

Energy Moments for Quark Jets at PETRA

PLUTO Collaboration

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Received 6 June 1983

Abstract. Measurements of energy moments for single quark jets at c.m. energies between 12 and

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¹¹ Partially supported by Department of Energy, USA

¹² Partially supported by the Israeli Academy of Sciences and Humanities – Basic Research Foundation

31.6 GeV are presented. The data, corrected for detector effects and initial state radiation, are compared to QCD predictions in the leading log approximation. Non perturbative effects are found to be moderate, and they strongly decrease with increasing c.m. energy. Once partly corrected for the presence of these fragmentation effects, our data agree well with all features of the leading log prediction, and in particular with the variation of the strong coupling constant over a wide range of energies and momentum transfers.

Hadron production in hadron-hadron, lepton-hadron and lepton-lepton interactions is described in perturbative QCD as the result of a two step pro-

cess. In the first step, occurring shortly after the interaction takes place, a parent parton (quark or gluon) radiates color and energy into a cone of finite aperture giving rise to a quark-gluon cascade, see Fig. 1a. In the second step, occurring after these virtual quanta have reached masses below a characteristic cut-off value, the radiated quarks and gluons condensate into colorless hadrons.

Experimental evidence supporting this picture is fragmentary in spite of the progress made in the last few years. Evidence for hard gluon bremsstrahlung comes from the observation of manifest three jet structures in e^+e^- annihilation at PETRA-PEP energies [1]. However the details of the parton shower formation which is believed to be due to multiple soft gluon emission are not yet well tested. Thus the important and universal feature characterising jet phenomena, transverse momentum damping, is not yet understood from first principles.

Even less understood is the transition from quarks and gluons to colorless hadrons. It has been observed that a considerable fraction of the final state hadrons are not directly produced but via the decay of hadron resonances (mainly pseudoscalar and vector mesons). These hadron resonances will be referred to from now on as primary hadrons, following the terminology used in the early quark fragmentation models [2].

Several observables have been proposed in the literature as means of testing soft gluon processes. Generally speaking these measures involve energy weighting factors to avoid infrared divergence problems. This is indeed the case for the same side [3] and opposite side [4] energy-energy correlations. Experimental studies [5], have shown that non-perturbative effects are important though it is encouraging to observe how data and theoretical predictions in the leading log approximation (LLA) come closer with increasing c.m. energy.

Konishi, Ukawa and Veneziano (KUV) have proposed an alternative measure as a possible way of testing the ideas underlying our current understanding of a parton shower, namely the jet energy moments [6]. Their idea is to decompose a quark (or antiquark) jet into cones of half opening angle δ , see Fig. 1b. Each cone contains a minijet whose origin is traced back to a parton in the quark-gluon cascade referred above, Fig. 1a. The energy moments are then defined as

$$C_n(\delta) = \frac{1}{\sigma} \int \frac{d\sigma(\delta)}{dE} \left(\frac{2E}{\sqrt{s}} \right)^n dE \quad (1)$$

where \sqrt{s} is the c.m. energy E , is the energy deposited inside a cone with half opening angle δ . The

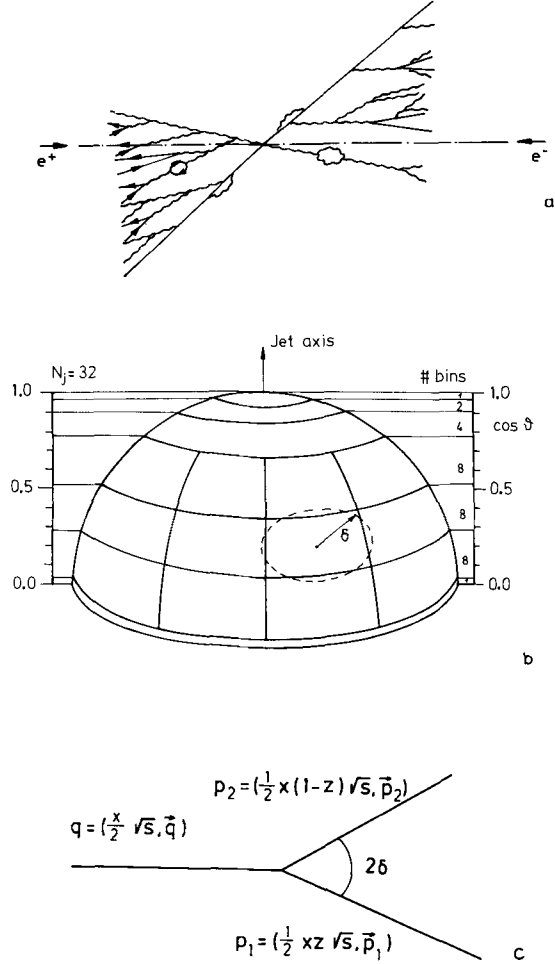


Fig. 1. a A quark-gluon cascade associated with the process $e^+e^- \rightarrow q\bar{q}$. b Decomposing a jet into 2^j ($j=5$) $\Delta\Omega$ bins of equal area. c Four momenta describing the decay of a virtual parton with invariant mass q into two others which are massless and form an angle 2δ

zeroth order moment has an infrared divergence, the first moment is trivially unity for all δ because of energy conservation. Higher moments are infrared stable and expected to be insensitive to fragmentation effects since by construction they involve a smearing over a finite size aperture.

The QCD prediction for these moments reads [6]

$$C_n(\delta) = a_n \left[\frac{\alpha_s(4q^2)}{\alpha_s(s)} \right]^{A_n^+} + (1 - a_n) \left[\frac{\alpha_s(4q^2)}{\alpha_s(s)} \right]^{A_n^-} \quad (2)$$

where a_n are constants, A_n^\pm are the anomalous dimensions of the theory which are calculable, α_s denotes, as usual, the strong coupling constant and q is the invariant mass of a parton whose decay products are confined within a cone of half opening angle δ . The

energy dependence of the moments C_n is implicitly contained in the energy dependence of the strong coupling constant. Thus measuring the moments at fixed angle δ and different energies can be used to test whether α_s does indeed run. Alternatively, measuring the moments at a fixed energy but varying the angle δ serves the same purpose. Such analysis was reported recently [10] for $\sqrt{s} = 29$ GeV. Ideally one would like to measure $C_n(\delta)$ both as a function of \sqrt{s} and δ . This is the aim of this paper.

The data used in the present study have been obtained with the PLUTO detector operating at the e^+e^- storage ring PETRA at DESY. Details of the detector have been published elsewhere [7]. The data selection criteria are similar to those described in [5a] and basically demand that a) the visible energy be greater than half the nominal c.m. energy, b) at least four charged tracks must belong to a common vertex, c) the reconstructed interaction vertex lies within ± 4 cm, of the center of the bunch-bunch collision, and d) the angle between the jet axis and the beam direction, ϑ , satisfies the condition $|\cos \vartheta| < 0.75$.

The analysis proceeds along the following steps

1. For a given event the sphericity axis [9] is determined with the purpose of defining a new coordinate system and partitioning the event into two hemispheres.

2. Each hemisphere, to which a jet is associated, is then subdivided into $N_j = 2^j$ bins of equal area $\Delta\Omega$ ($\Delta\Omega = \Delta \cos \vartheta \cdot \Delta\varphi$, $j = 1, 6$). For each of these par-

titions an equivalent angle δ is defined as the half opening angle of a cone which subtends the same solid angle (see Fig. 1b for an example corresponding to $j=5$).

3. The fractional energies, E_i , deposited in each of these "calorimeters" are measured. Only charged particles are used because of better energy and angular resolution. The energy carried by the charged tracks in the whole jet, E_v , is used as normalization factor.

4. We then calculate the moments for a single jet as

$$C_n = \sum_{i=1}^{n_j} \left(\frac{E_i}{E_v} \right)^n \quad (3)$$

where $n_j < N_j$ is the number of cones necessary to contain a jet, i.e. cones with $E_i > 0$.

5. In order to calculate the invariant mass squared, q^2 , see (2), of a parent parton giving rise to a mini-jet contained in a cone of half opening angle δ we follow the procedure first used by the MARK II Collaboration [10]. From Fig. 1c the relation between the invariant mass squared q^2 of a parton splitting into two other massless partons contained in a cone of opening angle 2δ is read off as

$$q^2 = x^2(1-z)z \cdot s \cdot \sin^2(\delta). \quad (4)$$

Not knowing the fractional energies x and z we maximize q^2 by assuming a) the decay depicted in Fig. 1c is symmetric that is, $z=1/2$ and b) the energy $\sqrt{s}/2$ emitted into one hemisphere is evenly shared by

Table 1. The second, fourth, sixth and eighth jet energy moments for different c.m. energies corrected at the level of final state hadrons

δ (deg.)	$4q^2$ GeV ²	$n = 2$	12 GeV $n = 4$	$n = 6$	$n = 8$	$4q^2$ GeV ²	$n = 2$	22 GeV $n = 4$	$n = 6$	$n = 8$
10.1	0.2	0.310 ± 0.010	0.092 ± 0.008	0.051 ± 0.010	0.036 ± 0.006	0.3	0.363 ± 0.031	0.168 ± 0.032	0.112 ± 0.028	0.082 ± 0.024
14.4	0.3	0.349 ± 0.011	0.125 ± 0.010	0.074 ± 0.010	0.053 ± 0.008	0.7	0.454 ± 0.033	0.245 ± 0.039	0.169 ± 0.037	0.128 ± 0.032
20.4	0.8	0.431 ± 0.012	0.196 ± 0.014	0.127 ± 0.012	0.095 ± 0.011	2.1	0.568 ± 0.037	0.366 ± 0.047	0.272 ± 0.045	0.214 ± 0.042
28.9	2.6	0.555 ± 0.014	0.327 ± 0.018	0.242 ± 0.018	0.196 ± 0.017	7.3	0.680 ± 0.034	0.490 ± 0.047	0.383 ± 0.047	0.312 ± 0.046
41.4	9.3	0.685 ± 0.014	0.480 ± 0.020	0.388 ± 0.009	0.332 ± 0.021	30.1	0.782 ± 0.029	0.620 ± 0.045	0.516 ± 0.048	0.441 ± 0.049
60.0	42.1	0.829 ± 0.011	0.682 ± 0.019	0.601 ± 0.007	0.547 ± 0.022	130.1	0.895 ± 0.021	0.795 ± 0.038	0.724 ± 0.044	0.667 ± 0.047
δ (deg.)	$4q^2$ GeV ²	$n = 2$	13 GeV $n = 4$	$n = 6$	$n = 8$	$4q^2$ GeV ²	$n = 2$	27.6 GeV $n = 4$	$n = 6$	$n = 8$
10.1	0.2	0.289 ± 0.012	0.076 ± 0.009	0.038 ± 0.007	0.025 ± 0.006	0.4	0.396 ± 0.013	0.181 ± 0.015	0.116 ± 0.013	0.085 ± 0.012
14.4	0.3	0.328 ± 0.013	0.105 ± 0.012	0.059 ± 0.009	0.040 ± 0.008	1.0	0.490 ± 0.015	0.275 ± 0.020	0.194 ± 0.019	0.150 ± 0.017
20.4	0.9	0.416 ± 0.016	0.179 ± 0.017	0.113 ± 0.015	0.084 ± 0.013	3.5	0.610 ± 0.016	0.397 ± 0.022	0.295 ± 0.022	0.233 ± 0.020
28.9	2.9	0.543 ± 0.018	0.309 ± 0.023	0.225 ± 0.022	0.180 ± 0.021	12.7	0.712 ± 0.015	0.520 ± 0.023	0.414 ± 0.024	0.343 ± 0.023
41.4	12.2	0.697 ± 0.018	0.494 ± 0.026	0.399 ± 0.027	0.341 ± 0.027	47.1	0.806 ± 0.014	0.657 ± 0.022	0.563 ± 0.024	0.494 ± 0.025
60.0	47.8	0.828 ± 0.015	0.682 ± 0.025	0.601 ± 0.028	0.547 ± 0.029	196.0	0.902 ± 0.010	0.810 ± 0.018	0.744 ± 0.021	0.690 ± 0.023
δ (deg.)	$4q^2$ GeV ²	$n = 2$	17 GeV $n = 4$	$n = 6$	$n = 8$	$4q^2$ GeV ²	$n = 2$	30.8 GeV $n = 4$	$n = 6$	$n = 8$
10.1	0.2	0.339 ± 0.015	0.127 ± 0.014	0.078 ± 0.012	0.056 ± 0.011	0.4	0.393 ± 0.009	0.181 ± 0.010	0.116 ± 0.009	0.085 ± 0.008
14.4	0.6	0.423 ± 0.018	0.210 ± 0.020	0.144 ± 0.018	0.110 ± 0.016	1.2	0.496 ± 0.011	0.286 ± 0.013	0.204 ± 0.012	0.158 ± 0.011
20.4	1.8	0.528 ± 0.020	0.311 ± 0.025	0.225 ± 0.023	0.177 ± 0.022	4.1	0.605 ± 0.011	0.398 ± 0.015	0.300 ± 0.014	0.240 ± 0.014
28.9	6.3	0.638 ± 0.020	0.434 ± 0.028	0.339 ± 0.028	0.280 ± 0.028	14.6	0.709 ± 0.010	0.524 ± 0.015	0.421 ± 0.016	0.352 ± 0.016
41.4	24.3	0.772 ± 0.018	0.612 ± 0.028	0.518 ± 0.031	0.453 ± 0.031	56.0	0.802 ± 0.009	0.656 ± 0.014	0.565 ± 0.016	0.498 ± 0.016
60.0	89.1	0.875 ± 0.015	0.769 ± 0.024	0.695 ± 0.028	0.639 ± 0.029	236.3	0.892 ± 0.007	0.797 ± 0.012	0.729 ± 0.014	0.676 ± 0.015

the n_j parent partons in that hemisphere so that $x = 1/n_j$. One thus has the relation

$$4q^2 = \frac{s \sin^2(\delta)}{n_j^2}. \quad (5)$$

6. The mean values for $C_n(\delta)$ and q^2 are obtained by averaging over all measured jets.

7. Using a jet simulation program [8] we correct for neutrals as well as for detector effects in the reconstruction of charged tracks. The correction also includes effects of initial state radiation, of photon conversion, detector resolution and of data selection. The corrections are $<5\%$ for any δ values, smaller than for most other jet variables [12]. We have repeated the analysis including neutrals and found

that the corrected moments are stable within the quoted statistical errors. Thus we estimate that the systematic uncertainties affecting our measurements are smaller than or of the same order as those of statistical nature.

In Table 1 we give moments obtained with the procedure described above for c.m. energies of 12, 13, 17, 22, 27.6 and 30.0–31.6 GeV. Fig. 2 displays as typical examples the δ dependence of the second and eighth moment. Note that the errors quoted are statistical only. We have fitted the δ -dependence of (2) to these moments after making use of the leading order expression relating α_s to the QCD scale parameter Λ . The fits for the only free parameter Λ were performed for each moment and energy sep-

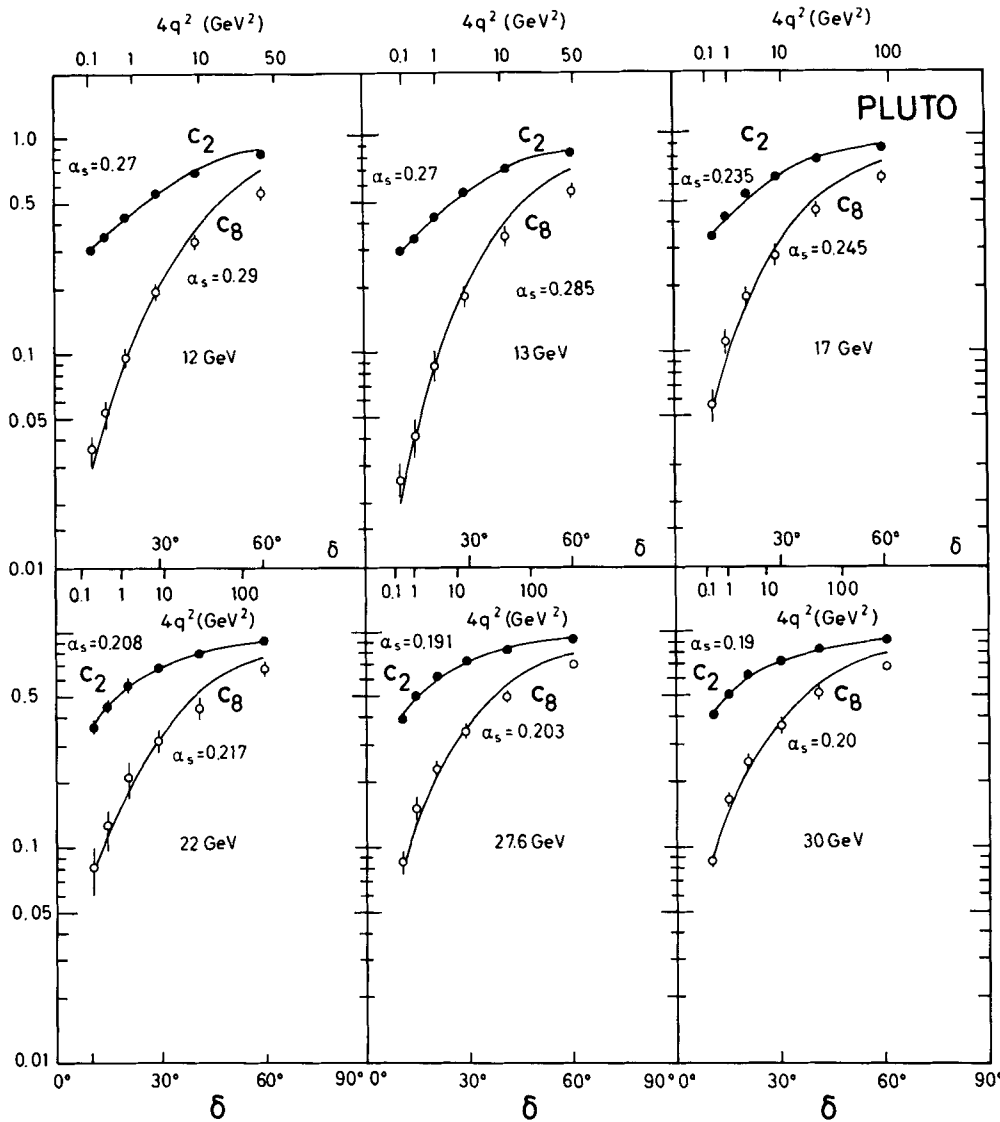


Fig. 2. Second (full circles) and eighth (open circles) order energy moments. The data have been corrected for detector acceptance and resolution, initial state radiation effects and effects due to track analysis. The solid lines represent the results of fits described in the text

arately. The results are represented by the solid curves in Fig. 2, and are labelled by the resulting values for α_s .

The following comments are in order

a) Our data at high energies shows good agreement with higher statistics measurements by the MARK II Collaboration [10].

b) The single jet energy moments can be roughly described by pure perturbative predictions.

c) The second order moments are indeed well described by KUV formula (2). Deviations which occur particularly for higher moments are found either at very small and/or very large angles.

d) Even if it is accepted that the LLA should not be taken seriously at these kinematical limits, it is worthwhile to note the following inconsistencies. At a fixed energy the value for α_s increases, slowly but monotonously with increasing order of the moment. Furthermore for a given moment the resulting values for α_s show a somewhat stronger energy dependence than the weak logarithmic behaviour expected in QCD.

Two reasons could explain the undesirable features of the analysis discussed above. It could be argued, for instance, that next to leading corrections would have different effects for different moments and different energies. That this might indeed be the case has been discussed recently in the literature [11]. On the other hand it could well be that fragmentation effects which have been found to be important for most jet variables at presently available

energies are also responsible for the inconsistencies discussed above. It is this second possibility that we want to investigate further.

As mentioned in the introduction the hadrons observed in the final states produced in e^+e^- annihilation, or in hadron-hadron and lepton-hadron interactions, are to a large extent the debris products of decaying meson (and baryon) resonances. Monte Carlo models like those described in [2] and [8] incorporate the empirical knowledge about their resonance spectrum and production cross sections. These Monte Carlo models are known to describe well existing data on meson resonance production [13]. Assuming these resonance decays to be well enough understood, one can correct the observed data so that they describe the final state at the level of these primary mesons. This procedure has been followed by the CELLO Collaboration in a recent study of energy-energy correlations [5b].

The results of our measurements corrected for resonance decay effects are presented in Table 2 and selectively in Fig. 3. Notice that though these effects might appear to be very large when looking at the variation of C_n with δ , they are in fact moderate when the translation from δ to q^2 is done. All moments can now be well described at all energies by (2) with the same value of A , the resulting fitted value being $A=464 \pm 10$ MeV, with a $\chi^2/\text{NDF}=1.2$. Since we have maximized q^2 in (5), the value of A determined here should then be understood as an upper limit. This is shown by the solid curves in Fig. 3.

Table 2. The second, fourth, sixth and eighth jet energy moments for different c.m. energies corrected at the level of primary mesons

δ deg.	$4q^2$ GeV ²	$n = 2$	12 GeV $n = 4$	$n = 6$	$n = 8$	$4q^2$ GeV ²	$n = 2$	22 GeV $n = 4$	$n = 6$	$n = 8$
10.1	0.6	0.559 ± 0.017	0.333 ± 0.027	0.268 ± 0.031	0.241 ± 0.033	1.7	0.558 ± 0.024	0.342 ± 0.037	0.272 ± 0.041	0.238 ± 0.043
14.4	1.3	0.601 ± 0.018	0.382 ± 0.028	0.311 ± 0.032	0.277 ± 0.034	4.1	0.640 ± 0.027	0.458 ± 0.043	0.387 ± 0.048	0.347 ± 0.050
20.4	3.0	0.663 ± 0.018	0.463 ± 0.031	0.389 ± 0.034	0.351 ± 0.036	10.8	0.714 ± 0.026	0.549 ± 0.043	0.472 ± 0.048	0.425 ± 0.051
28.9	8.4	0.748 ± 0.018	0.586 ± 0.030	0.518 ± 0.035	0.480 ± 0.038	30.4	0.789 ± 0.024	0.650 ± 0.041	0.576 ± 0.048	0.528 ± 0.051
41.4	24.6	0.831 ± 0.016	0.710 ± 0.028	0.653 ± 0.033	0.618 ± 0.036	89.3	0.890 ± 0.021	0.815 ± 0.037	0.765 ± 0.045	0.730 ± 0.050
60.0	63.9	0.917 ± 0.012	0.848 ± 0.022	0.813 ± 0.027	0.792 ± 0.030	214.8	0.931 ± 0.016	0.881 ± 0.028	0.844 ± 0.033	0.815 ± 0.037
δ deg.	$4q^2$ GeV ²	$n = 2$	13 GeV $n = 4$	$n = 6$	$n = 8$	$4q^2$ GeV ²	$n = 2$	27.6 GeV $n = 4$	$n = 6$	$n = 8$
10.1	0.6	0.524 ± 0.021	0.273 ± 0.031	0.201 ± 0.035	0.170 ± 0.036	1.6	0.562 ± 0.019	0.338 ± 0.027	0.253 ± 0.028	0.209 ± 0.028
14.4	1.4	0.564 ± 0.022	0.322 ± 0.034	0.245 ± 0.037	0.210 ± 0.039	3.9	0.630 ± 0.019	0.422 ± 0.029	0.332 ± 0.031	0.276 ± 0.031
20.4	3.5	0.638 ± 0.023	0.422 ± 0.036	0.344 ± 0.043	0.305 ± 0.045	11.1	0.729 ± 0.019	0.548 ± 0.030	0.449 ± 0.032	0.385 ± 0.033
28.9	9.7	0.732 ± 0.023	0.553 ± 0.039	0.480 ± 0.046	0.441 ± 0.049	33.1	0.802 ± 0.017	0.649 ± 0.028	0.559 ± 0.031	0.495 ± 0.033
41.4	29.6	0.846 ± 0.021	0.730 ± 0.037	0.671 ± 0.044	0.634 ± 0.048	93.1	0.865 ± 0.014	0.757 ± 0.025	0.688 ± 0.029	0.637 ± 0.031
60.0	77.4	0.916 ± 0.016	0.848 ± 0.030	0.813 ± 0.037	0.792 ± 0.041	278.9	0.935 ± 0.011	0.874 ± 0.019	0.832 ± 0.024	0.799 ± 0.027
δ deg.	$4q^2$ GeV ²	$n = 2$	17 GeV $n = 4$	$n = 6$	$n = 8$	$4q^2$ GeV ²	$n = 2$	30.8 GeV $n = 4$	$n = 6$	$n = 8$
10.1	0.7	0.572 ± 0.048	0.395 ± 0.076	0.333 ± 0.083	0.295 ± 0.084	1.8	0.559 ± 0.012	0.339 ± 0.013	0.255 ± 0.017	0.209 ± 0.017
14.4	1.7	0.643 ± 0.046	0.462 ± 0.074	0.382 ± 0.082	0.334 ± 0.086	4.7	0.638 ± 0.013	0.439 ± 0.016	0.348 ± 0.020	0.293 ± 0.020
20.4	4.3	0.724 ± 0.047	0.572 ± 0.073	0.486 ± 0.081	0.427 ± 0.083	13.0	0.724 ± 0.012	0.549 ± 0.019	0.457 ± 0.021	0.396 ± 0.021
28.9	12.0	0.800 ± 0.040	0.664 ± 0.083	0.575 ± 0.071	0.508 ± 0.074	38.2	0.800 ± 0.011	0.653 ± 0.018	0.567 ± 0.020	0.507 ± 0.021
41.4	38.1	0.865 ± 0.033	0.757 ± 0.055	0.682 ± 0.064	0.623 ± 0.070	111.4	0.860 ± 0.010	0.755 ± 0.016	0.689 ± 0.019	0.641 ± 0.021
60.0	113.8	0.943 ± 0.022	0.888 ± 0.042	0.849 ± 0.052	0.818 ± 0.058	339.5	0.924 ± 0.007	0.859 ± 0.013	0.816 ± 0.016	0.782 ± 0.017

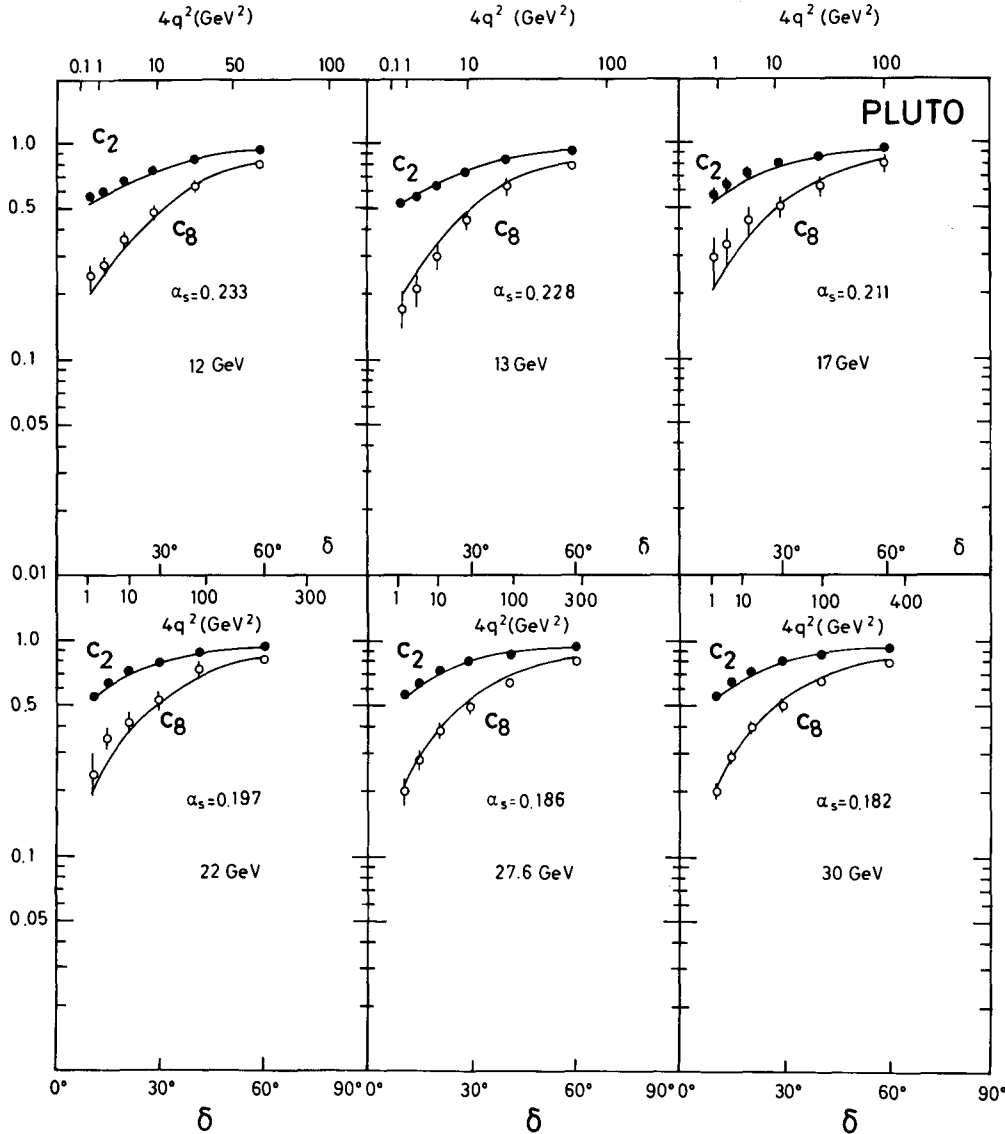


Fig. 3. Second (full circles) and eighth (open circles) order jet energy moments. The data have been corrected for effects due to resonance strong decays. The solid curve represents the result of a fit described in the text

We have investigated how sensitive the values for $C_n(\delta)$ corrected at the primary meson level are to the parameter A , which fixes the fraction of mesons in the Hoyer et al. Monte Carlo [8]. A recent study of ρ inclusive production in e^+e^- annihilation at PETRA energies [13], determines A to be $0.42 \pm 0.08 \pm 0.15$ thus favouring the original Field-Feynman ansatz for $A=0.5$ but not excluding the possibility that the cross sections for pseudoscalar and vector mesons are proportional to the spin factors $2S+1$ which would correspond to $A=0.25$. The fits described above which corresponded to a choice of $A=0.5$ were repeated using $A=0.25$ and turned out

to be similarly good, the resulting value for A being $A=402 \pm 10$ MeV.

We interpret these results as giving support to the jet calculus rules [6] or alternatively to the Altarelli-Parisi evolution equations for parton fragmentation functions [16] used to derive the energy and angular dependence of the moments given by (2). This of course implies that the strong coupling constant is running. We find it surprising that a perturbative approach in the LLA works well even at very small values of q^2 .

The following comments are however in order

1) The success of the perturbative QCD descrip-

tion discussed above relies on the choice of q^2 given by (4). If $4q^2$ is approximated by $s \cdot \sin^2(\delta)$ as suggested in [6], (2) would not describe the data.

2) Recently perturbative calculations for the moments of the energy flow through a single cone with aperture δ and axis fixed in space have been reported [11]. These calculations take into account the kinematical constraints in the branching vertices present in the parton cascade, thus incorporating to some extent next to leading corrections. The region of validity of these calculations are supposed to be restricted to high order moments and to δ values somewhere between 15 and 40 degrees. If interpreted as valid for the energy flow inside a cone oriented at random inside a jet [17], they describe correctly the order of magnitude of the moments, but seem to fail to describe the angular dependence, see Fig. 4. It is also worth noticing that this disagreement becomes less strong as the c.m. energy increases.

3) The moments of energy flow can be phenomenologically well described by fragmentation models with [8] and without [2] gluon emission. This indicates that in the angular range investigated here they are insensitive to the hard component of the gluon radiation.

Finally we discuss a simple consequence of (2).

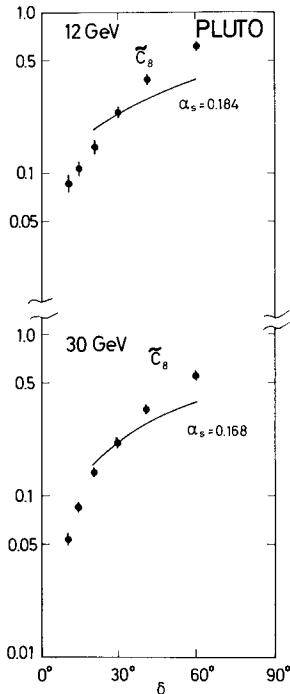


Fig. 4. The moments of order eight, $\tilde{C}(8, \delta)$, of the energy flow through a cone with half opening angle δ oriented at random inside a single jet. Notice that $\tilde{C}(n, \delta) = C(n, \delta)/n_j$. The solid line shows the prediction given in [11]

For high order moments, $n > 3$, the second term in the right hand side of (2) is negligible. From this it follows that a plot, at a fixed energy, of the log of the n th moment versus the log of the m th, results in a straight line whose slope is given by the ratios of the corresponding anomalous dimensions A_n^+ and A_m^+ i.e.

$$\frac{d \ln C_n(\delta)}{d \delta} = \frac{A_n^+}{A_m^+} \frac{d \ln C_m(\delta)}{d \delta}. \quad (6)$$

This relation has been verified in deep inelastic lepton hadron scattering experiments [14] and its significance has been the subject of intense debate [15]. In Fig. 5 we show the 4th vs the 8th moment at 12 and 30.0–31.6 GeV for both data samples i.e. that corrected at the final state hadron level and that at the primary meson level. As expected the data fall on straight lines. We note however that at lower energies the slopes of both data samples are slightly different while at high energies they converge. We interpret these results as a confirmation of the picture described above according to which fragmentation effects upon jet energy moments are moderate and strongly reduced with increasing c.m. energy.

To summarize, the jet energy moments have been

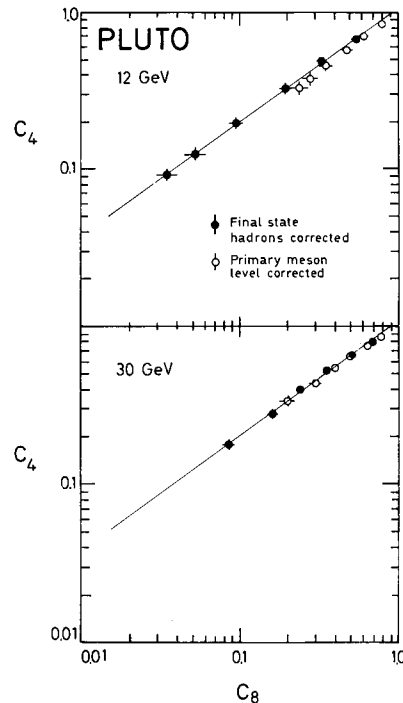


Fig. 5. Fourth vs eighth order jet energy moments for the data samples corrected at the final state hadron level (full circles) and at the primary meson level (open ones). The straight lines show the KUV predictions [6]

measured at 12, 13, 17, 22, 27.6 and 30.0–31.6 GeV. The data have been corrected both to the final state hadron and to the primary meson level, and are both presented in tabular form. At the primary meson level the jet energy moments are very well described by the QCD ‘jet calculus’, where both the angular and energy variation of the moments are determined by the variation of the ‘running’ coupling constant α_s . The absolute magnitude of α_s is consistent with the result of other determinations. However the attributed uncertainties in q^2 and in the next to leading corrections to (2) preclude a precise determination of Λ . These results may provide a deeper insight into our understanding of the limitation of transverse momenta in jet phenomena and suggest that the coupling constant responsible for strong interactions runs.

Acknowledgements. We wish to thank the members of the DESY directorate for the hospitality extended to the university groups. We warmly thank M. Greco and G. Veneziano for very helpful discussions and correspondence. We are indebted to the PETRA machine group and the DESY computer center for their excellent performance during the experiment. We gratefully acknowledge the efforts of all engineers and technicians who have participated in the construction and maintenance of the apparatus.

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