Nuclear Physics B225[FS9] (1983) 565–578

THE SCREENING OF COLOUR CHARGE IN THE NUMERICAL HOPPING-PARAMETER EXPANSION

H. JOOS

DESY, Hamburg

I. MONTVAY*

II. Institut für Theoretische Physik der Universität Hamburg, FRG

Received 23 June 1983

The screening of the quark-antiquark potential by dynamical quark-antiquark pairs is numerically investigated in SU(2) colour gauge theory with Wilson fermions. The expected qualitative behaviour is born out by a high-order hopping-parameter expansion. The screening length is about 0.8 fm and the dissociation energy of a heavy quark-antiquark pair is near 600–800 MeV.

1. Introduction

In this paper we report on an attempt to calculate the screening of static colour charges by dynamical quarks in the framework of the Monte Carlo evaluation of lattice QCD. As it turns out, the statistics of our numerical calculations allows only a rough determination of the screening parameters. However, the methodological experience and the qualitative insight gained seem worthwhile to be described.

Monte Carlo simulations of lattice gauge theories (in particular, lattice QCD) represent the most successful approach for dealing with the non-perturbative problems of hadron dynamics. The first results (relation between string constant and Λ-parameter [1], glueball masses [2, 3] or deconfinement temperature [4–6]) were obtained in pure gauge theories. In the next step, namely the tentative calculation of the hadron spectrum [7–9], the coupling of fermions to the gauge field had to be considered. However, fermion pair creation was neglected ("quenched approximation"). The effect of pair creation was included only recently in the effective action [10, 11], i.e. the so called "fermion determinant" was taken into account, and the corresponding corrections were estimated for some quantities where those are small.

* Supported by Bundesministerium für Forschung und Technologie, Bonn, Germany.

With this paper we want to open the discussion of problems for which the "fermion determinant" is decisive, because it allows the fragmentation of confined quarks into hadrons.

The screening of static colour sources by dynamical quarks is the simplest of these problems. As fragmentation processes in general [12], this is well studied in the Schwinger model. There it can be shown explicitly, how the linearly rising potential becomes a short range potential [13,14]: if and only if external and fermion charges match, the external charges are completely screened by fermion pair creation. In the intuitive language of QCD, this phenomenon is described as the breaking of the colour string between the static heavy quark-antiquark pair by the light dynamical $q\bar{q}$ pairs. By this process the static heavy quarks turn into heavy colourless mesons, therefore we may consider the screening mechanism as an adiabatic "fragmentation" of heavy quarks into heavy mesons. This can also be studied in the strong-coupling limit of lattice QCD [15].

The QCD potentials can be described by the expectation values of Wilson loops [16]. Screening is indicated by a transition of the area law to a perimeter law for very large Wilson loops. In other words, the local string tension expressed by the Creutz ratios [1] vanishes for large distances. Therefore, the study of screening in lattice QCD requires the calculation of the expectation values of large Wilson loops in the presence of the fermion determinant.

The lattice description of fermions has its own problems. Wilson fermions [17], Susskind fermions [18] and Dirac-Kähler fermions [19] represent competing lattice-approximation schemes. Because of the different treatment of "species-doubling" one would expect that the main differences of these schemes show up in problems for which the fermion determinant is decisive. From this point of view it would be interesting to treat the screening effect in different schemes. However, as a continuation of previous numerical calculations [20,21,9,22] the computational work of this paper is done for Wilson fermions. Some of our qualitative results might be transferred to the other schemes. For the geometric Dirac-Kähler fermions we plan to present it, together with the description of the "geometric hopping-parameter expansion", in a future paper. Dealing with the QCD fermion determinant in a Monte Carlo calculation is a difficult technical problem. There are, however, several possible numerical methods which seem to be promising. The best way is, presumably, to include the fermion determinant (at least approximately) in the updating. This can be achieved by the "pseudofermion method" (see ref. [10] and references therein). Estimates can also be obtained by evaluating the theory at negative flavour number (in some sense, replacing fermions by bosons [23,11]). Another interesting method is based on the stochastic calculation of the fermion determinant [24]. It is also possible to directly evaluate the full fermion determinant on a few (limited size) lattices [25]. As stated before, here we use the hopping-parameter expansion and hope to demonstrate convincingly the feasibility of the description of colour charge screening by this method.

2. Discussion of the calculations

The computation of the expectation values of large Wilson loops $U(\mathcal{C}) = \prod_{l \in \mathcal{C}} U(l)$ in SU(2) lattice QCD is based on the path integral over gluon configurations $U(l)$, $U(l) \in \mathrm{SU}(2)$, $\{l \equiv (x, \mu)\}$: links of the lattice Γ, and over the anticommuting quark fields ψ_x, $\bar{\psi}_y$:

$$W(\mathcal{C}) = \frac{1}{Z} \int \mathcal{D}U \int \mathcal{D}\psi \int \mathcal{D}\bar{\psi}\, \tfrac{1}{2}\mathrm{Tr}\, U(\mathcal{C}) \exp\Big(S_{\mathrm{G}}[U] + \sum_{x,y} \bar{\psi}_x Q_{xy} \psi_y \Big), \tag{1}$$

$$Z = \int \mathcal{D}U \int \mathcal{D}\psi \int \mathcal{D}\bar{\psi} \exp\Big(S_{\mathrm{G}}[U] + \sum_{x,y} \bar{\psi}_x Q_{xy} \psi_y \Big). \tag{1'}$$

Explicit "gaussian fermion integration" leads to

$$W(\mathcal{C}) = \frac{1}{Z} \int \mathcal{D}U\, \tfrac{1}{2}\mathrm{Tr} U(\mathcal{C}) \exp\big(S_{\mathrm{G}}[U] + S_{\mathrm{q}}^{\mathrm{eff}}[U] \big), \tag{2}$$

$$Z = \int \mathcal{D}U \exp\big(S_{\mathrm{G}}[U] + S_{\mathrm{q}}^{\mathrm{eff}}[U] \big), \tag{2'}$$

with the effective action describing virtual $\mathrm{q}\bar{\mathrm{q}}$ pairs:

$$S_{\mathrm{q}}^{\mathrm{eff}}[U] = \ln \det Q[U]. \tag{3}$$

For the gluon part of the action we assume the standard Wilson form ($g^{-2} \equiv \frac{1}{4}\beta$):

$$S_{\mathrm{G}}[U] = \frac{1}{g^2} \sum_{\mathrm{P} \in \Gamma} \mathrm{Tr}\big\{ U(\mathrm{P}) + U^{-1}(\mathrm{P}) - 2 \big\}, \qquad (\mathrm{P}: \text{ plaquettes}). \tag{4}$$

The quark matrix $Q[U]$ has the general form:

$$Q_{x_1 x_2}[U] = A \delta_{x_1 x_2} - \sum_{x, \mu} U(x, \mu) B_\mu \delta_{x_1, x + e_\mu} \delta_{x, x_2}. \tag{5}$$

A, B_μ are 4×4 matrices acting on the Dirac components of the Wilson fermions: $A = 4r + am$; $B_\mu = \frac{1}{2}(r + \gamma_\mu)$, $\gamma_{-\mu} = -\gamma_\mu$, or 16×16 matrices for Dirac-Kähler components given in ref. [19], respectively. This form of $Q[U]$ admits a hopping-parameter expansion of the quark part of the effective action $S_{\mathrm{q}}^{\mathrm{eff}}$, eq. (3). For this we make in the fermion integration the substitution $\bar{\psi} A \to \bar{\psi}$, then $S_{\mathrm{q}}^{\mathrm{eff}}$ becomes, up to an

additive constant:

$$S_q^{\text{eff}}[U] = \operatorname{Tr}\ln(1 - KM[U])$$

$$= -\sum_{R=2}^{\infty} \frac{K^R}{R} \sum_x \sum_{\mathcal{C}_R^x} \operatorname{Tr} \prod_{(x,\mu)\in\mathcal{C}_R^x} M_\mu U(x,\mu)$$

$$\equiv -\sum_{R=2}^{\infty} \frac{K^R}{R} \sum_x L_x^R[U] \equiv -\sum_{R=2}^{\infty} \frac{K^R}{R} L^R[U], \tag{6}$$

with $M_\mu = K^{-1}A^{-1}B_\mu$ ($K = (8r + 2am)^{-1}$, $M_\mu = r + \gamma_\mu$ for Wilson fermions with one flavour, $r = 1$ in our calculation) $\Sigma_{\mathcal{C}_R^x}$ sums over all closed paths $\mathcal{C}_R^x$ of length R with initial and end-point x.

The procedure of the numerical evaluation of eq. (2) with $S_q^{\text{eff}}[U]$ according to eq. (6) consists of the following steps.

(i) We consider $W(\mathcal{C})$ and Z as the expectation values of $\frac{1}{2}\operatorname{Tr} U(\mathcal{C})\exp S_q^{\text{eff}}[U]$, and $\exp S_q^{\text{eff}}[U]$ with respect to the probability measure $(1/Z_0)\exp S_G[U]$ of pure gluon dynamics. Therefore the first step consists in producing samples of gluon configurations in thermal equilibrium by the Metropolis method using the action $S_G[U]$, eq. (4). Altogether we produced 30 configurations on a 10^4 lattice with periodic boundary conditions, separated by 50 iterations with 3 Metropolis updatings per link. The value of the coupling constant is $\beta = 2.3$ which seems to lay in the middle of the scaling window. The average values of the Wilson loops $W_0(I, J; U)$ for length $I, J = 1, \ldots 5$ is calculated for all the configurations.

(ii) The main numerical effort goes into the calculation of $S_q^{\text{eff}}[U]$ for the different configurations. By standard methods [20,21,9,22], we calculate the coefficients $L_x^R[U]$ of the local hopping-parameter expansion up to the order $R = 32$ for a number of points x. These are ~ 300 points per configuration in a sample $\mathcal{A}$ of 20 configurations, and ~ 50 points per configuration in a sample $\mathcal{B}$ of 30 configurations; the points are chosen by random selection. Averaging $L_x^R[U]$ over x leads to an approximation of the hopping-parameter coefficients $L^R[U]$ of $S_q^{\text{eff}}[U]$.

(iii) In order to improve the convergence of the sum $S_q^{\text{eff}}[U] = -\Sigma_{R=2}^{\infty}(K^R/R)L^R[U]$ for hopping parameters K close to the critical value $K_{\text{crit}} \sim 0.16$, we transform these power series in Padé approximants. These calculations give average values $\bar{S}_q^{\text{eff}}$ of $S_q^{\text{eff}}[U]$ which increase from $S_q^{\text{eff}} \simeq 144$ for $K = 0.10$ up to $S_q^{\text{eff}} \simeq 1437$ for $K = 0.16$, (see table 1); the relative variances of the distributions of $S_q^{\text{eff}}[U]$ within the samples $\mathcal{A}$ and $\mathcal{B}$ are of order 0.5–1%. Relevant for the determination of $W(\mathcal{C})$ are the deviations $\Delta S_q^{\text{eff}}[U]$ of $S_q^{\text{eff}}[U]$ from the sample averages (see eq. (7)). Therefore, we represented $\Delta S_q^{\text{eff}}[U]$ as a function of K for all 20 configurations U of the sample $\mathcal{A}$ (fig. 1). Up to $K \simeq 0.14$ the 16th-order power series in K^2 gives usually a good value. In the range $K = 0.14$–0.16 the Padé

TABLE 1
Average values of the quark part of the effective action $\bar{S}_q^{eff}$, and their approximation by formula (11)

K	0.10	0.11	0.12	0.13	0.14	0.15	0.16
$\bar{S}_q^{eff}$	144	222	339	490	709	1015	1437
$\bar{S}_q^{eff}$ (approximation)	141	216	322	472	681	972	1357

approximation becomes important, but the central part of the Padé table is always stable (the deviation from the series is in most cases less than 10–20%).

(iv) Finally we calculate $W(\mathcal{C})$ as the quotient of weighted sample averages

$$W[I,J]=\frac{\Sigma_{U\in\mathcal{C}\text{ or }\mathcal{B}}W_0[I,J;U]\exp S_q^{eff}[U]}{\Sigma_U \exp S_q^{eff}[U]}=\frac{\Sigma_U W_0[I,J;U]\exp\Delta S_q^{eff}[U]}{\Sigma_U \exp\Delta S_q^{eff}[U]}. \tag{7}$$

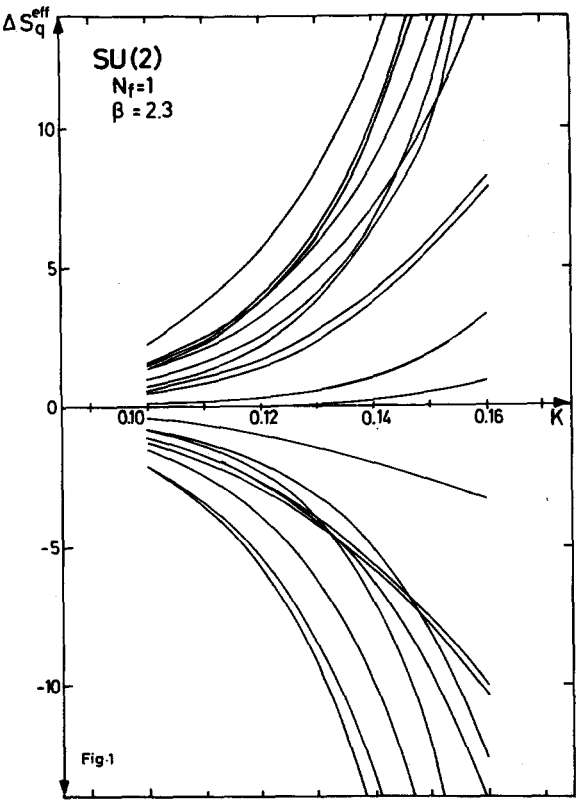

Fig. 1. The distribution of the fermion part of effective action S_q^{eff} (the average subtracted: $\Delta S_q^{eff} = S_q^{eff} - \bar{S}_q^{eff}$) over the 20 configurations of sample $\mathcal{C}$ as a function of the hopping parameter K.

A look at fig. 1 shows that at larger values of K only very few of our configurations actually contribute to this weighted average, a fact, which makes our statistics pure in this range. This shortcoming of our procedure is of course a consequence of the fact that $S_q^{\text{eff}}[U]$ is not included in the updating procedure of the Metropolis method. It can be partly cured with help of the following remark. The lowest non-vanishing term, $R = 4$, in the hopping-parameter expansion eq. (6) (with $r = 1$, we use) has the form

$$\tfrac{1}{4}K^4 \sum_x L_x^4[U] = 8K^4 \sum_{P \in \Gamma} \mathrm{Tr}\{U(P) + U^{-1}(P)\}, \tag{8}$$

similar to that of the pure gluon action, eq. (4). It can, therefore, be absorbed in $S_G[U]$ by a shift in the coupling constant $\beta \to \beta - 32K^4 = 2.3$, and thus included in the updating procedure. The fluctuations $\Delta S_q^{\text{effr}}[U]$ of the reduced action $S_q^{\text{effr}} = -\sum_{R=6}^{\infty} \ldots$ are smaller on the average by a factor of 2 compared to S_q^{eff}, which improves considerably the distributions of weights in eq. (7). The inclusion of the 6th-order term of eq. (6) in the updating procedure would not be difficult and would certainly lead to a further improvement. In this first calculation the results on $W(\mathcal{C})$ are represented in fig. 2. This concludes the first cursory description of our procedure. Next we shall give a theoretical interpretation, and we shall try a critical assessment of the numerical data gained in this way.

The numerical calculation of the coefficients $L^R[U]$ lead us to the discovery of an approximation formula for $S_q^{\text{eff}}[U]$. We observed that these are given to a surprisingly good approximation by

$$L^R[U] \simeq W^{R/2-1} L^R(1). \tag{9}$$

Here $L^R(1)$ are the coefficients of the hopping-parameter expansion of the logarithm of the free determinant:

$$S_q^{\text{eff}}(1, K) = -\sum_{R=4}^{\infty} \frac{K^R}{R} L^R(1)$$

$$= -2 \sum_{k_\rho = B}^{B+N-1} \ln\left\{\left[1 - 2K\sum_\mu \cos\frac{2\pi k_\mu}{N}\right]^2 + 4K^2 \sum_\mu \sin^2\frac{2\pi k_\mu}{N}\right\}, \tag{10}$$

($\rho, \mu = 1 \ldots 4$, $N = 10$), for a numerical evaluation see also ref. [26]. $W = W(1,1) = \frac{1}{2}\mathrm{Tr}\, U(\mathcal{C}(1,1))$ denotes the value of the smallest Wilson loop (plaquette). Formula (9) indicates that a lengthening of the paths $\mathcal{C}^R$ in (6) by two links corresponds on the average to the addition of an uncorrelated plaquette to the area enclosed by $\mathcal{C}^R$. For most of the $L^R[U]$ up to $R = 32$ eq. (9) is correct within 10–20%. Only a change of sign, which for $L^R[U]$ happens already for $R \simeq 16$ compared to $R = 24$ for $L^R(1)$,

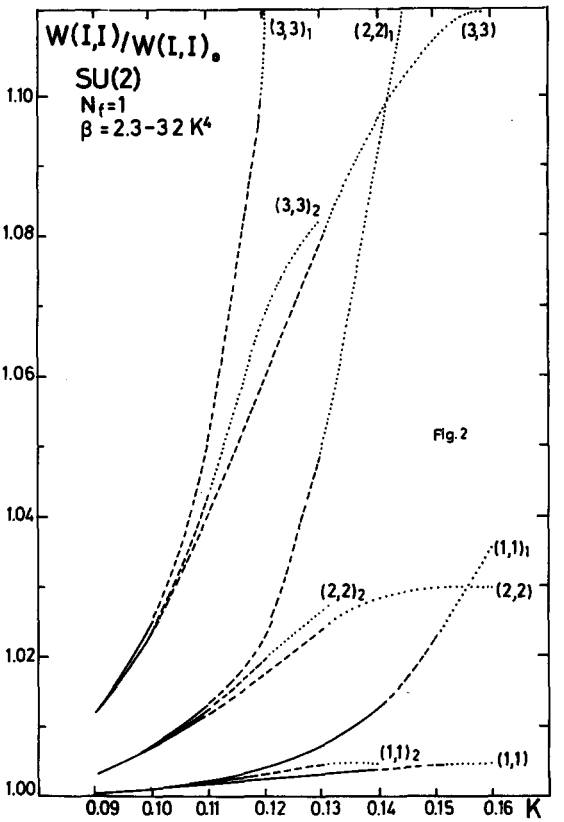

Fig. 2. The value of the Wilson loops $W(I, I)$ after taking into account the fermion determinant. $(W(I, I)_0$ is the value without the fermion determinant at $\beta = 2.3$.) Also shown are the results $W(I, I)_j$ of the expansion in eq. (12) for $j = 1, 2$. Full curves indicate a relative error less than 10% (of $W/W_0 - 1$), dashed curves have an error 10–20% and dotted ones an error 20–30%.

[26] induces major deviations from eq. (9) for $R = 14, \ldots 18$. Using eq. (9) for all orders R allows a summation of the hopping-parameter expansion (6)

$$S_q^{\text{eff}}(U, K) = \frac{1}{W} S_q^{\text{eff}}(1, \sqrt{W} K). \tag{11}$$

The quality of this formula when applied to $\bar{S}_q^{\text{eff}}$ is shown in table 1. It might be that such a formula is only valid for the particular range of coupling constants we consider. However, formulas like this, or improvements of it, may be very helpful for the calculation of more complicated quark fragmentation processes. It turns out, that for the calculation of the screening effects the approximation formula (11) is not good enough. We shall discuss another simple application of eq. (11) in sect. 3.

In order to gain insight into the mechanism of the quark part of the effective action, we discuss a sort of perturbation expansion for the expectation values of the

Wilson loops, eq. (2), in powers of $S_q^{\text{eff}}[U]$:

$$W(\mathcal{C}) = W_0(\mathcal{C}) + W_1(\mathcal{C}) + W_2(\mathcal{C}) + \cdots, \tag{12}$$

$$W_0(\mathcal{C}) = \left(\int \mathcal{D}U \tfrac{1}{2}\mathrm{Tr}\, U(\mathcal{C}) \exp S_G[U]\right) \Big/ \left(\int \mathcal{D}U \exp S_G[U]\right)$$

$$\equiv \langle \tfrac{1}{2}\mathrm{Tr}\, U(\mathcal{C})\rangle_0, \tag{13}$$

$$W_1(\mathcal{C}) = \langle \tfrac{1}{2}\mathrm{Tr}\, U(\mathcal{C}) S_q^{\text{eff}}\rangle_0 - \langle \tfrac{1}{2}\mathrm{Tr}\, U(\mathcal{C})\rangle_0 \langle S_q^{\text{eff}}\rangle_0, \tag{14}$$

$$W_2(\mathcal{C}) = \tfrac{1}{2}\Big(\langle \tfrac{1}{2}\mathrm{Tr}\, U(\mathcal{C})\big(S_q^{\text{eff}}\big)^2\rangle_0 - \langle \tfrac{1}{2}\mathrm{Tr}\, U(\mathcal{C})\rangle_0 \langle \big(S_q^{\text{eff}}\big)^2\rangle_0\Big)$$

$$- \langle S_q^{\text{eff}}\rangle_0 \Big(\langle \tfrac{1}{2}\mathrm{Tr}\, U(\mathcal{C}) S_q^{\text{eff}}\rangle_0 - \langle \tfrac{1}{2}\mathrm{Tr}\, U(\mathcal{C})\rangle_0 \langle S_q^{\text{eff}}\rangle_0\Big). \tag{15}$$

In a theory with N_F flavours, the $W_i(\mathcal{C})$ are the coefficients of an expansion of $W(\mathcal{C})$ in powers of N_F. This follows immediately from the fact that in such a theory the quark part of the effective action is $S_{q,N_F}^{\text{eff}} = N_F S_q^{\text{eff}}$. In our case the expansion (12) seems justified by the relative smallness of S_q^{eff} compared to $S_G \simeq 83000$. However, the second-order contribution turns out numerically important. This indicates a difficulty for the flavour-extrapolation estimates based on calculations at $N_F < 0$ [11].

The correction terms $W_1(\mathcal{C}), W_2(\mathcal{C}), \ldots$ have the form of correlation functions between the Wilson loops and the effective action. Writing $S_q^{\text{eff}}[U]$, according to eq. (6), as a sum over localized densities:

$$S_q^{\text{eff}}[U] = \sum_x S_q^{\text{eff}}[U; x], \qquad S_q^{\text{eff}}[U; x] = -\sum_R \frac{K^R}{R} L_x^R[U], \tag{16}$$

eq. (14) becomes

$$W_1(\mathcal{C}) = \sum_x \left\{\langle \tfrac{1}{2}\mathrm{Tr}\, U(\mathcal{C}) S_q^{\text{eff}}[U; x]\rangle_0 - \langle \tfrac{1}{2}\mathrm{Tr}\, U(\mathcal{C})\rangle_0 \langle S_q^{\text{eff}}[U; x]\rangle_0\right\}. \tag{14'}$$

This shows how in our formulation the screening appears as a correlation between the coefficients $L_x^R[U]$ calculated from the local paths $\mathcal{C}_x$ of the hopping-parameter expansion, and Wilson loops of fixed position. It is the finite correlation length between this Wilson loop and the local effective action which leads to a result for $W(\mathcal{C})$ independent of the size of large lattices. Similarly, the higher terms in expansion eq. (12) can be reduced to local higher-order correlations of $S_q^{\text{eff}}[U; x]$ and $U(\mathcal{C})$. The simplest numerical procedure is to average eq. (13)–(15) over position and orientation of the Wilson loops $\mathcal{C}$ of a given size (I, J). By this, however, the

basic local correlation expressed by eq. (14′) gets lost. As a consequence, our result is given by a difference of large numbers, i.e. by small fluctuations around large mean values. For our choice of the size of the Wilson loops, the size of the $\mathcal{C}_x$ of the hopping-parameter expansion, and the lattice size 10^4, this problem seems unavoidable. However, for larger lattices one should use the locality of the correlation for an effective arrangement of the calculation (e.g. by subdividing the large lattice in smaller pieces and proceeding with these smaller ones in the same way as we do here with the whole 10^4 lattice).

Similar to eq. (7), we calculate $W_1[I, J], W_2[I, J]$ as averages of the samples $\mathfrak{A}$ and $\mathfrak{B}$ of configurations. Since $W_i[I, J]$ have the form of correlations, the expressions become simplified by the subtraction of the sample averages:

$$W_1[I, J] = \sum_{U \in \mathfrak{A}, \mathfrak{B}} \Delta W_0(I, J; U) \Delta S_q^{\text{eff}}[U], \tag{17}$$

$$W_2[I, J] = \tfrac{1}{2} \sum_{U \in \mathfrak{A}, \mathfrak{B}} \Delta W_0(I, J; U) \left(\Delta S_q^{\text{eff}}[U]\right)^2, \tag{18}$$

with

$$\Delta W_0(I, J; U) = W_0(I, J; U) - W_0(I, J),$$

$$\Delta S_q^{\text{eff}}[U] = S_q^{\text{eff}}[U] - \bar{S}_q^{\text{eff}}. \tag{19}$$

As we mentioned above, the mean square average of $\Delta S_q^{\text{eff}}[U]$ is in the order of $\frac{1}{2}$–1%. Typical mean values and variances of the $W_0(I, J)$ are: $W_0(2,2) = 0.1803 \pm 0.0043$, $W_0(2,3) = 0.0886 \pm 0.0040$, $W_0(3,3) = 0.0348 \pm 0.0036$. The correlation coefficients between the $W_0(I, J)$ and the hopping-parameter coefficients L^R are typically 0.5...0.6 in sample $\mathfrak{A}$.

In fig. 2 we have compared the first- and second-order perturbation calculation to the result of the simple averaging according to eq. (7). We shall base our physical discussion in sect. 3 essentially on these data. For hopping parameters up to $K = 0.13$ there is reasonable agreement between the different approximations. For larger K we represent the accuracy of the different calculations by lines of different pattern. This estimate of the accuracy is based on comparing the results on different subsamples of configurations.

For planning future calculations we summarize the sources of errors in our procedure:

(a) the order of the hopping-parameter expansion, or Padé approximation, respectively;

(b) the number of points we choose for the calculation of $S_q^{\text{eff}}[U]$ on a given configuration;

(c) the number of configurations considered;

(d) the order of perturbation theory.

In order to illustrate the importance of higher orders in the hopping parameter K we give three examples. (a) $K = 0.12$: terms up to order $R = 6$ contribute 50–70%; up to $R = 12$ contribute 95% of $S_q^{\text{eff}}[U]$; the deviations between power series and Padé approximants are less than 1%. (b) $K = 0.15$: the corresponding values are: $R \simeq 10$ for 70%; $R = 14$–16 for 95%; Padé approximations indicate deviations up to 10%. (c) $K = 0.16$: the power series do not show convergence up to order $R = 32$; qualitatively the Padé approximants are more stable. (The 4th-order term is always included in $S_G[U]$ as described above, eq. (8).)

The fact that the determination of $S_q^{\text{eff}}[U]$ is based on a subset of lattice points x only, contributes an error to the variance σ of the $S_q^{\text{eff}}[U]$ distribution. Assuming, as usual, $\sigma = \sigma_0\sqrt{1 + d/N_x}$ for the distributions of the hopping-parameter coefficients $L^R[U]$, a comparison of the two samples $\mathfrak{A}$ with $N_x = 300$, and $\mathfrak{B}$ with $N_x = 50$ allows a determination of d. For the sample $\mathfrak{A}$ we get $\sqrt{1 + d/N_x} = 1.30, 1.25, 1.09, 1.05, 1.04$ for $R = 6, 8, 10, 12, 14$. From this we draw the conclusion that for sample $\mathfrak{A}$ we have chosen the minimal acceptable number of points. For higher-order coefficients less points are needed, because a large number of long closed curves contribute. This fact may be used to save computer time. The number of configurations N_{conf} in the samples $\mathfrak{A}$ and $\mathfrak{B}$ are too small to allow any quantitative statement on the dependence of the accuracy on N_{conf}. However, a look at fig. 1 clearly shows that a larger number of configurations is necessary for the larger values of K. This becomes particularly evident by considering the second-order perturbation contribution, eq. (18). On the one hand, according to fig. 2, these contributions are important. On the other hand, their dependence on $\Delta W(\Delta S_q^{\text{eff}})^2$ makes them very sensitive to bad statistics.

In conclusion, based on this experience in handling the quark part of the effective action, we think that one can get reliable results by roughly doubling the computer time. The present computation used the equivalent of 250 h of IBM 3081.

3. First physics results

Now we want to give a physical interpretation of our numerical results. From the limited statistics of the data it is evident that the physical conclusions derived from them are mainly qualitative. Quantitative statements should merely illustrate what type of results one might get from improved calculations.

First we may repeat that our calculations show a significant influence of the quark part of the effective action $S_q^{\text{eff}}[U]$ on large Wilson loops, fig. 2. The deviation goes in the direction of a screening of the static SU(2) colour charges. This can be seen from a comparison of the Creutz ratios

$$\chi(I, J) = -\ln\frac{W(I, J)W(I-1, J-1)}{W(I-1, J)W(I, J-1)} \tag{20}$$

TABLE 2
Creutz ratios calculated for different hopping parameters K with fermion determinant by sample averaging $\chi(I,J)$, first-order perturbation $\chi_1(I,J)$, and without fermion determinant $\bar{\chi}(I,J)$

K	0.10	0.11	0.12	0.13	0.14	0.15	0.16
$\beta(K)$	2.296	2.295	2.293	2.290	2.287	2.284	2.279
$\chi(2,2)$	0.317 ±0.002	0.315 ±0.002	0.312 ±0.003	0.309 ±0.004	0.307 ±0.005	0.305 ±0.006	0.304 ±0.009
$\chi(2,2)_1$	0.316 ±0.002	0.314 ±0.002	0.311 ±0.003	0.298 ±0.004	0.285 ±0.005	0.259 ±0.008	0.202 ±0.018
$\chi(3,3)$	0.216 ±0.006	0.211 ±0.009	0.204 ±0.010	0.195 ±0.012	0.183 ±0.015	0.164 ±0.020	0.146 ±0.028
$\chi(3,3)_1$	0.215 ±0.006	0.207 ±0.009	0.193 ±0.011	0.170 ±0.014	0.137 ±0.018	0.094 ±0.030	0.030 ±0.030
$\bar{\chi}(2,2)$	0.3336	0.3340	0.3366	0.3383	0.3416	0.3441	0.3536
$\bar{\chi}(3,3)$	0.2323	0.2330	0.2361	0.2378	0.2422	0.2465	0.2524

of different sized Wilson loops, with and without taking into account of $S_q^{\text{eff}}[U]$. At large hopping parameters the Creutz ratios of large loops become smaller with $S_q^{\text{eff}}[U]$, indicating that the potential becomes flatter at larger distances. (See table 2.)

It is more daring to construct from our numerical results the potential and derive its typical parameters. Besides the problems with statistics at large hopping parameters the main difficulty arises from the lack of information on the Wilson loops larger than 3×3. (They are very difficult to obtain even without fermion determinant.) We have to compare our screened potential with the unscreened one at distances where the confining potential itself is not purely linear, but has a decreasing slope due, for instance, to the short-distance Coulomb part. For illustration, we tried the following parametrization of the screened potential $V(r)$:

$$V(r) = \bar{V}(r) - \kappa r + \frac{\kappa}{\mu}(1 - e^{-\mu r}). \tag{21}$$

Here $\bar{V}(r)$ is the pure gauge theory confining potential at $\beta = 2.3\text{–}32K^4$ and κr is its linear part (κ = string tension). The exponential form of the screened linear potential chosen in (21) is suggested by the calculations in the Schwinger model [13,14].

According to ref. [16] we have at $\beta = 2.279$ (corresponding to $K = 0.16$) $\bar{V}(r) - \kappa r = \alpha/r$ with $\alpha = 0.164 \pm 0.008$ (fig. 3). Assuming that the Creutz ratios measure the slope of the potential: $\chi(n_1 n) = a^2 \, dV/dr|_{r=an}$ and denoting the lattice spacing in quenched QCD (at $\beta = 2.279$) by $\bar{a} = a/\xi$, from the values of $\chi(2,2)$ and $\chi(3,3)$ in table 2 and $\bar{a}^2\kappa = 0.19$ [16] we obtain $a\mu = 0.72$ and $\xi = 2.4$. In order to transform these values to physical units we need an estimate of the lattice spacing $\bar{a}$. From the

calculation of the hadron masses in quenched SU(2) gauge theory [27,28] we estimate $\bar{a} = 1.2$ GeV^{-1}. This gives $\mu^{-1} \simeq 0.8$ fm for the screening length and $\lim_{r\to\infty} V(r) = \kappa/\mu \simeq 0.6$ GeV for the dissociation energy of the heavy (external) quark-antiquark pair. These numbers depend somewhat on the K-value one uses. $K = 0.16$ is nearly equal to the critical value of the hopping parameter where the pion mass in quenched approximation vanishes. A preliminary estimate of the physical K-value, including the effect of the fermion determinant in the meson spectrum [28], shows that this remains roughly the correct value corresponding to zero quark masses. Besides the above procedure we also tried some other similar parametrizations for screening (e.g. by taking the parameters α, κ from our own data instead of the fit of ref. [16]). The obtained values of μ^{-1} ranged from $\mu^{-1} = 0.6$ to $\mu^{-1} = 1.0$ fm and those of κ/μ from 500 to 1000 MeV. The largest uncertainty was in the ratio of the lattice spacings $\xi = a/\bar{a}$ changing between 1.1 and 2.4. Thus our calculation leads to screening parameters of the expected order of magnitude. A better determination of the parameters and a check of validity of the parametrization (21) would be possible from data at different β values.

Finally, let us briefly comment on a simple use of the approximation formula (11). For free quarks the singularity corresponding to zero quark mass is at $K = (8r)^{-1}$. In the free theory this is the lowest singularity also in the multi-quark Green functions which couple in the interacting theory to the hadronic bound states. The

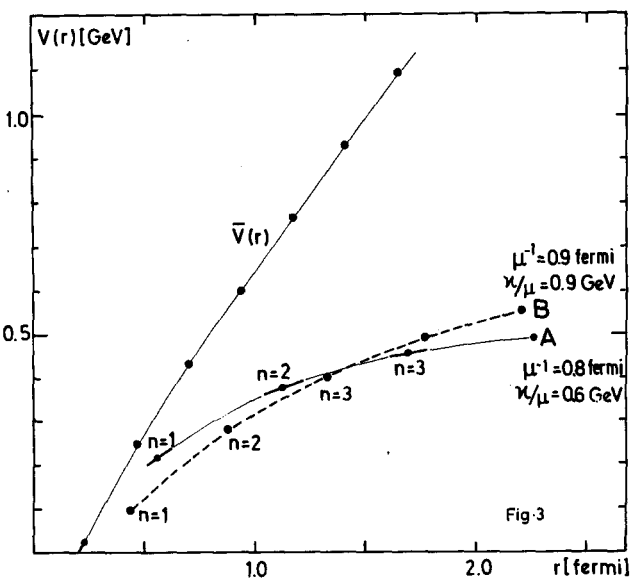

Fig. 3. The q$\bar{q}$-potential fit $\bar{V}(r)$ of ref. [16] compared to the screened potential eq. (21) with parameter values $\mu^{-1} = 0.8$ fm, $\kappa/\mu = 0.6$ GeV (curve A). The tangents at $n = 1,2,3$ indicate the values $\chi_{1,2,3}$ of the Creutz ratios, where the parameters are derived from. The dashed curve (B) is the screened potential at some other typical parameter values; $\mu^{-1} = 0.9$ fm, $\chi/\mu = 0.9$ GeV.

"critical hopping-parameter" K_c in the interacting theory is defined as the K-value, where the lowest hadron (the pion) becomes massless [29]*. According to the approximation (11) K_c in the quenched approximation is connected to the free-quark singularity by

$$K_c \simeq \frac{1}{8r\sqrt{W}} . \tag{22}$$

This formula is well satisfied by the known estimates of K_c in SU(2) [27,28]. For instance, at $\beta = 2.3$ (and $r = 1$) eq. (22) looks like $0.159 \simeq 0.161$. In a recent SU(3) calculation on 16^4 lattices [30,31] we have at $\beta = 5.4$: $0.193 \simeq 0.182$ and at $\beta = 5.7$: $0.169 \simeq 0.169$.

References

[1] M. Creutz, Phys. Rev. Lett. 43 (1979) 553; Phys. Rev. D21 (1980) 2308
[2] B. Berg and A. Billoire, Phys. Lett. 113B (1982) 65; 115B (1982) 151
[3] K. Ishikawa, G. Schierholz and M. Teper, Phys. Lett. 116B (1982) 429
[4] L.D. McLerran and M. Svetitsky, Phys. Lett. 98B (1981) 195
[5] J. Kuti, J. Polónyi and K. Szlachányi, Phys. Lett. 98B (1981) 199
[6] J. Engels, F. Karsch, I. Montvay and H. Satz, Phys. Lett. 101B (1981) 89
[7] H. Hamber and G. Parisi, Phys. Rev. Lett. 47 (1981) 1792;
E. Marinari, G. Parisi and C. Rebbi, Phys. Rev. Lett. 47 (1981) 1795
[8] D.H. Weingarten, Phys. Lett. B109 (1982) 57
[9] A. Hasenfratz, P. Hasenfratz, Z. Kunszt and C.B. Lang, Phys. Lett. 110B (1982) 282
[10] H.W. Hamber, E. Marinari, G. Parisi and C. Rebbi, Phys. Lett. 124B (1983) 99
[11] W. Duffy, G. Guralnik and D. Weingarten, Phys. Lett. 125B (1983) 311
[12] A. Casher, J. Kogut and L. Susskind, Phys. Rev. D10 (1974) 732
[13] S. Coleman, R. Jackiw and L. Susskind, Ann. of Phys. 93 (1975) 267
[14] P. Becher, Ann. of Phys. 146 (1983) 223
[15] H.G. Dosch and V.F. Müller, Nucl. Phys. B158 (1979) 419
[16] J.D. Stack, Phys. Rev. D27 (1983) 412
[17] K.G. Wilson, Phys. Rev. D14 (1974) 2455; in New phenomena in subnuclear physics, Erice, 1975, ed. A. Zichichi (Plenum Press, New York, 1977)
[18] L. Susskind, Phys. Rev. D16 (1977) 3031, and references therein
[19] P. Becher, Phys. Lett. 104B (1981) 221;
P. Becher and H. Joos, Z. Phys. C15 (1982) 343
[20] C.B. Lang and H. Nicolai, Nucl. Phys. B200[FS4] (1982) 135
[21] A. Hasenfratz and P. Hasenfratz, Phys. Lett. 104B (1981) 489
[22] P. Hasenfratz and I. Montvay, Phys. Rev. Lett. 50 (1983) 309
[23] S. Anthony, C. Llewellyn Smith and J. Wheater, Phys. Lett. 116B (1982) 287
[24] J. Kuti, Phys. Rev. Lett. 49 (1982) 183; in Proc. Johns Hopkins Workshop, Florence, 1982
[25] I.M. Barbour, J.P. Gilchrist, H. Schneider, G. Schierholz and M. Teper, Phys. Lett. 127B (1983) 433
[26] B. Berg, A. Billoire and D. Foerster, Lett. Math. Phys. 6 (1982) 293

* Stehr and Weisz in this reference give corrected numerical values.

[27] M. Fukugita, T. Kaneko and A. Ukawa, Monte Carlo study of meson spectroscopy in SU(2) lattice gauge theory, KEK-preprint TH.58 (1982)
[28] I. Montvay, Estimate of the effect of virtual quark loops on meson masses in lattice QCD, DESY preprint 83-074 (1983)
[29] N. Kawamoto, Nucl. Phys. B190[FS3] (1981) 617;
J. Stehr, P. Weisz, Nuovo Cim. Lett., to appear
[30] F. Gutbrod, P. Hasenfratz, Z. Kunszt and I. Montvay, Phys. Lett. 128B (1983) 415
[31] P. Hasenfratz and I. Montvay, Meson spectrum in quenched QCD on a 16^4 lattice, DESY preprint 83-072 (1983)