $SU(3)\times SU(3)'\times U(1)\times U(1)'$ | Quark and lepton candidates (for positive index) |
|---|---|---|
| $l_1$ | $TLL' = (1,1)_{-1/2,1/2}$ | $\nu_L, \nu_R^c$ |
| $l_2$ | $TLQ' = (1,\bar{3})_{-1/2,-1/6}$ | $u_R^c$ |
| $l_3$ | $TL'Q = (3,1)_{1/6,1/2}$ | $u_L$ |
| $l_4$ | $TQQ' = (3,\bar{3})_{1/6,-1/6}$ | contains $\nu_L, \nu_R^c$ |
| $k_1$ | $T(LL')^+ = (1,1)_{1/2,-1/2}$ | $\nu_L, \nu_R^c$ |
| $k_2$ | $T(LQ')^+ = (1,3)_{1/2,1/6}$ | $u_L$ |
| $k_3$ | $T(L'Q)^+ = (\bar{3},1)_{-1/6,-1/2}$ | $u_R^c$ |
| $k_4$ | $T(QQ')^+ = (\bar{3},3)_{-1/6,1/6}$ | contains $\nu_L, \nu_R^c$ |
| $m_1$ | $(TL')^+L = (1,1)_{-1/2,-1/2}$ | $e_L$ |
| $m_2$ | $(TQ')^+L = (1,3)_{-1/2,1/6}$ | $d_L$ |
| $m_3$ | $(TL')^+Q = (3,1)_{1/6,-1/2}$ | $d_L$ |
| $m_4$ | $(TQ')^+Q = (3,3)_{1/6,1/6}$ | contains $d_R^c$ |
| $n_1$ | $(TL)^+L' = (1,1)_{1/2,1/2}$ | $e_R^c$ |
| $n_2$ | $(TL)^+Q' = (1,\bar{3})_{1/2,-1/6}$ | $d_R^c$ |
| $n_3$ | $(TQ)^+L' = (\bar{3},1)_{-1/6,1/2}$ | $d_R^c$ |
| $n_4$ | $(TQ)^+Q' = (\bar{3},\bar{3})_{-1/6,-1/6}$ | contains $d_L$ |

The color representations are of course obtained from a decomposition of the Kronecker product of the two chiral SU(3) representations, the electromagnetic charge is the sum of the two U(1) quantum numbers. We indicate in the table which composite fermions are candidates for quarks and leptons with respect to the color and charge quantum numbers. The states are left-handed for positive index and right-handed for negative index. In the latter case they have to be replaced by their charge conjugates in order to conform with our left-handed Weyl notation.

Given the symmetry (38), the set of preons (39) and the candidates for massless composite fermions in table 1, we single out the composite fermions kept massless by the unbroken chiral $SU(3)\times SU(3)'\times U(1)\times U(1)'$ by means of 't Hooft's anomaly matching equations. It is straightforwardly derived that all triangle anomalies involving three SU(3) type currents or three U(1) type currents or two SU(3) type and one U(1) type currents on the preon and on the composite level are matched if the following set of equations is satisfied

$$
\begin{aligned}
L_2 - M_2 &= L_1 - M_1 ,\\
L_2 + M_2 &= L_4 + M_4 ,\\
L_3 + M_3 &= L_1 + M_1 ,\\
L_3 - M_3 &= L_4 - M_4 ,\\
L_1 + M_1 + 3(L_2 + M_2) &= 2 ,\\
L_1 - M_1 + 3(L_3 - M_3) &= 2 ,
\end{aligned}
\tag{40}
$$

where

$$L_i = l_i - k_i\,, \qquad M_i = m_i - n_i\,, \quad \text{for } i = 1, 2, 3, 4\,. \tag{41}$$

These anomaly matching equations turn out to have only *two* solutions, if the weak condition is added that

$$\text{all } L_i \text{ and } M_i \text{ take only the values } 0\,,\ \pm 1\,,\ \pm 2\,. \tag{42}$$

This is certainly a reasonable condition in view of the fact that $L_i$ and $M_i$ are *differences* of indices. The two solutions are given by

$$\begin{aligned}
&\text{(i) } L_1 = l_1 - k_1 = 2\,, \quad L_2 = l_2 - k_2 = 1\,, \quad L_3 = l_3 - k_3 = 1\,, \quad L_4 = l_4 - k_4 = 0\,,\\
&M_1 = m_1 - n_1 = 0\,, \quad M_2 = m_2 - n_2 = -1\,, \quad M_3 = m_3 - n_3 = 1\,, \quad M_4 = m_4 - n_4 = 0\,,
\end{aligned} \tag{43}$$

$$\begin{aligned}
&\text{(ii } L_1 - l_1 - k_1 = -1\,, \quad L_2 = l_2 - k_2 = 0\,, \quad L_3 = l_3 - k_3 = 0\,, \quad L_4 = l_4 - k_4 = 1\,,\\
&M_1 = m_1 - n_1 = 0\,, \quad M_2 = m_2 - n_2 = 1\,, \quad M_3 = m_3 - n_3 = -1\,, \quad M_4 = m_4 - n_4 = 0\,.
\end{aligned} \tag{44}$$

Case (ii) is discussed in appendix A. It is also potentially interesting; however, we have not yet fully explored it. Let us concentrate in the following on case (i). Under the restriction to positive indices only, eq. (43) admits the following simple solution.

$$\begin{gathered}
l_1 = 2\,, \qquad k_1 = 0\,,\\
m_1 = 1\,, \qquad n_1 = 1\,,\\
l_3 = 1\,, \qquad k_3 = 0\,, \qquad l_2 = 1\,, \qquad k_2 = 0\,,\\
m_3 = 1\,, \qquad n_3 = 0\,, \qquad n_2 = 1\,, \qquad m_2 = 0\,,\\
l_4 = k_4 = m_4 = n_4 = 0\,.
\end{gathered} \tag{45}$$

With the exception of $l_1$ (see below), all non-vanishing indices ("multiplicities") are $+1$, which is a necessary condition [30] that the states be "ground states". The anomaly constraints (43) are obviously invariant under any of the following replacements

$$\begin{aligned}
l_1 &= 2\,, & k_1 &= 0 \rightarrow l_1 = 1\,, & k_1 &= -1\,,\\
l_2 &= 1\,, & k_2 &= 0 \rightarrow l_2 = 0\,, & k_2 &= -1\,,\\
l_3 &= 1\,, & k_3 &= 0 \rightarrow l_3 = 0\,, & k_3 &= -1\,,\\
m_1 &= 1\,, & n_1 &= 1 \rightarrow m_1 = -1\,, & n_1 &= -1\,,\\
m_2 &= 0\,, & n_2 &= 1 \rightarrow m_2 = -1\,, & n_2 &= 0\,,\\
m_3 &= 1\,, & n_3 &= 0 \rightarrow m_3 = 0\,, & n_3 &= -1\,.
\end{aligned}$$

The replacement of an index $+1$ by an index $-1$, however, turns a "ground state" into an excited state, typically involving derivatives in the composite fermion operators. As usual we shall discard such operators throughout. In our left-handed notation, the spectrum of massless composite fermions corresponding to solution (45), consists of the following set of states classified with respect to $SU(3) \times SU(3)' \times U(1) \times U(1)'$ (see table 1)

$$\begin{aligned}
&\nu \equiv \nu_L = (TL')L = (1,1)_{-\frac{1}{2},\frac{1}{2}}\,, &&\nu' \equiv \nu_R^c = (TL)L' = (1,1)_{-\frac{1}{2},\frac{1}{2}}\,,\\
&e \equiv e_L = (TL')^+L = (1,1)_{-\frac{1}{2},-\frac{1}{2}}\,, &&e' \equiv e_R^c = (TL)^+L' = (1,1)_{\frac{1}{2},\frac{1}{2}}\,,\\
&u \equiv u_L = (TL')Q = (3,1)_{\frac{1}{6},\frac{1}{2}}\,, &&u' \equiv u_R^c = (TL)Q' = (1,\bar{3})_{-\frac{1}{2},-\frac{1}{6}}\,,\\
&d \equiv d_L = (TL')^+Q = (3,1)_{\frac{1}{6},-\frac{1}{2}}\,, &&d' \equiv d_R^c = (TL)^+Q' = (1,\bar{3})_{\frac{1}{2},-\frac{1}{6}}\,,
\end{aligned} \tag{47}$$

each one appearing with a "multiplicity" (index) of 1. Thus our aim, to obtain a *left–right symmetric spectrum* of *composite* quarks and leptons, has been achieved. The quarks and leptons (47) obviously make up one standard family*. *No* additional massless composite fermions with *exotic color or charge* assignments appear. Moreover, the leptons are formed from the colorless preons, $T$, $L$ and $L'$. Thus, the composite leptons have zero color radius, which is reassuring. The neutrinos are built from charged preons (electromagnetic charge of $T$, $L$, and $L'$ is $0$, $-\frac{1}{2}$ and $+\frac{1}{2}$ respectively). This implies a non-vanishing charge radius for the neutrino which is necessary [12] for obtaining the correct low-energy effective weak interaction lagrangian.

A further remark concerning $\nu_L$ and $\nu_R^c$ is in place. We chose to interpret the result $l_1 = 2$ of anomaly matching, eq. (45), such that $\nu_L$ and $\nu_R^c$ appear with multiplicity 1 each. This implies identical preon content and hence identical internal quantum numbers for $\nu_L$ and its partner $\nu_R^c$. The Pauli principle then requires that the corresponding composite operators differ in their spin couplings (once derivatives are discarded). This fact will play an important rôle in sect. 5, in connection with parity violation for $p < \Lambda_L$.

One (critical) comment is in order concerning the electrons $e_L$ and $e_R^c$ in the spectrum (47). Our solution (45) represents a minimal saturation of the anomaly equations (43) with the exception of the choice $m_1 = n_1 = 1$ instead of zero. The electrons are, correspondingly, not protected by a chiral symmetry from acquiring a mass. In order to overcome this shortcoming one has to resort to left-over discrete symmetries. The most attractive possibility is to replace the $TT$ condensate, breaking the $U(1)_A$ symmetry, by a $TTTT$ condensate which leaves a discrete $Z(4)$ subgroup of $U(1)_A$ unbroken. Besides keeping the electrons massless, it also opens up the possibility of solutions of the anomaly equations (43) with more than one light family. This issue of higher generations will be discussed elsewhere. The replacement

* We recall that $SU(3)_{color}$ is the diagonal subgroup of $SU(3) \times SU(3)'$ and $U(1)_{e.m.}$ the diagonal subgroup of $U(1) \times U(1)'$.

$\langle\psi\psi\rangle \neq 0$ by $\langle(\psi\psi\psi\psi)^+\rangle \neq 0$ in eq. (24), has the important advantage that all our previous considerations of breaking the global SU(6) symmetry remain untouched, since the totally antisymmetrized representation of $(\psi\psi\psi\psi)^+$ suggested by the Fermi principle is again the $(\overline{15})$ of SU(6).

Next, let us discuss a further neat feature of the spectrum (47). It exhibits a *higher global classification symmetry*, namely one enlarged by an "accidental" *global* $\mathrm{SU(2)\times SU(2)'}$ symmetry. We rewrite the generators of the global U(1) groups as follows

$$\begin{aligned} \mathrm{U(1)}: &\quad T'_{3\,\mathrm{global}} + \tfrac{1}{2}(B-L)\,, \\ \mathrm{U(1)'}: &\quad T_{3\,\mathrm{global}} + \tfrac{1}{2}(B-L)'\,, \end{aligned} \tag{48}$$

where $T_{3\,\mathrm{global}}$ and $T'_{3\,\mathrm{global}}$ denote the diagonal generators of $\mathrm{SU(2)_{global}}$ and $\mathrm{SU(2)'_{global}}$ respectively. Moreover,

$$(B-L)_{\mathrm{vectorial}} = (B-L) + (B-L)' = \text{physical baryon minus lepton number} \tag{49}$$

and as usual

$$Q_{\mathrm{e.m.}} = (T_3 + T'_3)_{\mathrm{global}} + \tfrac{1}{2}(B-L)_{\mathrm{vectorial}} = \text{electro-magnetic charge}\,. \tag{50}$$

Then we obtain a classification of the quarks and leptons of eq. (47) with respect to

$$\begin{aligned} &(\mathrm{SU(2)_L\times SU(2)_R})_{\mathrm{gauge}} \times (\mathrm{SU(3)\times U(1)}_{\frac{1}{2}(B-L)} \times \mathrm{SU(3)'\times U(1)'}_{\frac{1}{2}(B-L)'} \\ &\qquad \times \mathrm{SU(2)\times SU(2)'})_{\mathrm{global}} \end{aligned} \tag{51}$$

as follows

$$\begin{pmatrix} \nu;\, u_1u_2u_3 \\ e;\, d_1d_2d_3 \end{pmatrix}_{\mathrm{L}} = \begin{pmatrix} TL' \\ (TL')^+ \end{pmatrix} \underbrace{(L;\, Q_1Q_2Q_3)}_{F} = (1, 1|1_{-1/2}\oplus 3_{1/6}, 1_0|2, 1)\,, \tag{52}$$

$$\begin{pmatrix} \nu;\, u_1u_2u_3 \\ e;\, d_1d_2d_3 \end{pmatrix}^{\mathrm{c}}_{\mathrm{R}} = \begin{pmatrix} TL \\ (TL)^+ \end{pmatrix} \underbrace{(L';\, Q'_1Q'_2Q'_3)}_{F'} = (1, 1|1_0, 1_{1/2}\oplus \bar{3}_{-1/6}|1, 2)\,. \tag{53}$$

Since the preons are singlets with respect to the global $\mathrm{SU(2)\times SU(2)'}$ groups, their U(1) and U(1)′ charges just correspond to $\frac{1}{2}(B-L)$ and $\frac{1}{2}(B-L)'$ respectively. Thus, the classification of the preons $F$ and $F'$ with respect to the symmetry group (51) reads

$$F = (L;\, Q_1Q_2Q_3) = (2, 1/1_{-1/2}\oplus 3_{1/6}, 1_0/1, 1)\,, \tag{54}$$

$$F' = (L';\, Q'_1Q'_2Q'_3) = (1, 2/1_0, 1_{1/2}\oplus \bar{3}_{-1/6}/1, 1)\,. \tag{55}$$

We observe a striking one-to-one correspondence of the preon representations (54), (55) with those of the quark-lepton family (52), (53). In fact, the representations with respect to $[\mathrm{SU(2)_L\times SU(2)_R}]_{\mathrm{gauge}}$ and $[\mathrm{SU(2)\times SU(2)'}]_{\mathrm{global}}$ are just interchanged for the preons and the composite fermions! Moreover, it is most intriguing that *with respect to this higher global classification symmetry (51) all anomalies are matched with index + 1. All* composites are kept massless. Furthermore, the anomalies are matched *separately* between preons $F$ and the *left-handed* composites and preons $F'$ and the *right-handed* composites. The way how anomalies are matched is most obvious and elegant. This pattern of anomaly matching appeared already in Abbott and Farhi's confining $SU(2)_L$ model [3], however, for the left-handed sector only. Here we find the left–right symmetric realization of it.

The crucial question is, of course, to what extent the global $[SU(2)\times SU(2)']$ classification symmetry, or part of it, is also a symmetry group of the composite quark and lepton interactions [for $e\to 0$]. It is obvious that this global $[SU(2)\times SU(2)']$ symmetry coincides with the habitual chiral SU(2) symmetry of the QCD lagrangian involving "pointlike" *massless* quarks and their interactions with gluons which are charge and flavor neutral. (It will be spontaneously broken by a $q\bar{q}$ condensate at low energies, when $SU(3)_{color}$ becomes strong, giving rise to a Goldstone boson to be identified with the $\pi$ meson . . . .)

The non-trivial question of whether the global SU(2) classification group of the *left-handed* quarks and leptons turns out to be the weak isospin $SU(2)_{WI}$ *symmetry* group of the residual V − A weak interactions for $p<\Lambda_L$ will be addressed in sect. 5.

## 5. Global weak isospin symmetry and parity violation

At low energies, $p\ll\Lambda_L$, *residual* $[SU(2)_L\times SU(2)_R]_{gauge}$ interactions among the massless composite quarks and leptons take place. The residual four-fermion interactions should reproduce the properties of the measured weak interactions, in particular the global $SU(2)_{WI}$ symmetry of weak isospin (for $e\to 0$), the $(V-A)\times(V-A)$ dominance and quark-lepton universality.

First, let us consider our Abbott and Farhi type solution, obtained from separate anomaly saturation and valid for $\xi=\Lambda_R/\Lambda_L>\xi_{crit}$. As pointed out already, in this solution the global $SU(2)_{WI}$ symmetry is exact. It is present already on the preon level as an (unbroken) subgroup of the global Sp(4). Moreover, since there is only a left-handed composite quark-lepton multiplet, its four-fermion interactions are automatically restricted to a $(V-A)\times(V-A)$ form [3] (of course, at the expense of having point-like right-handed quarks and leptons). Quark-lepton universality is a consequence of the SU(4) symmetry, in analogy to ref. [3].

As emphasized already, the situation is much less simple in the case of the solution (47) with a left–right symmetric spectrum of composite quarks and leptons, as obtained from joint anomaly saturation and valid for $1\leq\xi=\Lambda_R/\Lambda_L<\xi_{crit}$. Here, the $SU(2)_{WI}$ of global weak isospin is *not* a symmetry of the fundamental preon lagrangian. Moreover, since now both the left-handed and right-handed quarks and leptons are composites in distinct representations of chiral symmetry, their four-fermion interactions may, a priori, comprise besides $(V-A)\times(V-A)$ also $(V-A)\times(V+A)$ and $(V+A)\times(V+A)$ terms. For $\Lambda_R=\Lambda_L$ we even face $V-A\leftrightarrow V+A$ symmetry instead of $(V-A)\times(V-A)$ dominance.

In this section we study this solution with a left–right symmetric spectrum for $\Lambda_R$ substantially larger than $\Lambda_L$. The crucial question is, whether the global $SU(2)_{WI}$ symmetry and the $(V-A)\times(V-A)$ structure may then arise *approximately.* However, this section also serves a more general purpose. It is to illustrate the type of information, one may obtain on the dynamics of a generic class of composite models with a hierarchy of scales by *combining* effective lagrangian techniques [18, 25, 26] with the requirement of anomaly saturation [5–7]. We recall that the crucial function of the latter is to select a natural set of preferred fields [26] in terms of which the effective lagrangian is written down (c.f. sect. 3)!

First, let us attack the question of an approximate $SU(2)_{WI}$ by constructing the intermediate interaction lagrangian, $\mathscr{L}_{\mathrm{eff(R),gauge(L)}}$ appropriate for the momentum range $\Lambda_L<p<\Lambda_R$.

The first step is to write down $\mathscr{L}_{\mathrm{eff(R)}}$ for $g_L=0$ in terms of the (composite) Goldstone bosons $\chi$ and $\varphi$ only (i.e. with preons $F$ still decoupled). According to eq. (30) $\chi$ and $\varphi$ transform *linearly* with respect to the stability group $SU(2)_L\times SU(4)'$ as $(2,4)$ and $(1,1)$ respectively and have to realize the original global SU(6)symmetry *non-linearly.* Following a classical method by Coleman, Wess and Zumino [25] it is convenient in our case to first define a non-linear function $M=M(\chi,\varphi)$ of $\chi$ and $\varphi$ as follows*.

(i) $M(\chi,\varphi)$ transforms *linearly* with respect to SU(6) like an antisymmetrical tensor ($\begin{smallmatrix}\square\\\square\end{smallmatrix}=15$).

(ii) This $6\times6$ antisymmetrical matrix satisfies the usual constraint that a general SU(6) invariant potential, depending on powers of the SU(6) invariants $\mathrm{Tr}MM^+$ and $\mathrm{Tr}MM^+MM^+$, becomes a constant.

Then

$$\mathscr{L}_{\mathrm{eff(R)};g_L=0}=-\tfrac{1}{2}\mathrm{Tr}\,(\partial_\mu M(\partial^\mu M)^+)\,,\qquad M=M(\chi,\varphi)\,. \tag{56}$$

Since $M$ decomposes with respect to $SU(2)\times SU(4)'$ like

$$M(15)\to S(1,1)+\chi(2,4)+m(1,6)\,, \tag{57}$$

$M$ may be parametrized in terms of the $2\times4$ Goldstone boson matrix $\chi$ and the singlet Goldstone boson $\varphi$ as

$$M(\chi,\varphi)=\sqrt{\tfrac{1}{2}}\begin{pmatrix}-i\tau_2S(\chi,\varphi) & \chi\\ -\chi^{\mathrm{T}} & m(\chi,\varphi)\end{pmatrix}=-M^{\mathrm{T}}\,, \tag{58}$$

* As a reminder we note that for the $SU(2)\times SU(2)$ non-linear $\sigma$-model $M=M(\boldsymbol{\pi})$ takes the following familiar form $M(\boldsymbol{\pi})=\sqrt{f^2-\boldsymbol{\pi}^2}+i\boldsymbol{\tau}\cdot\boldsymbol{\pi}=(2,\bar{2})$, with $\mathrm{Tr}\,MM^+=f^2$. In general, the representation of $M$ is restricted to contain a singlet when decomposed with respect to the invariant subgroup [25]. This leads to $M=(N,\bar{N})$ for $SU(N)_L\times SU(N)_R\to SU(N)_{L+R}$ and to $M=(15)$ for our case of $SU(6)\to SU(2)_L\times SU(4)'$.

with auxiliary fields

$$m(\chi,\varphi)=-m^{\mathrm{T}}=\chi^{\mathrm{T}}(-i\tau_2)\chi/S(\chi,\varphi)\,,$$
$$S(\chi,\varphi)=\mathrm{e}^{i\varphi/f}\cdot\sigma(\chi)\,, \tag{59}$$

and $\sigma(\chi)$ being a solution of the constraint equation

$$\sigma^4+(\mathrm{Tr}\,(\chi\chi^+)-f^2)\sigma^2+\det\,(\chi\chi^+)=0\,,\qquad \sigma_{\chi=0}=f, \tag{60}$$

equivalent to $\mathrm{Tr}\,MM^+=f^2$. Eqs. (58)–(60) imply $\mathrm{Tr}\,MM^+MM^+\propto\mathrm{Tr}\,MM^+$. The decay constant $f$ is the large parameter in the game,

$$f=\mathrm{O}(\Lambda_{\mathrm{R}})\,. \tag{61}$$

Upon inserting the parametrization (58) for $M(\chi,\varphi)$ into eq. (56), $\mathscr{L}_{\mathrm{eff(R)},g_{\mathrm{L}}=0}$ takes the following non-linear form in terms of $\chi$ and $\partial\mu\varphi$

$$\begin{aligned}\mathscr{L}_{\mathrm{eff(R)};g_{\mathrm{L}}=0}&=-\tfrac{1}{2}\mathrm{Tr}\,[\partial_\mu\chi(\partial^\mu\chi)^+]-\tfrac{1}{2}\partial_\mu\varphi\,\partial^\mu\varphi\\&+\mathscr{L}_{\mathrm{int}}(\mathrm{Tr}\,\chi\chi^+;\,\det\chi\chi^+;\\&\chi^{\mathrm{T}}(-i\tau_2)\chi;\,\partial_\mu\,\mathrm{Tr}\,\chi\chi^+;\,\partial_\mu\,\det\chi\chi^+;\partial_\mu\chi^{\mathrm{T}}(-i\tau_2)\chi;\,\partial_\mu\varphi)\,.\end{aligned} \tag{62}$$

To leading order $1/f^2\sim1/\Lambda_{\mathrm{R}}^2$ we find for $\mathscr{L}_{\mathrm{int}}$

$$\begin{aligned}\mathscr{L}_{\mathrm{int}}&=\frac{1}{4f^2}\{\tfrac{1}{2}\partial_\mu(\mathrm{Tr}\,\chi\chi^+)\,\partial^\mu(\mathrm{Tr}\,\chi\chi^+)\\&+\mathrm{Tr}\,[\partial_\mu(\chi^{\mathrm{T}}(-i\tau_2)\chi)\cdot\partial^\mu(\chi^{\mathrm{T}}(-i\tau_2)\chi)^+]\}+\frac{1}{f^3}(\dim 7-\text{operators})+\cdots\end{aligned} \tag{63}$$

Note that terms involving the "axion" $\varphi$ only appear to order $1/\Lambda_{\mathrm{R}}^3$.

Next, we switch on the $\mathrm{SU(2)_L}$ gauge interactions ($g_{\mathrm{L}}\neq0$), whence the fermion multiplet $F$ is locked in. The meson sector is gauged by simply performing in eq. (56) the replacement

$$\partial_\mu M\rightarrow(\mathscr{D}_\mu M)=\partial_\mu M-ig_{\mathrm{L}}A^i_\mu(\lambda_iM+M\lambda_i^{\mathrm{T}})\,, \tag{64}$$

with

$$\lambda_i=\left(\begin{array}{c|c}\tau_i&0\\\hline0&0\end{array}\right),\qquad i=1,2,3, \tag{65}$$

and adding the usual Yang–Mills kinetic term $-\frac{1}{4}\mathscr{F}_{i,\mu\nu}\cdot\mathscr{F}^{i,\mu\nu}$ for the $\mathrm{SU(2)_L}$ gauge fields $A^i_\mu$. Expressed in terms of the Goldstone boson fields $\chi$ and $\varphi$ eq. (64) induces the replacement

$$\partial_\mu\chi\rightarrow(\mathscr{D}_\mu\chi)=\partial_\mu\chi-ig_{\mathrm{L}}A^i_\mu\tau_i\chi\,, \tag{66}$$

in the *kinetic* term for the $\chi$ field *only* (see eq. (62)). In particular $\mathscr{L}_{\mathrm{int}}$ in eq. (62) remains unchanged although it also involves derivatives of the $\chi$ fields. The reason is that these derivatives only act on the auxiliary, $SU(2)_L$ *singlet* fields $\sigma(\chi)$ and $m(\chi, \varphi)$ which, according to eqs. (59), (60), only involve $\chi$ fields in *locally* $SU(2)_L$ invariant combinations.

Before writing down the final form of $\mathscr{L}_{\text{eff(R),gauge(L)}}$, let us introduce a notation, more suitable for our investigation of an approximate $SU(2)_{WI}$ invariance. For guidance, we first identify the *left-handed* quarks and leptons, appearing as three-preon composites in the left–right symmetrical solution (47), as bound states of the preon multiplet $F$ and the composite Goldstone boson multiplet $\chi$ (using the correspondence (32)). In fact, as concerns the left-handed quarks and leptons, their composition is like in the Abbott and Farhi model [3]: we find, in compact matrix notation,

$$(\nu, u_1u_2u_3)_L = \phi^+F, \qquad (e, d_1d_2d_3)_L = (i\tau_2\phi^*)^+F, \tag{67}$$

with $F = (L, Q)$ written as a $2\times4$ matrix. All left-handed composites involve the same *complex* scalar $SU(2)_L$ doublet

$$\phi = \begin{pmatrix}\phi_1\\ \phi_2\end{pmatrix} = \text{“}(TL')^{+}\text{”}, \tag{68}$$

appearing in the decomposition of the $2\times4$ Goldstone boson matrix

$$\chi = (\phi, \Delta) = \phi(2,1)_{-\frac{1}{2}} \oplus \Delta(2,3)_{\frac{1}{6}} \tag{69}$$

with respect to $SU(2)_L\times SU(3)'\times U(1)'$. The Goldstone boson

$$\Delta(2,3)_{\frac{1}{6}} = \text{“}(TQ')^{+}\text{”}, \tag{70}$$

does not appear as a constituent of the left-handed quarks and leptons. As in ref. [3] we may now exhibit the global $SU(2)_{WI}$ symmetry by introducing the $2\times2$ matrix

$$\Omega = (\phi, i\tau_2\phi^*), \tag{71}$$

in terms of which eq. (67) turns into the analogue of eq. (52)

$$\begin{pmatrix}\nu, u_1u_2u_3\\ e, d_1d_2d_3\end{pmatrix}_L = \Omega^+F. \tag{72}$$

$\Omega$ transforms like a $(2,\bar{2})$ of $SU(2)_L\times SU(2)_{WI}$

$$\begin{aligned}&\Omega\to U\Omega, &&U\in SU(2)_L,\\ &\Omega\to\Omega V^+, &&V\in SU(2)_{WI},\end{aligned} \tag{73}$$

such that the composite fermion operator (72) is $SU(2)_L$*invariant* and a *doublet* of $SU(2)_{WI}$. By means of the new field $\Omega$ we are now ready to examine explicitly to what extent $SU(2)_{WI}$ is a symmetry of $\mathscr{L}_{\text{eff(R),gauge(L)}}$. Expressing the $\chi$ field in terms of $\Omega$ and $\Delta$ using eqs. (62), (63), (64), (66), (69), (71)) we finally obtain

$$\mathscr{L}_{\mathrm{eff(R),gauge(L)}} = -\tfrac{1}{2}\partial_\mu\varphi\,\partial^\mu\varphi - \tfrac{1}{4}\mathscr{F}_{i,\mu\nu}\mathscr{F}^{i,\mu\nu} - \bar{F}_{\mathrm{L}}\gamma_\mu\mathscr{D}^\mu F_{\mathrm{L}} - \tfrac{1}{2}\mathrm{Tr}\,[\mathscr{D}_\mu\Delta(\mathscr{D}^\mu\Delta)^+]$$
$$-\tfrac{1}{4}Tr\,[\mathscr{D}_\mu\Omega(\mathscr{D}^\mu\Omega)^+] + \mathscr{L}_{\mathrm{int}}^{\mathrm{SU(2)_{WI}}} + \mathscr{L}_{\mathrm{int}}^{\mathrm{BR}}\,, \tag{74}$$

where we have split $\mathscr{L}_{\mathrm{int}}$ of eq. (62) into an $\mathrm{SU(2)_{WI}}$ invariant part $\mathscr{L}_{\mathrm{int}}^{\mathrm{SU(2)_{WI}}}$ and a breaking part $\mathscr{L}_{\mathrm{int}}^{\mathrm{BR}}$. Obviously, all other terms in (74) are $\mathrm{SU(2)_{WI}}$ invariant.

In leading order $1/f^2 \sim 1/\Lambda_{\mathrm{R}}^2$, eq. (63) gives

$$\mathscr{L}_{\mathrm{int}}^{\mathrm{SU(2)_{WI}}} = \frac{1}{4f^2}\{\mathrm{Tr}\,(\partial_\mu(\Omega^{\mathrm{T}}(-i\tau_2)\Delta)\partial^\mu(\Omega^{\mathrm{T}}(-i\tau_2)\Delta)^+)$$
$$+\mathrm{Tr}\,(\partial_\mu(\Delta^{\mathrm{T}}(-i\tau_2)\Delta)\partial^\mu(\Delta^{\mathrm{T}}(-i\tau_2)\Delta)^+)$$
$$+\tfrac{1}{2}\partial_\mu\,\mathrm{Tr}\,(\tfrac{1}{2}\Omega\Omega^+ + \Delta\Delta^+)\,\partial^\mu\,\mathrm{Tr}\,(\tfrac{1}{2}\Omega\Omega^+ + \Delta\Delta^+)\} + \cdots\,, \tag{75}$$

$$\mathscr{L}_{\mathrm{int}}^{\mathrm{BR}} = \frac{1}{4f^2}\mathrm{Tr}\,(\tau_3\partial_\mu(\Omega^{\mathrm{T}}(-i\tau_2)\Delta)\partial^\mu(\Omega^{\mathrm{T}}(-i\tau_2)\Delta)^+) + \cdots\,. \tag{76}$$

Because of the presence of $\tau_3$, $\mathscr{L}_{\mathrm{int}}^{\mathrm{BR}}$ is only invariant under the global transformations

$$\Omega \to \Omega V^+\,, \qquad V = \mathrm{e}^{i\alpha\tau_3} \in \mathrm{U(1)}\,, \tag{77}$$

corresponding to the U(1) subgroup of $\mathrm{SU(2)_{WI}}$.

Let us summarize. The dim $\leq 4$ operators are $\mathrm{SU(2)_{WI}}$ symmetric; violations appear on the level of dim $\geq 6$ operators. In conclusion, for $\Lambda_{\mathrm{R}}$ sufficiently large, we have an *approximate* $SU(2)_{\mathrm{WI}}$ *symmetry*, at least at this *classical level* (ignoring loops). Its origin is twofold.

(a) The scalar field $\chi = (\phi, \Delta)$ is a *complex* $\mathrm{SU(2)_L}$ doublet, allowing for the introduction of the scalar multiplet $\Omega = (\phi, i\tau_2\phi^*)$ which transforms like $(2, \bar{2})$ with respect to $\mathrm{SU(2)_L \times SU(2)_{WI}}$ as in ref. [3].

(b) The scalar fields $\chi$ and $\varphi$ are *Goldstone* bosons, in terms of which a *higher* symmetry, the SU(6) symmetry, is realized non-linearly in the lagrangian for $g_{\mathrm{L}} = 0$. This fact prevents, besides mass terms, $\mathrm{SU(2)_L \times SU(4)'}$ invariant interaction terms of dimension 4, like $\mathrm{Tr}\,\chi\chi^+\chi\chi^+$ or $\det\chi\chi^+$, to appear already at the classical level with coefficients of order 1. Such terms would violate the $\mathrm{SU(2)_{WI}}$ symmetry of order 1 instead of order $(p/\Lambda_{\mathrm{R}})^2$.

Of course, one has to worry about quantum effects. Since the $\mathrm{SU(2)_L}$ gauge interactions break the global SU(6) symmetry intrinsically, loops containing $\mathrm{SU(2)_L}$ gauge bosons will generate mass terms of the Goldstone fields $\chi$ as well as $\mathrm{SU(2)_{WI}}$ violating terms (which vanish for $g_{\mathrm{L}} \to 0$). The scale $\Lambda_{\mathrm{R}}$ plays the rôle of a cut-off. However, since the $\mathrm{SU(2)_L}$ gauge interactions are confining, it is difficult to state precisely, to which approximation the $\mathrm{SU(2)_{WI}}$ symmetry will feed through into the regime $p < \Lambda_{\mathrm{L}}$. Nevertheless, it is suggestive that the SU(6) symmetry will leave its traces even below $\Lambda_{\mathrm{L}}$, such that $\mathrm{SU(2)_{WI}}$ violations of order $(p/\Lambda_{\mathrm{R}})^2$ for $p > \Lambda_{\mathrm{L}}$

will turn into $\mathrm{SU}(2)_{\mathrm{WI}}$ violations of order

$$\left(\frac{\Lambda_{\mathrm{L}}}{\Lambda_{\mathrm{R}}}\right)^2, \qquad \text{for } p<\Lambda_{\mathrm{L}}. \tag{78}$$

These are the violations we expect for the full low-energy effective lagrangian. We recall that the breaking of $\mathrm{SU}(2)_{\mathrm{WI}}$ is associated with the interactions of the colored $\Delta$ field which is *not* a constituent of the left-handed quarks and leptons. Correspondingly one expects the breaking of global weak isospin symmetry in their residual $(V-A)\times(V-A)$ *four-fermion* interactions to be much weaker.

Next, we address the question of an approximate $(V-A)\times(V-A)$ structure of the (residual) four-fermion interactions at low energies $p<\Lambda_{\mathrm{L}}$.

Again, within the two-step confinement picture, a neat (though not conclusive) mechanism offers itself. Also in this case, the mechanism to be discussed is not specific to our model.

Let us start by reinterpreting the preon content of the *right-handed* quarks and leptons (47) in the language of the two-step confinement picture. This reinterpretation will provide the key to the suppression mechanism.

Consider first the right-handed $\mathrm{SU}(4)'$ multiplet of the "down" type (see eq. (47))

$$(e, d_1 d_2 d_3)_R = L(TF'^{+}). \tag{79}$$

The $\mathrm{SU}(2)_{\mathrm{R}}$ singlet, *mesonic* operator $TF'^{+}$ involves strong $\mathrm{SU}(2)_{\mathrm{R}}$ forces and transforms like

$$\mathscr{L}(\tfrac{1}{2},\tfrac{1}{2}) \tag{80}$$

under the homogeneous Lorentz group. It is very suggestive to associate with $TF'^{+}$ a *vector-meson-multiplet* dominating the $\mathrm{SU}(2)_{\mathrm{L}}\times\mathrm{SU}(4)$ current

$$\overline{F'_{\mathrm{L}}}\gamma_\mu T_{\mathrm{L}} = (2,4). \tag{81}$$

Its divergence is related to the Goldstone boson multiplet $\chi=(2,4)$, since the current (81) transforms like the corresponding broken generators of SU(6). However, in contrast to the relatively light (pseudo-) Goldstone boson multiplet $\chi$ (with $\mathrm{SU}(2)_{\mathrm{L}}$ *constituent* mass $\sim\Lambda_{\mathrm{L}}$) its *vector-meson partner* "$TF'^{+}$" will have a canonical $\mathrm{SU}(2)_{\mathrm{R}}$ *mass* $\sim\Lambda_{\mathrm{R}}$. Consider next the right-handed $\mathrm{SU}(4)'$ multiplet of the "up" type (see eq. (47))

$$(\nu, u_1 u_2 u_3)_{\mathrm{R}} = L^{+}(TF')^{+}. \tag{82}$$

A priori, there are *two* possibilities for the $\mathrm{SU}(2)_{\mathrm{R}}$ singlet mesonic operator $(TF')^{+}$ to transform with respect to the homogeneous Lorentz group

$$\mathscr{L}(0,0)+\mathscr{L}(1,0). \tag{83}$$

The scalar part of "$(TF')^{+}$" just corresponds to the Goldstone boson multiplet $\chi$. In order to decide whether the right-handed composite quark-lepton operator (82) may involve the same scalar multiplet as the left-handed one, eq. (67), we make use of an important observation stated in sect. 4. It concerned $\nu_L$ and $\nu_R^c$ having *identical* preon composition and thus identical internal quantum numbers. The Pauli principle, therefore, requires the operators coupling to $\nu_L$ and $\nu_R^c$ to differ in their spin structure, which means that

$$\nu_R = L^+(TL')^+ \text{ involves "}(TL')^+\text{" transforming like } \mathscr{L}(1,0)\,. \tag{84}$$

In view of the SU(4)′ symmetry we are thus led to associate again with $(TF')^+$ in the "up" quark-lepton multiplet (82) an *effective spin-1 field of (canonical) mass* ~ $\Lambda_R$ transforming like $(2,4)\times\mathscr{L}(1,0)$.

Altogether, from the two-step point of view the following picture has emerged. The *left-handed* quark-lepton multiplet is composed of the massless preon $F$ and a *relatively light (pseudo-) Goldstone scalar* $\Omega$. The *right-handed* quark-lepton multiplet, in contrast, involves *heavy effective spin-1 fields* of mass ~ $\Lambda_R$ together with the massless preon $L$. In the light of Preskill and Weinberg's [31] results, the *massive* vector mesons may well be constituents of massless fermionic bound states *below* a phase boundary at some critical value $\xi_{crit}$ of $\xi = \Lambda_R/\Lambda_L$ (enforced by the Symanzik-Applequist-Carazzone theorem [20]). This interpretation of the right-handed composite quarks and leptons also provides a natural explanation for the existence of our second Abbott and Farhi type solution (see sect. 3) for $\xi > \xi_{crit}$: at $\xi = \xi_{crit}$ the system has to undergo a phase transition to a solution *without right-handed*, massless composite quarks and leptons since the vector fields of mass ~ $\Lambda_R = \xi\Lambda_L$ become too heavy and must decouple completely. The left-handed massless composite fermions involve the light Goldstone bosons only and therefore persist also for $\xi > \xi_{crit}$.

Finally, let us explore the consequences of this picture for the residual interactions of the right-handed quark-lepton multiplet. Instead of the *non*-linear form (74) of $\mathscr{L}_{eff(R),gauge(L)}$ for $\Lambda_L < p < \Lambda_R$ let us imagine a *linear* realization involving also the heavy, effective fields of spin-1. According to the decoupling theorem [20] their effect in calculating Green functions with light external fields for $\Lambda_L < p \ll \Lambda_R$ is suppressed by powers of $p/\Lambda_R$ (in the framework of perturbation theory). For precisely this reason heavy fields are usually omitted in the *non*-linear version of $\mathscr{L}_{eff(R),gauge(L)}$ at the tree-level. However, in our case, for $p \sim \Lambda_L$ these vector fields, being $SU(2)_L$doublets, experience strong $SU(2)_L$ forces and may be invoked in $SU(2)_L$ singlet bound states. Some of those, the right-handed quarks and leptons are kept massless through chiral symmetry and anomaly matching. However, *all other $SU(2)_L$ singlet bound states* involving the heavy fields of mass ~ $\Lambda_R$ will themselves *have masses* ~ $\Lambda_R$. (This is analogous to heavy "quarkonium" states in QCD the mass of which is proportional to the heavy quark mass and not related to $\Lambda_{QCD}$.) This fact is crucial for the following argument. The residual interactions between left-handed and right-handed massless composite quarks and leptons may be intuitively pictured by drawing connected constituent rearrangement diagrams in analogy to the familiar duality diagrams of strong interactions. It is easy to see that for the case of interest, of four external chiral fermion multiplets, left-handed and right-handed ones must occur in pairs. Thus if *right-handed* quarks or leptons are involved their residual interaction proceeds always through the exchange of a $SU(2)_L$ singlet boundstate of *mass* $\sim \Lambda_R$ *made up from at least one heavy spin-1 field.* In contrast, if all four external fermions are left-handed quarks or leptons the exchange of the composite $SU(2)_{WI}$ triplet $W^{\pm}_{\mu}$, $W^3_{\mu}$

$$\Omega^+ \mathscr{D}_\mu \Omega - \tfrac{1}{2}\mathrm{Tr}\,(\Omega^+ \partial_\mu \Omega) = \begin{pmatrix} W^3_\mu & \sqrt{2}\,W^+_\mu \\ \sqrt{2}\,W^-_\mu & -W^3_\mu \end{pmatrix} = \boldsymbol{\tau} \cdot \boldsymbol{W}_\mu \,, \tag{85}$$

is involved. Since it is a $SU(2)_L$ singlet *boundstate of Goldstone bosons $\Omega$ its mass should be only $O(\Lambda_L)$*. By comparison, the residual four-fermion interactions involving the right-handed massless composite quarks and leptons should be suppressed relative to those with only left-handed ones by (positive) powers of

$$\left(\frac{\Lambda_L}{\Lambda_R}\right)^2 . \tag{86}$$

Let us emphasize again: we have been forced to include heavy vector meson fields of mass $O(\Lambda_R)$ in the effective lagrangian $\mathscr{L}_{\mathrm{eff(R),gauge(L)}}$, in contrast to usual experiences with effective lagrangians, because *anomaly matching* enforces the right-handed quarks and leptons containing these vector mesons to be *massless* on the scale $\Lambda_L$.

The resulting picture is quite appealing. The *existence* of the right-handed massless composite quarks and leptons, harbouring a heavy vector meson constituent, is a matter of "yes" or "no", i.e. a *discontinuous function of* $\xi = \Lambda_R/\Lambda_L$: they exist for $\xi < \xi_{\mathrm{crit}}$ and do not exist for $\xi > \xi_{\mathrm{crit}}$. Ther *effective residual interactions*, however, are *continuous functions of* $\xi$, becoming weaker for increasing $\xi$.

Let us finally address the question of quark-lepton universality in the effective low-energy lagrangian ($p \ll \Lambda_L$). The $(V-A)\times(V-A)$ four-femion term has to exhibit the original SU(4) symmetry even though the SU(4) is spontaneously broken to $SU(3)\times U(1)$ (see eq. (35)). The low-energy effect of this spontaneous breakdown only appears in terms of higher dimension involving the associated Goldstone boson fields besides the four-fermion fields. These terms are suppressed by additional powers of $p/\Lambda_L$. The SU(4) symmetry of the leading $(V-A)\times(V-A)$ four-fermion term then implies quark-lepton universality as in the Abbott and Farhi model [3].

## 6. Summary and conclusions

The purpose of this paper was to explore further the attractive idea that the weak interactions at present energies are residual hypercolor interactions among composite quarks and leptons. One main objective was to find a realization of this picture satisfying certain desirable requirements, such as the absence of fundamental scalar fields and the possibility for a *left–right symmetric spectrum* of composite quarks and leptons. This has led us to systematically investigate a $SU(2)_L \times SU(2)_R$ hypercolor gauge theory with "double"-confinement [34]* corresponding to the *two* scales $\Lambda_L$ and $\Lambda_R$ with $\Lambda_L \sim G_F^{-1/2} < \Lambda_R$. The smallest possible global chiral symmetry one encounters at the preon level for e, $g_{\text{color}} \to 0$ is $SU(4)_L \times SU(4)_R \times U(1)_{\text{axial}} \supset SU(3)_{\text{color}} \times U(1)_{\text{e.m.}}$. As concerns the number of fundamental fields, this model, in fact, represents a *minimal* composite model with only fermionic preons in fundamental hypercolor representations. (For instance, it involves fewer hypercolor gauge bosons and fewer fundamental Weyl fermions than the Rishon model of ref. [1].) On the other hand it has revealed a wealth of dynamical structure due to the two confinement scales $\Lambda_L$ and $\Lambda_R$. Keeping $\Lambda_L \sim G_F^{-1/2}$ fixed we have presented a detailed analysis of this $SU(2)_L \times SU(2)_R$ model as a function of the parameter $\xi = \Lambda_R/\Lambda_L$. Effective lagrangian techniques were employed *in combination* with the general requirement of anomaly saturation. The important function of the latter is to select a natural set of (composite) "preferred fields [26]" and their representations, in terms of which (non-linear) effective lagrangians may be constructed and analyzed. In fact, a second objective of this paper was to illustrate the type of dynamical information one may gain by means of such a general strategy for a class of generic composite models with a hierarchy of confinement scales.

Our analysis was performed along two complementary lines. For $\xi \sim 1$ $SU(2)_L \times SU(2)_R$ was treated as a confining product group ("one-step" confinement). For the simplest case of a global chiral SU(4) symmetry we find one standard family of massless left-handed *and* right-handed composite quarks and leptons. *No* light exotics appear. Moreover there is the possibility of higher generations, if discrete symmetries are invoked. The quarks and leptons are three preon bound states and satisfy 't Hooft's anomaly matching conditions after a necessary partial symmetry breaking $SU(4) \times SU(4)' \times U(1)_{\text{axial}} \to SU(3) \times SU(3)' \times U(1) \times U(1)'$. Furthermore, the composite spectrum exhibits a *higher* classification symmetry. More precisely, the original symmetry is enlarged by a global $SU(2) \times SU(2)'$ symmetry, which coincides with the familiar "accidental" chiral SU(2) symmetry of the QCD lagrangian with massless quarks. It is an intriguing result that all anomalies are matched with respect to this *higher* global symmetry in an obvious and most elegant way.

For $\xi = \Lambda_R/\Lambda_L \gg 1$ an important dynamical insight comes from treating confinement in *two steps*, at $\Lambda_R$ and $\Lambda_L$ consecutively. Around $p \sim \Lambda_R SU(2)_R$ confines while $SU(2)_L$ interactions are still negligibly weak. All $SU(2)_R$ *singlet* bound states are *mesons*, since they necessarily involve an *even* number of preons. Therefore, in the intermediate region $\Lambda_L < p < \Lambda_R$ we face an effective lagrangian involving the $SU(2)_L$ interactions of composite meson fields with left-over massless preons. Some of these mesons are *massless Goldstone scalars*, *enforced* by the requirement of anomaly

* An $SU(N) \times SU(2)_L \times SU(2)_R$ extension of our $SU(2)_L \times SU(2)_R$ model was recently studied in ref. [35]. The structure of the resulting quark and lepton spectrum remains (largely independent of $N$) similar to our solution (47).

saturation around $\Lambda_R$. All $SU(2)_L$ non-singlet Goldstone bosons are $SU(2)_L$ *doublets.* They serve as "preferred fields" in a non-linear effective lagrangian and finally are subject to $SU(2)_L$ confinement together with the left-over preons. Thus, a picture *complementary to technicolor emerges.*

We find two distinct solutions of composite quarks and leptons separated by a phase boundary at some value $\xi_{crit} > 1$. Phase II($\xi > \xi_{crit}$) corresponds to *separate* anomaly saturation, phase I ($\xi < \xi_{crit}$) to *joint* anomaly saturation around the scales $\Lambda_L$ and $\Lambda_R$. *For* $\xi > \xi_{crit}$ *(phase II)*, an Abbott and Farhi type solution [3] results, where only the *left-handed* quarks and leptons are composite fermions, containing preons and composite (pseudo) Goldstone bosons. The latter replace the fundamental scalars of the Abbott and Farhi model. The residual (four-fermion) weak interactions are pure$(V-A)\times(V-A)$, global $SU(2)_{WI}$ symmetry of weak isospin is exact and quark-lepton universality holds because of SU(4) symmetry as in ref. [3]. For $\xi < \xi_{crit}$ *(phase I)*, the solution found in the "one-step" analysis, with massless left-handed *and* right-handed composite quarks and leptons is recovered. However, in the "two-step" language they appear as $SU(2)_L$ singlet bound states of fermionic preons and composite meson fields. Those involved in the *left-handed* quarks and leptons are all relatively *light* (pseudo) *Goldstone bosons* (with $SU(2)_L$ *constituent* mass $\sim \Lambda_L$) as in phase II. In contrast, the *right-handed* quarks and leptons involve *heavy effective vector fields of mass* $\sim \Lambda_R$. We face a situation as discussed by Preskill and Weinberg [31]. For a *range* $\xi < \xi_{crit}$ massive fields may be forced by chiral symmetry and anomaly matching to occur in massless composite fermions. If $\xi$ gets too large, $\xi > \xi_{crit}$, the system has to undergo a phase transition into phase II, where the heavy meson fields have decoupled and no light composite right-handed quarks and leptons can exist.

This characteristically different composition of the left-handed and right-handed quarks and leptons also provides the key for a neat suppression mechanism of the residual $(V+A)\times(V\pm A)$ four-fermion interactions relative to those of the pure $(V-A)\times(V-A)$ type by (positive) powers of $(\Lambda_L/\Lambda_R)^2$. Moreover, by analyzing the *non*-linear effective lagrangian involving only preons and Goldstone fields for $\Lambda_L < p < \Lambda_R$ we were able to argue for an *approximate* validity of the global $SU(2)_{WI}$ symmetry and quark-lepton universality in the left-handed quark-lepton sector.

Both solutions (phase I and phase II) are interesting in their own right and possible candidates for describing weak interactions. Unfortunately, we have no control on the actual value of

$$\xi_{crit} = (\Lambda_R/\Lambda_L)_{crit} > 1.$$

In this paper we have refrained from discussing the question of quark-lepton mass generation. Let us emphasize that a possible mechanism is at hand through radiative effects of the electromagnetic and color gauge interactions, which we have considered to be switched off up to now. This type of mechanism has been advocated by Weinberg [32] and also by Fritzsch [33].

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## Appendix A

We choose to satisfy the solution (44) of the anomaly matching equations (40) by indices taking only the values $0, +1$:

$$\begin{aligned} &l_1 = 0,\ k_1 = 1\,, && m_1 = n_1 = 1\,, \\ &l_2 = k_2 = 0\,, && m_2 = 1,\ n_2 = 0\,, \\ &l_3 = k_3 = 1\,, && m_3 = 0,\ n_3 = 1\,, \\ &l_4 = 1,\ k_4 = 0\,, && m_4 = n_4 = 0\,. \end{aligned} \tag{A.1}$$

This is minimal except for $m_1 = n_1 = 1$ and $l_3 = k_3 = 1$ instead of 0. The resulting spectrum is again one generation of left-handed as well as right-handed quarks and leptons, accompanied, however, by an exotic, charge neutral, color octet quark:

$$\begin{aligned} &\nu \equiv \nu_{\mathrm{L}} \subset (TQ')Q = (3, \bar{3})_{\frac{1}{6},-\frac{1}{6}}\,, && \nu' \equiv \nu^{\mathrm{c}}_{\mathrm{R}} = (TL'^{+})L^{+} = (1,1)_{\frac{1}{2},-\frac{1}{2}}\,, \\ &e \equiv e_{\mathrm{L}} = (TL')^{+}L = (1,1)_{-\frac{1}{2},-\frac{1}{2}}\,, && e' \equiv e^{\mathrm{c}}_{\mathrm{R}} = (TL'^{+})^{+}L^{+} = (1,1)_{\frac{1}{2},\frac{1}{2}}\,, \\ &u \equiv u_{\mathrm{L}} = (TL')Q = (3,1)_{\frac{1}{6},\frac{1}{2}}\,, && u' \equiv u^{\mathrm{c}}_{\mathrm{R}} = (TL'^{+})Q^{+} = (\bar{3},1)_{-\frac{1}{6},-\frac{1}{2}}\,, \\ &d \equiv d_{\mathrm{L}} = (TQ')^{+}L = (1,3)_{-\frac{1}{2},\frac{1}{6}}\,, && d' \equiv d^{\mathrm{c}}_{\mathrm{R}} = (TL'^{+})^{+}Q^{+} = (\bar{3},1)_{-\frac{1}{6},\frac{1}{2}}\,. \end{aligned} \tag{A.2}$$

Again, in order to keep the electron and in addition the $u$-quark massless one has to invoke discrete symmetries.

Notice, this spectrum of composite fermions is again left–right symmetric as concerns color and charge quantum numbers, but it is asymmetric as concerns the preon content, in contrast to solution (47) of case (i). The left-handed neutrino is different from the right-handed one in that it is the color singlet component of a chiral $(3, \bar{3})$ representation. It thus has a color radius.

In contrast to solution (47) there is no obvious way, how the global SU(2) symmetry of weak isospin could arise on the level of the composite fermions.

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