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Higher Order QCD Corrections to the Three-Jet Cross Section: Bare Versus Dressed Jets

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Abstract. We report results for the three-jet cross section to order α_s^2 using a jet resolution criterion depending on the jet mass and study the sum of three- and four-jet cross sections in $O(\alpha_s^2)$ as a function of the resolution parameters in order to obtain the limit of infinite resolution.

1. Introduction

We believe that QCD can be tested unambiguously at short distances, that is on perturbative grounds without having to solve the confinement and bound state problems, and that e^+e^- annihilation into hadron jets is a unique laboratory for doing so.

In the past five years many shape variables have been suggested to describe the jet structure of the final state hadrons. Since in perturbation theory we calculate the shape of an e^+e^- hadronic event due to the production of massless quarks and gluons instead of the observed hadrons with finite mass, the jet variable should be insensitive to the nonperturbative process of hadronization. It is generally accepted that this will be the case only at infinite energy, so that model calculations are needed to correct for these effects at finite energies. But already at the perturbative level problems arise. Here the variables must be insensitive to the emission of soft and/or collinear radiation. Most of the (once) popular jet variables do not satisfy this criterion. This has been emphasized repeatedly over the last two years [1], and the large $O(\alpha_s^2)$ corrections to the three-jet cross section proclaimed by the Caltech group [2] give evidence for that. To be brief we shall confine our discussion to bare thrust [3]

$$T = \max_{\mathbf{n}} \sum_i |\mathbf{p}_i \mathbf{n}| / \sum_i |\mathbf{p}_i| \tag{1}$$

The conclusions we shall reach are equally valid for spherocity, acoplanarity, the Caltech shape variables C and D [2] and most other bare shape variables though.

Before we go into calculational details in the next sections, let us make a few remarks to explain on a more qualitative level what is meant by the notations of bare jets and dressed jets [1].

Consider a 4-parton $q\bar{q}gg$ final state and let us assume, for example, $x_1 = 0.8$, $x_2 = 0.7$, $x_3 = 0.4$ and $x_4 = 0.1$, where $x_i = 2E_i/\sqrt{q^2}$ are the scaled parton energies for massless partons using the notation of ref. 4. For this event thrust as calculated from (1), may take many different values depending on the relative angles of the 4 partons. For example, if the two gluons or the soft gluon and the antiquark are close in angle, we have $T = 0.8$, while in case the soft gluon is emitted along the direction of the quark, we find $T = 0.9$. These T values computed from (1) on the basis of the 4-parton momenta define the so-called bare thrust of the event. At this point one can ask whether the soft gluon with $x_4 = 0.1$ has enough energy to be considered as a separate jet. If this is not so, then the two cases, the soft gluon being combined either with the hard gluon or the antiquark and the soft gluon combined with the quark, respectively, cannot be distinguished. Therefore we have advocated [4] to average over the momentum of the soft, i.e. $x_4 \leq \varepsilon$, gluon à la Sterman and Weinberg [5]. This leads us to à three-jet event whose "dressed" thrust $x_{\max}$ is $x_{\max} = 0.84$ [4] in both cases. A similar possibility occurs for collinear partons. For definiteness let us

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assume that again $x_1 = 0.8, x_2 = 0.7$, but $x_3 = 0.25$ and $x_4 = 0.25$. For the two gluons being parallel we have $T = 0.8$. Suppose now the two gluons are emitted at a relative angle of say 40°. Then we find that the bare thrust calculated from (1) can be as large as $T = 0.86$. Introducing, however, an angle resolution parameter δ larger than 40° the two gluons are considered as one jet, which yields for the "dressed' thrust $x_{max} = 0.8$ and this for all cases with relative gluon angle less than δ. Thus, as has been emphasized already some time ago [6], it is possible that an event can have on the parton level, i.e. in a perturbative treatment, rather different thrust values depending on which resolution criteria are applied, either infinite resolution giving bare thrust or finite resolution which gives the dressed thrust value.

These two examples illustrate that integrating the soft and collinear parton momenta up to the resolution parameters of the jets, which is also necessary to get rid of the large logarithms that signal the breakdown of finite order perturbation theory, will involve smearing thrust over a relatively wide range of values. If, on the other hand, bare thrust is used as jet measure, we are forced to cut off the integrals at rather soft and collinear parton momenta, so that we encounter large (left-over) logarithmic corrections which lead to unphysical cross sections for a specific number of jets.

In view of this situation, we have chosen to employ an explicit jet resolution criterion in calculating the $O(\alpha_s^2)$ corrections to the three-jet cross section [4]. This is the only way to correctly define cross sections for a fixed number of jets, i.e. exclusive multi-jet cross sections. In our published work we have employed ε, δ Sterman-Weinberg type [5] resolution parameters and found small radiative corrections for appropriately chosen values of ε and δ. This stands in contrast to the large $O(\alpha_s^2)$ corrections to the event shape distributions of the bare thrust type [2, 8, 12]. These calculations correspond to adding both three- and four-jet cross sections and letting $\varepsilon, \delta \to 0$.

The purpose of this work is twofold. First, we supplement our earlier calculations [4] by a calculation of the three-jet cross section using a jet resolution criterion depending on the jet mass. This is of interest for comparison with the data in order to secure that the final results on coupling constants etc. do not depend on how jets are defined. Secondly, we want to explicitly demonstrate that the large $O(\alpha_s^2)$ corrections to the bare thrust-like distributions arise indeed primarily from rather soft and collinear partons. To do so we only have to add three- and four-jet cross sections and take the resolution parameters to zero. In this limit we should then recover the large radiative corrections of the other groups.

After having given the three-jet cross sections for ε, δ and invariant mass cut-offs in Sect. 2, we shall study the limit of small resolution parameters in Sect. 3. We finish with some concluding remarks in Sect. 4.

2. Three-Jet Cross Sections to $O(\alpha_s^2)$

Most of the content of this section can be found in our earlier publications [4]. We nevertheless consider it useful, for later reference and for reasons of comparison, to list both three-jet cross sections for ε, δ [7] and invariant mass cut-offs in a coherent fashion and briefly discuss its physical impact. After all, these are the physically meaningful jet cross sections.

2.1. *ε, δ-Cut-Off*

In this case we say that two partons are irresolvable if either parton has energy less than $\varepsilon\sqrt{q^2}/2$ or the (full) angle between two partons is less than δ. By three-jet cross section we then understand the cross section for events which have all but a fraction $\varepsilon/2$ of the total energy distributed within three separated cones of opening angle δ. We define the "dressed" jet variables x_1, x_2 and x_3 to be equal twice the energy flowing into the parent quark, antiquark and gluon jet cone, respectively, divided by the total energy flowing into all three cones (and only the cones). Obviously $x_1 + x_2 + x_3 = 2$. We like to emphasize that this definition of jet variables eliminates the ambiguities in the treatment of soft final state partons seen by Gottschalk [1].

The three-jet cross section can be written, in the MS renormalization scheme,

$$\frac{1}{\sigma_0}\frac{d^2\sigma^{\text{3-jet}}(\varepsilon,\delta)}{dx_1 dx_2} = \frac{\alpha_s(q^2)}{2\pi} \cdot C_F \left\{ B^v(x_1,x_2)\left[1 + \frac{\alpha_s(q^2)}{2\pi}(J_1 + J_2 + J_3)\right] + \frac{\alpha_s(q^2)}{2\pi} f(x_1,x_2) \right\} + O(\varepsilon,\delta), \tag{2}$$

where

$$B^0(x_1,x_2) = \frac{x_1^2 + x_2^2}{(1-x_1)(1-x_2)} \tag{3}$$

and

$$\begin{aligned} J_1 = C_F\Bigg[&\left(-2\ln\frac{\varepsilon}{x_1} - 2\ln\frac{\varepsilon}{x_2} - 3\right)\ln\left(\frac{1-\cos\delta}{2}\right) \\ &+ 4\ln\varepsilon\ln\left(\frac{x_1+x_2-1}{x_1x_2}\right) + 2\left(\frac{\varepsilon}{x_1} + \frac{\varepsilon}{x_2}\right)\ln\left(\frac{1-\cos\delta}{2}\right) \\ &+ \ln^2\left(\frac{x_1+x_2-1}{x_1x_2}\right) + 2\ln^2 x_1 + 2\ln^2 x_2 \\ &- 3\ln x_1 - 3\ln x_2 - \ln^2(1-x_3) \\ &- 2L_2\left(\frac{x_1+x_2-1}{x_1x_2}\right) - \frac{\pi^2}{3} + 5\Bigg], \end{aligned}$$

$$J_2 = N_c\left[\left(-2\ln\frac{\varepsilon}{x_3} - \frac{11}{6}\right)\ln\left(\frac{1-\cos\delta}{2}\right)\right.$$

$$+2\ln\varepsilon\left(\ln\left(\frac{x_1+x_2-1}{x_1x_3}\right)+\ln\left(\frac{x_2+x_3-1}{x_2x_3}\right)\right.$$
$$\left.-\ln\left(\frac{x_1+x_2-1}{x_1x_2}\right)\right)+\frac{2\varepsilon}{x_3}\ln\left(\frac{1-\cos\delta}{2}\right)$$
$$+\tfrac{1}{2}\ln^2\left(\frac{x_1+x_3-1}{x_1x_3}\right)+\tfrac{1}{2}\ln^2\left(\frac{x_2+x_3-1}{x_2x_3}\right)$$
$$-\tfrac{1}{2}\ln^2\left(\frac{x_1+x_2-1}{x_1x_2}\right)+2\ln^2 x_3-\tfrac{11}{3}\ln x_3$$
$$+\tfrac{1}{2}\ln^2(1-x_3)-\tfrac{1}{2}\ln^2(1-x_1)-\tfrac{1}{2}\ln^2(1-x_2)$$
$$+L_2\left(\frac{x_1+x_2-1}{x_1x_2}\right)-L_2\left(\frac{x_1+x_3-1}{x_1x_3}\right)$$
$$-L_2\left(\frac{x_2+x_3-1}{x_2x_3}\right)-\frac{\pi^2}{6}+\frac{137}{18}-\frac{x_3^2}{x_1^2+x_2^2}$$
$$\left.+\frac{1}{3}\frac{(1-x_1)(1-x_2)}{x_1^2+x_2^2}\right],$$
$$J_3=\frac{N_f}{2}\left[\tfrac{2}{3}\ln\left(\frac{1-\cos\delta}{2}\right)+\tfrac{4}{3}\ln x_3-\frac{26}{9}+\frac{1}{3}\frac{x_3^2}{x_1^2+x_2^2}\right] \tag{4}$$

The function $f(x_1, x_2)$ is identical to that in (2.14) in [4] (second reference). We have brought (2) into a common form with the three-jet cross section for invariant mass cut-off here, which we will discuss next.

2.2. Invariant Mass Cut-Off

In this case we say that two partons are irresolvable if $s_{ij}=(p_i+p_j)^2\leqq yq^2$. By three-jet cross section we then understand the cross section for events which consist of three clusters, each having an invariant mass squared smaller than yq^2. We define the dressed jet variables x_1, x_2 and x_3 to be equal twice the energy of parent quark, antiquark and gluon cluster, respectively, divided by the total energy ($x_1+x_2+x_3=2$). It may happen that a wee parton, which is too soft to be independently resolved, can be attached to either two or three of the jets and give an invariant jet mass smaller than the resolution limit. In this case the energy of the soft parton is given randomly to the two or three jets it fits into.

We obtain the three-jet cross section, in the $\overline{\rm MS}$ scheme,

$$\frac{1}{\sigma_0}\frac{d^2\sigma^{3\text{-jet}}(y)}{dx_1dx_2}=\frac{\alpha_s(q^2)}{2\pi}$$
$$\cdot C_F\left\{B^v(x_1,x_2)\left[1+\frac{\alpha_s(q^2)}{2\pi}(J_1+J_2+J_3)\right]\right.$$
$$\left.+\frac{\alpha_s(q^2)}{2\pi}f(x_1,x_2)\right\}+O(y), \tag{5}$$

where $f(x_1, x_2)$ is the same as above and

$$J_1=C_F\left[-2\ln^2\frac{y}{1-x_3}-3\ln y-1+\frac{\pi^2}{3}\right.$$
$$\left.+\frac{2y}{1-x_3}\ln\frac{y^2}{1-x_3}\right],$$
$$J_2=N_c\left[\ln^2\frac{y}{1-x_3}-\ln^2\frac{y}{1-x_1}-\ln^2\frac{y}{1-x_2}\right.$$
$$-\tfrac{11}{6}\ln y+\frac{67}{18}+\frac{\pi^2}{6}-\frac{y}{1-x_3}\ln\frac{y^2}{1-x_3}$$
$$\left.+\frac{y}{1-x_1}\ln\frac{y^2}{1-x_1}+\frac{y}{1-x_2}\ln\frac{y^2}{1-x_2}\right],$$
$$J_3=\frac{N_f}{2}\left[\tfrac{2}{3}\ln y-\tfrac{10}{9}\right] \tag{6}$$

The derivation of (5) and (6) proceeds in an analogous fashion to that of (2), (3) and (4) which has discussed in detail in [4] (second reference). One starts with the individually infrared- and collinear-divergent 3- and 4-parton cross sections

$$d\sigma^{3\text{-jet}}(y)=d\sigma^{3\text{-parton}}+d\sigma^{4\text{-parton}}(y), \tag{7}$$

where $d\sigma^{3\text{-parton}}$ sums the $O(\alpha_s)$ and $O(\alpha_s^2)$ cross sections with 3 partons in the final state (which by now has been calculated by three independent groups [2, 4, 9] with identical results), and where $d\sigma^{4\text{-parton}}$ stands for the cross section for $e^+e^-\to q\bar{q}gg$ and $e^+e^-\to q\bar{q}q\bar{q}$, in which two of the partons are irresolvable, i.e. have an invariant mass squared smaller than yq^2. The integration over the irresolvable parton configurations is similar to the case of the ε, δ cut-off [4] and will not be reported here. Since the boundaries are completely different, the two-cross sections can, however, not be obtained from one another by a simple substitution. As in our calculation in [4], terms of order y cannot be calculated analytically, but terms $\sim y\ln y$ have been included. Therefore the cross section formula (5) does not apply for too large values of y. In practice terms of $O(y)$ are irrelevant for $y\lesssim 0.05$.

Kunszt [8] also has calculated the analogue of the Sterman–Weinberg formula for three-jet production with invariant mass resolution. We differ from his result in the subdominant (constant) terms in J_1 and J_2, while we agree in the leading (for $y\to 0$) logarithmic terms and, entirely, in J_3.

2.3. Discussion of Results

The cross sections (2) and (5) show a great similarity. A little algebraic calculation furthermore shows that the leading logarithmic terms in (2) and (5) (being proportional to $\ln\varepsilon\ln((1-\cos\delta)/2)$, $\ln\varepsilon\ln((1-\cos\delta)/2)$, $\ln^2 y$ and $\ln y$) become identical if we identify

$$\frac{y}{1-x_i}=\frac{\varepsilon^2}{1-x_i}=\frac{1-\cos\delta}{2},\quad i=1,2,3. \tag{8}$$

It should be stressed that (8) connects only the leading terms in (2) and (5). The (renormalization group improved) running coupling constant $\alpha_s(q^2)$ in (2) and (5) has been evaluated at the overall energy q^2, which provides the only scale if the resolution parameters and the angles between the three jets are all large. This changes of course in the limit of small ε, δ and y and if we approach the 2-jet limit. The logarithms in J_2, J_3

$$\tfrac{11}{6}N_c \ln\left\{\begin{matrix}\dfrac{1-\cos\delta}{2}\\ y\end{matrix}\right\}, \quad \tfrac{1}{3}N_f \ln\left\{\begin{matrix}\dfrac{1-\cos\delta}{2}\\ y\end{matrix}\right\} \tag{9}$$

and, in addition, for the ε, δ cut-off

$$-\tfrac{11}{3}N_c \ln x_3, \quad \tfrac{2}{3}N_f \ln x_3 \tag{10}$$

give evidence of this change of scales. As can readily be seen, (9) and (10) can be absorbed into α_s by

$$\varepsilon, \delta\,\text{cut-off}: q^2 \to x_3^2\left(\frac{1-\cos\delta}{2}\right)q^2,$$

$$\alpha_s(q^2) \to \alpha_s\left(x_3^2\frac{1-\cos\delta}{2}q^2\right)$$

$$y\,\text{cut-off}: q^2 \to y q^2, \quad \alpha_s(q^2) \to \alpha_s(y q^2) \tag{11}$$

This is to say, if the renormalization is performed at $x_3^2((1-\cos\delta)/2)q^2$ and yq^2, respectively, the logarithmic terms (9) and (10) are exactly cancelled. One should be aware that the Λ parameter will in practice depend on *where* the renormalization is performed (and not only on the renormalization *scheme*) due to the finite order of calculation.

It is clear that the three-jet cross sections (2) and (5) are applicable only for such ε, δ and y cut-off's, which leave the effective expansion parameter

$$(2C_F + N_c)\frac{\alpha_s}{2\pi}\left\{\begin{matrix}\ln^2\varepsilon \ln\dfrac{1-\cos\delta}{2}\\ \ln^2 y\end{matrix}\right\} \ll 1\,. \tag{12}$$

If ε, δ and y, respectively, are too small, the three-jet cross section may become negative, which signals the breakdown of $O(\alpha_s^2)$ perturbation theory for a cross section with a fixed number of jets. Thus ε, δ and y must be chosen in accord with (12).

But in any case y and ε, δ should not be smaller than the nonperturbative jet mass and jet opening angle given by the finite transverse momenta of the fragmentation process. A measure for the nonperturbative jet mass is the slim jet mass, which at PETRA energies is roughly 6 GeV [10]. This corresponds to $y \simeq 0.04$, which translates roughly into $\varepsilon \simeq 0.2$, $\delta \simeq 40°$ (assuming $x_{1,2,3} \simeq \frac{2}{3}$).

In order to give an idea of the magnitude of the $O(\alpha_s^2)$ corrections to the three-jet cross sections, we have calculated $d\sigma^{3\text{-jet}}(\varepsilon,\delta)/dx_{\max}$ and $d\sigma^{3\text{-jet}}(y)/dx_{\max}$ for some values of ε, δ and y, respectively. In Fig. 1 we show $d\sigma^{3\text{-jet}}/dx_{\max}$ for ε, $\delta = 0.2$, 40° and

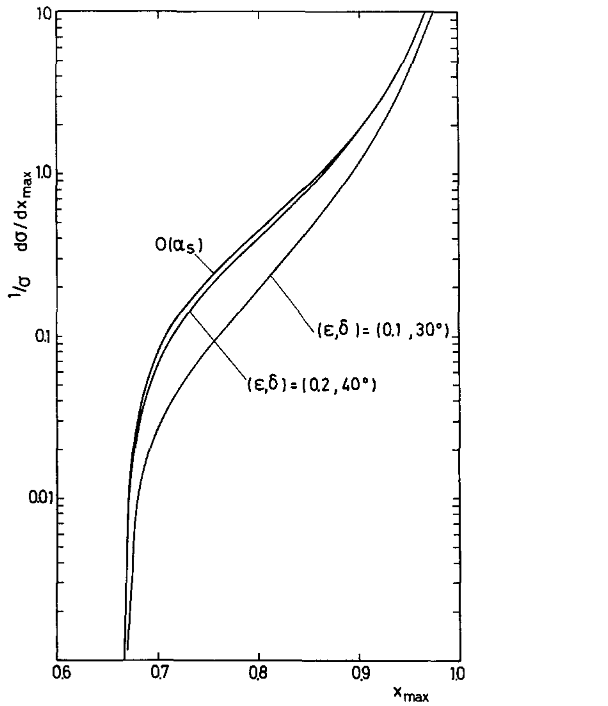

Fig. 1. Three-jet cross section for $(\varepsilon,\delta) = (0.2, 40°)$ and $(\varepsilon,\delta) = (0.1, 30°)$ with the Born cross section $(O(\alpha_s))$ as a function of $x_{\max}$ for $\alpha_s = 0.16$

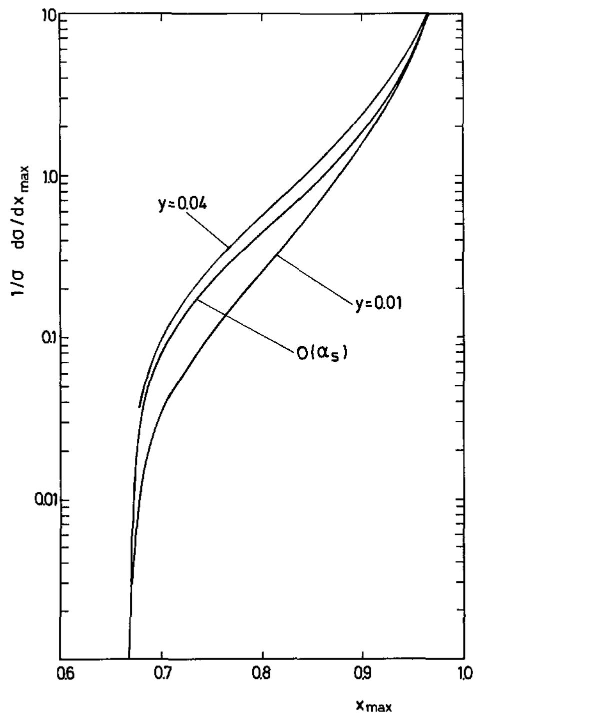

Fig. 2. Three-jet cross section for $y = 0.04$ and $y = 0.01$ with the Born cross section $(O(\alpha_s))$ as a function of $x_{\max}$ for $\alpha_s = 0.16$

$\varepsilon, \delta = 0.1$, $30°$, $\alpha_s = 0.16$ and $N_f = 5$ together with the $O(\alpha_s)$ curve. In Fig. 2 $d\sigma^{3\text{-jet}}(y)/dx_{max}$ is shown for $y = 0.04$ and $y = 0.01$ again for $\alpha_s = 0.16$ together with the $O(\alpha_s)$ curve. We see that for the larger values of ε, δ and y the $O(\alpha_s^2)$ corrections are small, while the smaller values lead already to large negative $O(\alpha_s^2)$ corrections, which indicates that these values are outside the perturbative regime.

The cross sections $d\sigma^{3\text{-jet}}(\varepsilon,\delta)/dx_{max}$ and $d\sigma^{3\text{-jet}}(y)/dx_{max}$ have been tested recently by the JADE-Collaboration and two of us [11]. In this paper it is also described how the ε, δ and y cut-offs are realized experimentally. The three-jet cross sections (2) and (5) were also used to determine the value of α_s in the $\overline{\text{MS}}$ scheme.

3. Bare Thrust Distribution to $O(\alpha_s^2)$

To make contact to the bare thrust distribution now, we have to complete our $O(\alpha_s^2)$ calculation by adding the proper 4-jet cross sections to the three-jet cross section $d\sigma^{3\text{-jet}}(\varepsilon, \delta)/dx_{max}$. This means we shall have to calculate

$$\frac{d\sigma(\varepsilon,\delta)}{dT} = \left(\frac{d\sigma^{3\text{-jet}}(\varepsilon,\delta)}{dx_{max}}\right)_{T=x_{max}} + \frac{d\sigma^{4-\text{jet}}(\varepsilon,\delta)}{dT} \tag{13}$$

and

$$\frac{d\sigma(y)}{dT} = \left(\frac{d\sigma^{3\text{-jet}}(y)}{dx_{max}}\right)_{T=x_{max}} + \frac{d\sigma^{4\text{-jet}}(y)}{dT}, \tag{14}$$

where $d\sigma^{4\text{-jet}}(\varepsilon,\delta)$ and $d\sigma^{4\text{-jet}}(y)$ are the 4-jet cross sections with all jet energies (angles) larger than $(\varepsilon/2)\sqrt{q^2}(\delta)$ and $s_{ij} \geqq yq^2$ (as already defined above), respectively, and where $T = x_{max}$ in case of the three-jet distribution. We obtain the bare thrust distribution, if we let ε, δ and y go to zero in (13) and (14), respectively.

Before we present our results, a few comments about the numerical calculations are in order. The three-jet and four-jet cross sections are individually divergent for ε, δ and y going to zero. While the relevant part of the three-jet cross section has been calculated analytically, the calculation of the four-jet cross section is done with numerical integration methods, what then requires high accuracy to successfully cancel the divergent contributions. Since the dominant contribution comes from the rather small region of phase space of soft and small angle partons, hardly any standard Monte Carlo program will meet this requirement.

We therefore have split the phase space into a part, where the differential cross section varies less dramatically (in a way defined below), which we call the Monte Carlo (MC) region, and a second part (singular region), which contains the collinear maxima of the cross section. The integral over the first region was performed with the iteratively improving Monte Carlo program VEGAS [12], whereas in the singular region a five dimensional generalized Gaussian technique has been applied, which takes into account the form of the nearby singularities of the differential cross section. It has been used previously in a QED-calculation [13].

The above separation of phase space is done with respect to angles Θ_{ij} between pairs of partons, summed over all pairings with singular matrix element for $\Theta_{ij} \to 0$. The admissible opening angles of the cones produced in this way (they must not overlap) depend on T. Inside these cones the differential cross section varies like $1/\Theta_{ij}$, the lower limit of Θ_{ij} depending on y and the parton energies. This behaviour is chosen as a weight function for the generalized Gaussian integration over Θ_{ij}. After this integration being done, the energy (E_i) integration behaves like $\ln(E_i E_j/E_0^2)/E_i$, where E_0 is the minimal energy (again depending on y). The integration over the remaining variables is smooth and can be done by standard Monte Carlo methods.

In the MC region the cross section still behaves like $1/E_i$ due to gluons being soft, but this singularity is removed easily by a logarithmic mapping for the gluon energies.

The relative contribution from the singular region varied between 10% and 30%, depending on y, and has an error less than 1%. The error quoted by VEGAS for the MC region was usually slightly less than 1%.

Since we are interested in the $O(\alpha_s^2)$ corrections, it is appropriate to write

$$\frac{1}{\sigma_0}\frac{d\sigma}{dT} = \frac{\alpha_s}{\pi}A_0(T) + \left(\frac{\alpha_s}{\pi}\right)^2 A_1(T). \tag{15}$$

We have calculated $A_1(T)$ now for various ε, δ and y. The result is shown in Figs. 3 and 4, varying ε, δ and y from about the border line of the nonperturbative regime to the smallest values that are feasible by the Monte Carlo method. In case of the ε, δ cut-off we have chosen $\varepsilon = \frac{1}{2}(1 - \cos\delta)$ to reduce the plot to one parameter. Any other fixed ratio between ε and δ would be possible as well. The dependence on T is shown in Fig. 5 for the two cases $y = 0.04$ and $y = 0.01$ together with the lowest order term $A_0(T)$.

We find that A_1 increases drastically as $\varepsilon = \frac{1}{2}(1 - \cos\delta)$ and y decrease. The increase is most prominant for larger T, which fits in with the qualitative arguments in the introduction. We furthermore reproduce the bare thrust distribution found by the other groups [2, 8, 14]. Numerically we find, however, that this limit is not achieved until $y \leqq 10^{-4}$, which corresponds to an invariant jet mass of about 1 GeV. For example at $T = 0.8$ we find that A_1 changes from $y = 0.04$ to $y = 10^{-4}$ roughly by a factor of 2. Given $A_1/A_0 \simeq 10$ at $y = 0.04$ from Fig. 5, this has the effect that the $O(\alpha_s^2)$ corrections enhance the thrust distri-

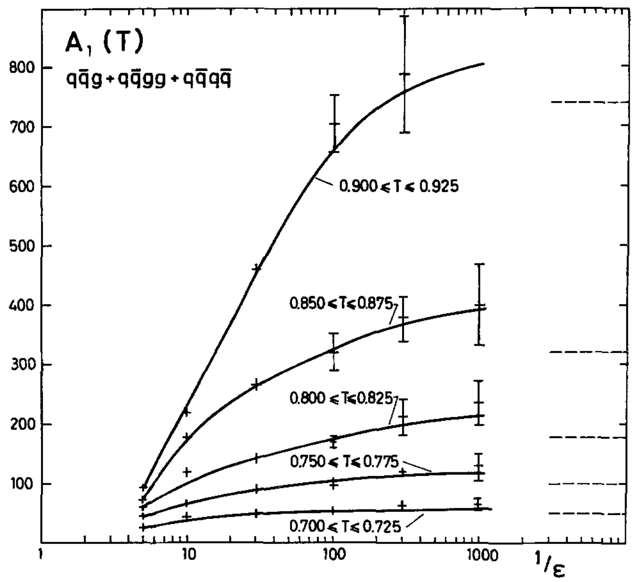

Fig. 3. The sum of three-jet ($O(\alpha_s^2)$) and four-jet contribution to $A_1(T)$ for different thrust bins as a function of $1/\varepsilon$. The dashed lines give the asymptotic values obtained in [14]

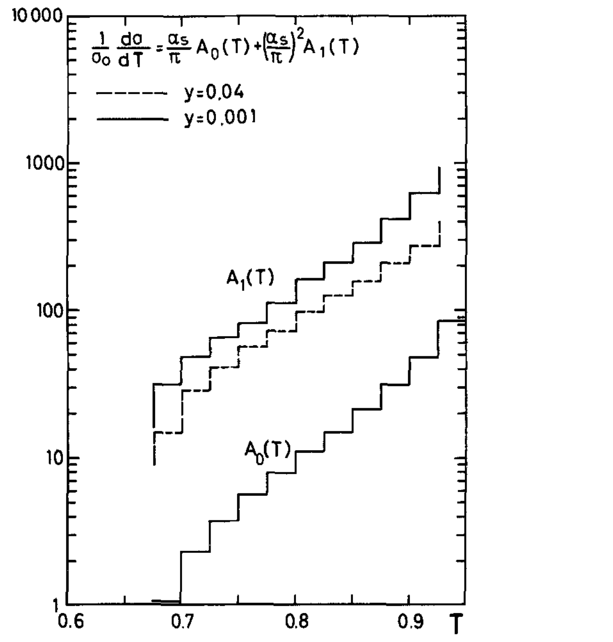

Fig. 5. $A_1(T)$ as a function of thrust T for $y=0.04$ and $y=0.001$ together with lowest order contribution $A_0(T)$

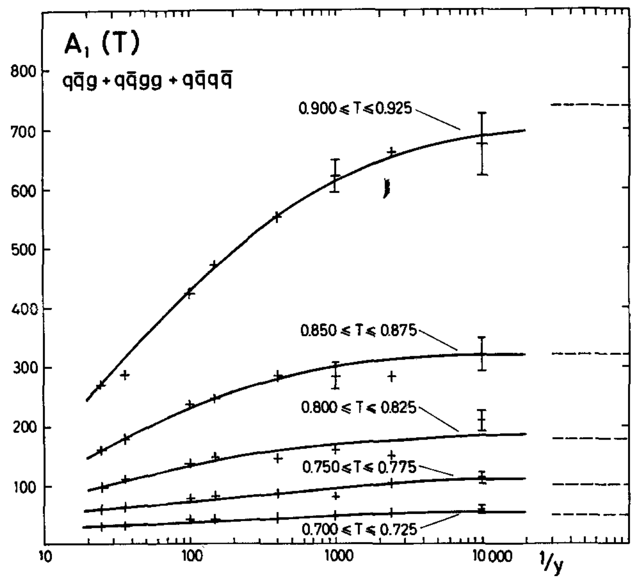

Fig. 4. Same as Fig. 3 as a function of $1/y$

bution by 50% with respect to the lowest order result at $y=0.04$, and assuming $\alpha_s/\pi=0.05$, and by 100% at $y=10^{-4}$. For other thrust values the situation is similar, although A_1/A_0 is somewhat different there (see Fig. 5).

Although the four-jet part had an error not larger than 1% for the invariant mass cut-off and 1.5% for the ε, δ cut-off, the error in $A_1(T)$ is still large in particular for the larger thrust bins. This comes from the strong cancellation between a large positive contribution from four jets and the correspondingly large negative contributions for three jets. This could be improved by other numerical methods. We see that the increase of $A_1(T)$ is not very dramatic for $T \leqq 0.825$ for y values in the physically interesting interval $0.02 \leqq y \leqq 0.04$ and $0.15 \leqq \varepsilon \leqq 0.2$.

4. Conclusions

We conclude that the large $O(\alpha_s^2)$ corrections to the thrust and thrust-like distributions are due to low-mass parton pairs (or equivalently rather soft and collinear partons), being much smaller than the non-perturbative jet mass. This is what we expected on the grounds discussed in the introduction.

The Sterman–Weinberg type cross sections discussed in Sect. 2 are perfectly insensitive to the emission of soft and/or collinear radiation but depend on a resolution parameter. If this fixed resolution is also incorporated into the analysis of the experimental data, as has been done for example with the cluster algorithm, this does not introduce any ambiguity. One may hope that the (theoretical) resolution parameter dependence will also be borne out by the high statistics data to come.

Concerning adding hadronization of quarks and gluons to the perturbative three- and four-jet cross sections, we would argue that the perturbative resolution should not be too much smaller than the nonperturbative jet spread. But this should be seen in relation to the fragmentation mechanism used and the adjustment of fragmentation parameters by the data. Certainly in this field further studies are called for.

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