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MONTE CARLO CALCULATION OF SU(2) GLUEBALL STATES WITH MANTON'S ACTION

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Using Manton's action we carry out a Monte Carlo variational calculation of the SU(2) glueball spectrum. We find a scaling window for the 0^+ state, but no scaling for 1^-, 2^+, 2^- and 3^+ states. The 0^+ result is consistent with universality.

1. Introduction and results

In ref. [1] Creutz carried out a lattice Monte Carlo (MC) calculation and established a *scaling window* for the SU(2) and SU(3) string tension. For SU(2) and SU(3) glueballs 0^{++} (mass gap) similar scaling windows (albeit on a finer scan) were first found in refs. [2, 3]. With some reservations in mind such scaling windows allow predictions about the continuum limit. For instance a reasonable estimate of the SU(2) string tension (close to that of refs. [1, 4]) is

$$\sqrt{K^{\mathrm{W}}} = (79 \pm 12)\Lambda_{\mathrm{L}}^{\mathrm{W}}, \tag{1}$$

and the 0^+ SU(2) glueball estimate of ref. [2] is

$$m_{\mathrm{g}}^{\mathrm{W}} = (170 \pm 30)\Lambda_{\mathrm{L}}^{\mathrm{W}}. \tag{2}$$

In eqs. (1), (2) the index W indicates that the calculations were done with the Wilson action [5], and the Λ-scale is defined as usual:

$$\Lambda_{\mathrm{L}} = a^{-1}\left(\tfrac{6}{11}\pi^2\beta\right)^{51/121}\exp\left(-\tfrac{3}{11}\pi^2\beta\right)\left(1+\mathrm{O}\left(\frac{1}{\beta}\right)\right). \tag{3}$$

If predictions such as (1), (2) are supposed to represent true continuum physics, the question of universality becomes important. This means a wide class of actions has to yield the same continuum theory, when the coupling constant approaches the scaling critical point. In the present paper we use Manton's action [6] and carry out a MC variational (MCV) [2,7,8] calculation of the SU(2) glueball spectrum. Manton's action is given by

$$S^{\mathrm{M}} = \tfrac{1}{2}\sum_{\mathrm{p}}\tfrac{1}{2}\theta_{\mathrm{p}}^2, \tag{4}$$

where θ_{p} is the plaquette angle, related to the plaquette variable U_{p} through

$$U_{\mathrm{p}} = \cos\theta_{\mathrm{p}} + i\boldsymbol{\sigma}\cdot\hat{\boldsymbol{n}}\sin\theta_{\mathrm{p}}.$$

The ratio between the Λ scales $\Lambda_{\mathrm{L}}^{\mathrm{M}}$ and $\Lambda_{\mathrm{L}}^{\mathrm{W}}$ has been calculated perturbatively [9,10]:

$$\frac{\Lambda_{\mathrm{L}}^{\mathrm{M}}}{\Lambda_{\mathrm{L}}^{\mathrm{W}}} = r = 3.07. \tag{5}$$

If the universal (2-loop) part of the β function is already dominant in a MC calculation (of the string tension or the mass gap), then the MC calculation would give a similar result for r. Lang et al. [9] have calculated the string tension with Manton's action. Their result

$$\sqrt{K^{\mathrm{M}}} = (16.2 \pm 0.6)\Lambda_{\mathrm{L}}^{\mathrm{M}}, \tag{6a}$$

gives (combined with (1))

$$r_{\sqrt{K}} = 4.9 \pm 1.0, \tag{6b}$$

and indicates relevant three-loop corrections to (at least one of) the Λ scales (3).

Our present mass-gap calculation gives

$$m_{\mathrm{g}}^{\mathrm{M}} = (49 \pm 6)\Lambda_{\mathrm{L}}^{\mathrm{M}}. \tag{7a}$$

Combined with (2) this yields

$$r_{m_{\mathrm{g}}} = 3.5 \pm 1.0. \tag{7b}$$

Contrary to (6b) eq. (7b) is well consistent with the analytic result (5). Definite conclusions are difficult because of the large error bars in (6b), (7b). Higher perturbative corrections to the Λ scales drop out in mass ratios like

$$m_{\mathrm{g}}^{\mathrm{W}}/\sqrt{K^{\mathrm{W}}} = 2.2 \pm 0.8\,, \tag{8a}$$

as obtained from (1), (2), and

$$m_{\mathrm{g}}^{\mathrm{M}}/\sqrt{K^{\mathrm{M}}} = 3.0 \pm 0.5\,, \tag{8b}$$

as obtained from (6a), (7a). Clearly eqs. (8) are consistent with universality, but they are not very restrictive.

In the present paper we use the MCV method [2,7,8]*. By a different method Mütter and Schilling [12,13] investigated scaling for the SU(2) mass gap. For the Wilson as well as for the Manton action they found scaling windows. Their results are

$$m_{\mathrm{g}}^{\mathrm{W}} = (185 \pm 25)\Lambda_{\mathrm{L}}^{\mathrm{W}}, \qquad \text{ref. [12]}, \tag{9}$$

$$m_{\mathrm{g}}^{\mathrm{M}} = (46 \pm 6.3)\Lambda_{\mathrm{L}}^{\mathrm{M}}, \qquad \text{ref. [13]}, \tag{10a}$$

leading to

$$r_{m_{\mathrm{g}}} = 4.0 \pm 1.0\,. \tag{10b}$$

These results are in good agreement with (2), (7a, b) (consequently also with eqs. (8)).

Our mass gap $m(0^+)$ results are presented in sect. 2. The MCV method is also suitable for calculating excited glueball states [7, 8, 14]. Using all Wilson loops up to length 6 we obtain MC results for 1^-, 2^+, 2^- and 3^+ states, which are presented in sect. 3. Finally a summary and conclusions are given in sect. 4.

2. The mass gap

We use the MCV 2-point correlation procedure as described in ref. [2]. We measure Wilson loops W_i ($i = 1,\ldots,4$) up to length 6 as depicted in fig. 1. The operator

$$\mathcal{O} = \sum_{\lambda=1}^{4} c_i W_i\,, \tag{11}$$

* For reviews of glueball calculations in lattice gauge theories see ref. [11].

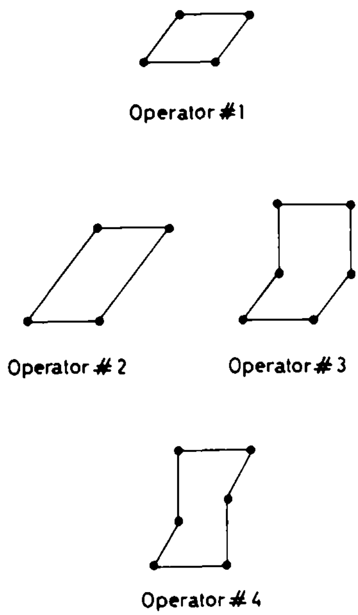

Fig. 1. Wilson loops up to length 6.

is constructed, such that the signal

$$c(t) = \langle \mathcal{O}(t)\mathcal{O}(0)\rangle - \langle \mathcal{O}(0)\rangle^2, \tag{12}$$

is maximized for correlations from distance $t = 1$. By summing each Wilson loop over spacelike translations and orientations, momentum $p = 0$ and angular momentum 0^+ (more precisely A_1^+ [14]) is obtained.

Our MCV calculation is carried out on a $4^3 \cdot 16$ lattice (4^3 is the space-like box). To speed up the calculation we use the 120-element icosahedral subgroup [15] and as in ref. [9] a table for Manton's action (4) is used. Metropolis upgrading with 6 trials for each matrix is done. One sweep through the lattice is defined by upgrading each link matrix precisely once (S-upgrading in the notation of ref. [14]). At each considered β value we have first spent 3000 sweeps for equilibrium without measurements, and then assembled the following statistics:

25 000 sweeps at $\beta = 1.20, 1.30$;
40 000 sweeps at $\beta = 1.40, 1.45, 1.50, 1.55, 1.60, 1.65, 1.70$;
25 000 sweeps at $\beta = 1.80, 1.90$.

All our error bars are computed with respect to 10 bins. The total calculation took 76 CYBER 175 hours, this is roughly equivalent to 38 CDC 7600 hours.

Our final mass-gap results are summarized in fig. 2 and fig. 3. We use the glueball mass definition

$$\tilde{m}(t) = -\ln \frac{C(t)}{C(t-1)}, \tag{13}$$

with $c(t)$ defined by eq. (12).

The educated reader may be worried, because it has been demonstrated in ref. [16] that Manton's action violates Osterwalder-Schrader positivity (more precisely: reflection positivity with respect to *bond* planes), and the transfer matrix has negative eigenvalues for any value of β. In our MC calculation we maximize the correlation function (12) for $t = 1$. Because of $C(1) = \langle \mathcal{O}(0)\mathcal{O}(1)\rangle_c = \langle \mathcal{O}(0)T\mathcal{O}(0)\rangle_c$ (c = connected) only the positive eigenvalues of the transfer matrix are important. It could, however, happen that in the maximization of $C(2) = \langle \mathcal{O}(0)\mathcal{O}(2)\rangle_c = \langle \mathcal{O}(0)T^2\mathcal{O}(0)\rangle_c$ the negative eigenvalues dominate the positive eigenvalues. In such a case the theory could be very pathological and have different wave functions and eigenvalues for T^n (n = odd) and T^n (n = even). At large enough β, where the continuum limit is recovered, such a behaviour can certainly be excluded, and for β small the situation could be checked in strong-coupling expansion.

As our wave functions are not exact eigenfunctions of the transfer matrix T, the negative eigenvalues of T may give rise to (small) oscillations in the correlation

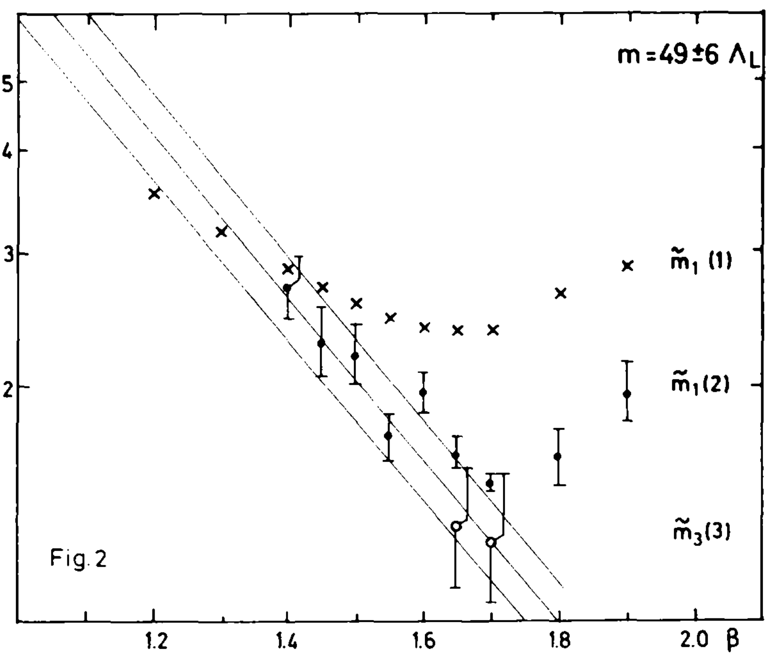

Fig. 2. Mass gap $m(0^-)$ results from the 1-plaquette operator.

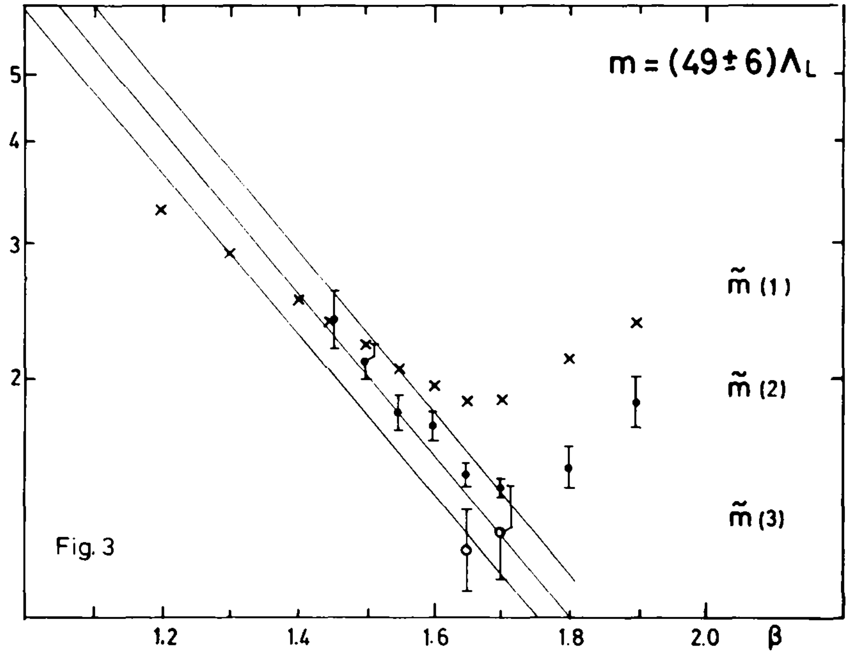

Fig. 3. Minimized mass gap $m(0^+)$ results.

functions. The formula

$$m(t) = -\frac{1}{t}\ln\left|\frac{C(t)}{C(0)}\right|, \tag{13'}$$

will still give upper bounds for the glueballs, but eqs. (13) may have problems. In the present investigation the oscillations are numerically negligible: we have checked all our results with eq. (13′) and found the difference similar as for the corresponding MC investigation with the Wilson action [2]. Therefore we trust eq. (13), which gives better (= lower) results, but leads to t-times larger error bars than eq. (13′).

In fig. 2 we give results as obtained from plaquette-plaquette correlations alone. Glueball mass definitions for single operators are denoted $\hat{m}_i(t)$. They are defined by eq. (13) with $c_j = \delta_{ij}$ in eq. (11).

Fig. 3 gives our results after minimization. Minimization lowers considerably the results from correlations at distance $t = 1$. $\hat{m}(1)$ is already a good upper bound. The final estimate takes $\hat{m}(2)$, $\hat{m}_i(2)$ into account and some results for $\hat{m}(3)$, $\hat{m}_i(3)$ are also out of the statistical noise. Similar to ref. [13] a *scaling window* is found for

$$1.4 \lesssim \beta \lesssim 1.7. \tag{14}$$

TABLE 1
Comparison of our results on an $4^3 \cdot 16$ lattice with those of ref. [7] on an 8^4 lattice ($\beta = 1.55$)

	$4^3 \cdot 16$	8^4 ref. [7]
$\tilde{m}(t=1)$	2.07 ± 0.02	%
$\tilde{m}(t=2)$	1.83 ± 0.09	1.85 ± 0.23
$\tilde{m}_1(t=1)$	2.47 ± 0.02	2.54 ± 0.06
$\tilde{m}_1(t=2)$	1.73 ± 0.12	2.13 ± 0.46

Comparison of $\tilde{m}_1(2)$ and of $\hat{m}(2)$ shows that these values can hardly be distinguished within statistical errors, and within the scaling window these values are argued to exhibit the asymptotic behaviour $t \to \infty$ quite accurately. Putting all our information together, we obtain the estimate (7a) of the introduction.

At $\beta = 1.55$ we can compare with the results from 9000 sweeps on an 8^4 lattice as given in ref. [7]. This is done in table 1. Within the statistical errors only small finite-size effects are indicated.

Finally we would like to mention that our high-statistics data, show that the distance-zero plaquette-plaquette correlation function $C_{11}(0)$ is a very smooth function of β (see fig. 4). This may well be in contradiction with the claim made in ref. [9] of a break in the slope of the derivative of the average plaquette energy $-\partial E/\partial\beta$ at $\beta = 1.6$ using the Manton action. Both quantities are defined by eqs. (11), (12) with $c_i = \delta_{i1}$. For $C_{11}(0)$ one uses space-like plaquettes at time $t = 0$ (summed over space translation and rotations) whereas for $-\partial E/\partial\beta$ all the plaquettes of the lattice are used. This should not make much difference as far as qualitative properties, like the presence of phase transitions, are concerned.

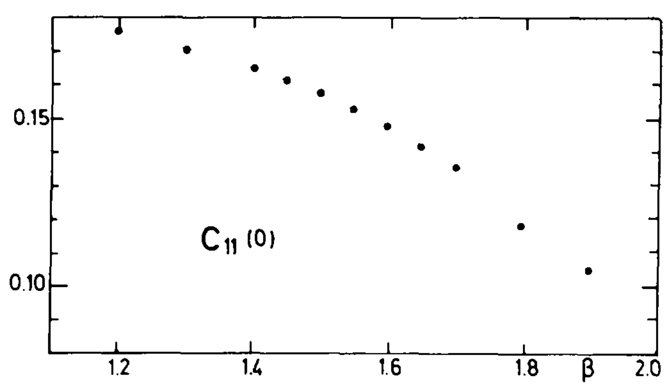

Fig. 4. The connected correlation function $c_{11}(0)$ as function of m.

3. Excited states

As in sect. 2 we consider the spacelike Wilson loops of fig. 1. In ref. [14] we have explicitly constructed all irreducible representations of the full cubic group on each of these loops. For the convenience of the reader we repeat some of the results.

In the standard notation for point groups the irreducible representations of the full cubic group are A_1^P(1d), T_1^P(3d), E^P(2d), T_2^P(3d) and A_2^P(1d) ($P = \pm$parity and in brackets the dimension of the representation). In the continuum limit of an MCV calculation these representations become respectively related to spins: 0^P, 1^P, 2^P, 2^P and 3^P. For SU(2) the representations R(OP) of the full cubic group on the operators OP of fig. 1 decompose as follows:

$$R(1) = A_1^+ \oplus E^+ , \tag{15a}$$

$$R(2) = A_1^+ \oplus A_2^+ \oplus 2E^+ , \tag{15b}$$

$$R(3) = A_1^+ \oplus E^+ \oplus T_2^+ \oplus T_1^- \oplus T_2^- , \tag{15c}$$

$$R(4) = A_1^+ \oplus T_2^+ . \tag{15d}$$

In sect. 2 we investigated the A_1^+ representation. Now we consider T_1^-, E^+, T_2^+, T_2^- and A_2^+. Our MC statistics is that of sect. 2.

From our SU(3) experience [14] we expect high-lying excited states (in units of $m(0^+)$) and severe problems with scaling. However, for SU(2) with Manton's action it was claimed in ref. [7] that (at $\beta = 1.55$ on an 8^4 lattice) spin-2 and spin-3 states are obtained with masses of the order $m(0^+) \pm 30\%$. With our present high-statistics analysis such a result can be ruled out. It is consistent to attribute it to statistical fluctuations. As for SU(3) [14] we find the lowest mass values for the E^+ representation. Details are, however, quite different. Our present main results for the E^+ representation are summarized in fig. 5. Minimization at distance $t = 1$ lowers significantly the mass values as obtained from the 1-plaquette operator ($m_1^{E^+}(1)$) alone. For the optimized wave function the signal is supposed to be maximal. We have results for $m^{E^+}(2)$, which are clearly out of the statistical noise, contrary to the results $m_i^{E^+}(2)$ ($i = 1, \ldots, 4$) for single operators. It is remarkable that the $m^{E^+}(2)$ results are well consistent with the $m^{E^+}(1)$ results. This means we do not expect a very drastic lowering for this state by going to larger distances. In fig. 5 we see no signal for scaling, albeit the $m^{E^+}(2)$ results do not necessarily exclude scaling. In table 2 we have for the considered values of β collected the fractions $m^{E^+}(1)/m^{A_1^+}(1)$. In the scaling window of $m^{A_1^+}(1)$ we find a steady increase. But contrary to SU(3) all fractions are below the two-particle 0^+ threshold. Therefore our present 2^+ data are consistent with rather late scaling of the 2^+ state and a mass value $1.5\, m(0^-) < m(2^+)$ Unfortunately the lack of scaling prevents any serious prediction of the continuum limit. Our results from T_2^+ are consistent with those from E^+.

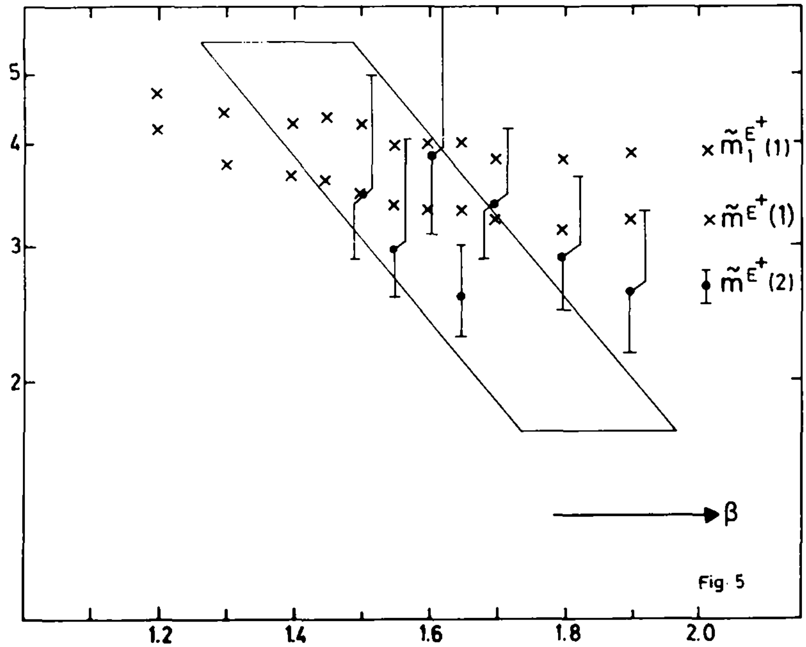

Fig. 5. MCV results for $m(2^+)$ with the representation E^+.

Finally we have collected in table 3 our results for 1^-, 2^- and 3^+. As is obvious from eqs. (15) each of the results relies on a single operator. The results at distance $t = 1$ are much higher than those for 2^+ from single operators. At distance $t = 2$ only noise is obtained. With insufficient statistics, accidentally low-lying results may be obtained. For instance $m^{T_2^-}(2) = 1.0 \pm 0.3$ ($\beta = 1.55$) or $m^{A_2^+}(2) = 1.1 \pm 0.5$ ($\beta = 1.60$). In view of an unstable overall pattern such values have to be discarded.

TABLE 2
Mass fractions $0^+/2^+$ from minimized results at distance $t = 1$

β	$m^{E^+}(1)/m^{A_1^+}(1)$
1.20	1.27
1.30	1.31
1.40	1.45
1.45	1.52
1.50	1.58
1.55	1.63
1.60	1.68
1.65	1.75
1.70	1.70
1.80	1.46
1.90	1.36

TABLE 3
Excited glueball states from correlations at distance $t = 1$

β	$m(1^-)$	$m(2^-)$	$m(3^+)$
1.20	6.1 ± 0.4	5.7 ± 0.7	5.5 ± 0.4
1.30	7.2 ± 0.9	5.2 ± 0.4	5.3 ± 0.4
1.40	6.3 ± 0.6	4.9 ± 0.2	5.0 ± 0.2
1.45	5.8 ± 0.4	5.0 ± 0.3	4.8 ± 0.2
1.50	5.8 ± 0.3	4.5 ± 0.2	4.8 ± 0.2
1.55	5.8 ± 0.4	4.6 ± 0.1	4.5 ± 0.1
1.60	6.1 ± 0.4	4.6 ± 0.2	4.9 ± 0.2
1.65	5.9 ± 0.3	4.6 ± 0.2	4.4 ± 0.1
1.70	6.1 ± 0.4	4.4 ± 0.2	4.3 ± 0.1
1.80	6.4 ± 0.6	4.5 ± 0.2	4.3 ± 0.2
1.90	6.2 ± 0.4	4.3 ± 0.1	4.3 ± 0.3

Conclusions are difficult; at least it is indicated that all these states are much higher than 0^+ and also 2^+.

4. Summary and conclusions

Our MCV calculation for SU(2) lattice gauge theory with Manton's action has given a scaling window for the $m(0^+)$ state. Comparing the continuum estimate (7a) with the corresponding one for Wilson's action (2), consistency with universality is found. A warning comes, however, from the 2d σ-model. In this model mass gap results with the 1-loop improved action [17] are very different from those with the standard action, whereas mass gap results with the tree-level improved action [18] are rather close to those with the standard action.

Our results for 2^+ vary between $1.2m(0^+) < m(2^+) < 1.8m(0^+)$ (table 2). The upper value is much lower than the corresponding one in case of SU(3) [14] with Wilson's action. Lack of scaling, however, prevents a continuum estimate. The results for 1^-, 2^- and 3^+ states are again much higher, and we do not have a good explanation. A discussion would be similar to that given in ref. [14].

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