HIGH-ENERGY PION–PROTON ELASTIC SCATTERING AND THE PION FORM FACTOR

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Recent data on π p elastic scattering at 200 GeV/c from Fermilab allow a further application of the Chou–Yang relation between scattering amplitudes and form factors. Using a carefully selected parametrization of the elastic amplitude, we find that the pion form factor has a *zero* for $|t|$ between 15 and 20 (GeV)^2. In order to locate this zero accurately, more experimental data at large momentum transfers are needed.

With the close agreement [1] between the CERN $\bar{p}p$ data [2] and the early theoretical predictions [3] [‡1]. $\bar{p}p$ elastic scattering at high energies is reasonably well understood. The same cannot yet be said of pion–proton scattering. It is the purpose here to investigate phenomenologically this latter problem.

The present considerations are motivated by the excellent experimental data [4] from Fermilab for π^- of 200 GeV/c. Since the theoretical prediction [5] and the subsequent experimental verification [6] of increasing cross sections for pp scattering, phenomenological analyses of high-energy scattering have been carried out at two levels. On one level, the increasing cross section is taken into account together with its implications such as the fact that the scattering amplitudes are necessarily complex. Alternatively, the increasing cross section is ignored, and the scattering amplitudes are considered to be approximately purely imaginary. This second approach is much simpler and may be justifiable when the energy is not very high so that the total cross section is not yet increasing significantly. In this paper, we take this second, simpler approach, and use, as the basis of our phenomenological analysis, the Chou–Yang method [7].

One of the original motivations of the Chou–Yang analysis was the observed relation [8] between pp elastic scattering cross section, and the proton form factor. It is therefore natural to use this method to deduce the pion form factor from π^-p elastic scattering cross section. Specifically, we make the following assumptions:

(1) On the theoretical side, the Chou–Yang relation [eq. (1) below] between the elastic amplitudes and the form factors are assumed to be valid to a large momentum transfer of about 4 GeV/c.

(2) On the experimental side, the Fermilab data [4] are assumed to imply that, in the πp elastic cross section, there is a second dip at around $|t|$ = 8 (GeV)^2 followed by a third maximum of the observed magnitude.

Our result is sensitive to both assumptions. In particular, future confirmation of the data point at $|t|$ = 10.25 (GeV)^2 is of great importance. This problem of extracting the pion form factor from the Fermilab data has been considered in two recent papers [9,10] that we shall compare to the present work.

The Chou–Yang relation expresses the πp scattering amplitude $a(t)$ in terms of the form factors F_π and F_p as follows.

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[‡1] The slope of the cross section found in ref. [1] is slightly lower near the forward direction than the value of the UA4 experiment.

$$a(t) = \mathrm{i} \int_0^\infty b\, \mathrm{d}b\, J_0(b\sqrt{-t})$$
$$\times \left[1 - \exp\left(-C \int_0^\infty \sqrt{-t'}\, \mathrm{d}\sqrt{-t'}\right.\right.$$
$$\left.\left.\times F_\pi(t') F_\mathrm{p}(t') J_0(b\sqrt{-t'})\right)\right], \quad (1)$$

where C is a normalization constant. Therefore

$$CF_\pi(t)F_\mathrm{p}(t) = -\int_0^\infty b\, \mathrm{d}b\, J_0(b\sqrt{-t}) \ln[1 - \widetilde{a}(b)]\,, \quad (2)$$

with

$$\widetilde{a}(b) = \int_0^\infty \sqrt{-t}\, \mathrm{d}\sqrt{-t}\, J_0(b\sqrt{-t}) a(t)\,.$$

Since we have to compute a two-fold Hankel transform with a very high accuracy, it is desirable to use a convenient form for $a(t)$ which not only fits the experimental data but also agrees with the known qualitative features including the asymptotic behavior allowed by the analytic properties [11]. After considering various possibilities, we have chosen

$$a(t) = \alpha_1 \exp[-\beta_1(|t| + \gamma_1)^{1/2}]$$
$$+ \alpha_2 \exp(-\beta_2|t|) J_1(\gamma_2\sqrt{|t|})/\sqrt{|t|}\,. \quad (3)$$

The first term which survives when $|t| \to \infty$ has been used in a previous analysis of high-energy hadron–hadron scattering [3]. The second term in (3) leads to a change of sign in the amplitude in order to produce the dips of the cross section related to the zeroes of the Bessel function J_1. When $\beta_2 = 0$, this term corresponds to the scattering by a black sphere. However, we know that physical particles cannot have such a sharp edge, so β_2 must be positive. The parameters corresponding to the best fit of the data are given in table 1 and the result of the fit is shown by the solid curve in fig. 1. Here some comments are in order on the role of the different parameters. The first term dominates up to $|t| \sim 3\ (\mathrm{GeV})^2$ because $\alpha_1 \gg \alpha_2$ and the value of β_1 is related to the rapid fall off of the cross section near the forward direction. In the second term the value of β_2 is strongly related to the decrease between the second and the third maximum. Moreover the value of γ_2 could be anticipated to reproduce the position of the two dips. For comparison we also show in fig. 1 (dashed line) the result of

Table 1
Values of the parameters and corresponding errors of our fit with $\mathrm{d}\sigma/\mathrm{d}t = \pi|a(t)|^2$.

$\alpha_1 \pm \Delta\alpha_1$	705.67	$\pm$ 0.98 GeV^{-2}
$\beta_1 \pm \Delta\beta_1$	6.979	$\pm$ 0.001 GeV^{-1}
$\gamma_1 \pm \Delta\gamma_1$	0.5053	$\pm$ 0.0004 GeV^{-2}
$\alpha_2 \pm \Delta\alpha_2$	0.00386	$\pm$ 0.00026 GeV^{-1}
$\beta_2 \pm \Delta\beta_2$	0.080	$\pm$ 0.016 GeV^{-2}
$\gamma_2 \pm \Delta\gamma_2$	2.616	$\pm$ 0.040 GeV^{-1}

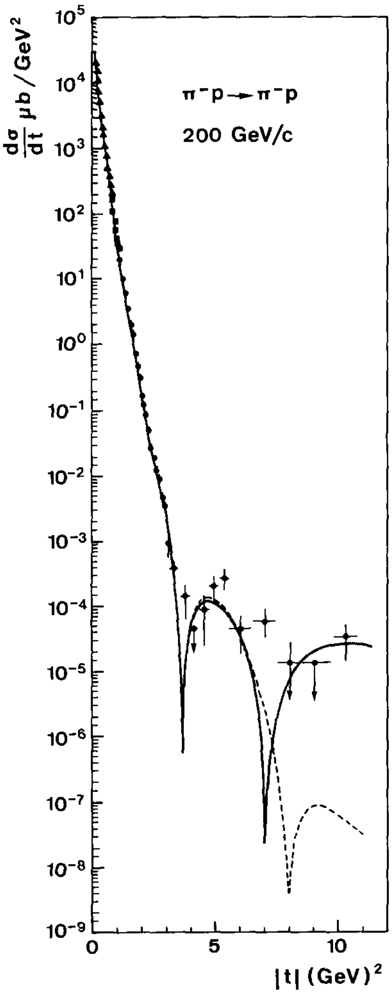

Fig. 1. π^-p elastic scattering data at 200 GeV/c (ref. [4] and ref. [15]). Solid curve from parametrization (3). Dashed curve from ref. [10].

the fit obtained by using the parametrization of ref. [10]. This fit is very similar to the previous one, except at large t where it does not match the last data point. In addition, in the parametrization of ref. [10], the dominant term $\exp(-\beta t)$ gives an asymptotic t behavior in conflict with analyticity and the term $J_0(\gamma|t|)$ seems hard to justify.

Once the parametrization of the cross section is determined, its impact parameter representation $\widetilde{a}(b)$ is easily computed by using eq. (2) and the resulting curves are displayed in fig. 2. We see that the $\widetilde{a}(b)$ corresponding to ref. [10] decreases for large b like a gaussian, that is much faster than in our case where we only have an exponential fall off. This reflects different small t behaviour of the two parametrizations. The pion form factor can be deduced from (2) and we have assumed that the proton form factor is $F_p(t) = [1 + |t|/0.71]^{-2}$. We show in fig. 3 the absolute value of the resulting F_π. The solid line which corresponds to our parametrization of $a(t)$ has a zero for $|t| = 15.8\ (\text{GeV})^2$. The position of this zero is related to the value of the cross section around $|t| = 10\ (\text{GeV})^2$. For example, if one reduces this

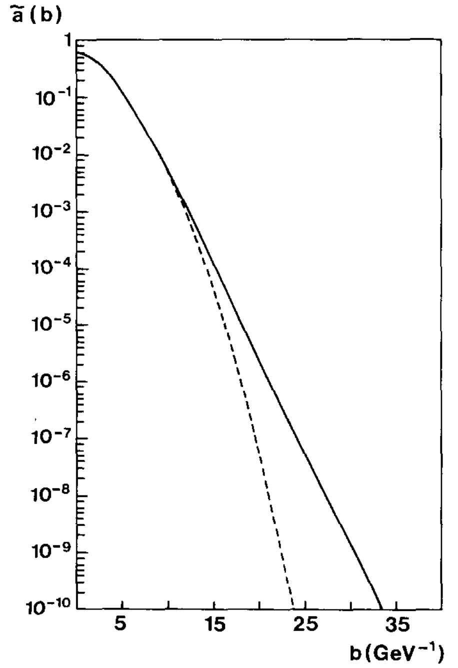

Fig. 2. $\widetilde{a}(b)$ impact parameter representation of $a(t)$. Solid curve from parametrization (3). Dashed curve from ref. [10].

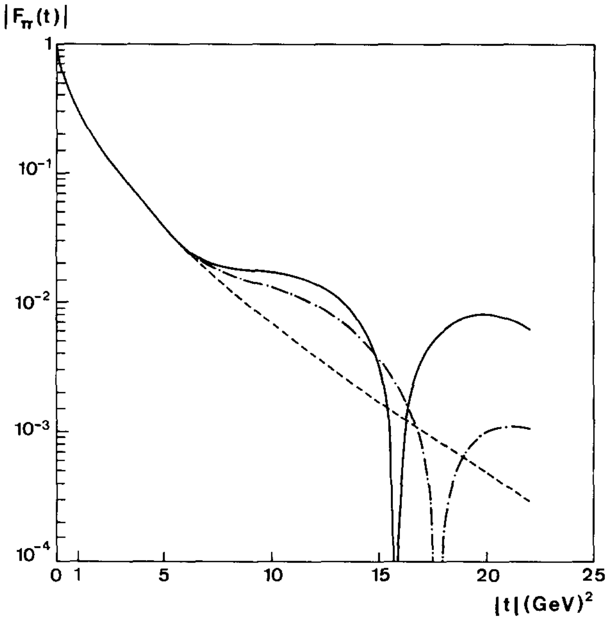

Fig. 3. The resulting pion form factor. Solid and dotted–dashed curves from parametrization (3). Dashed curve from ref. [10].

value by a factor of three, one obtains the dotted–dashed line where the zero moves out and occurs at $|t| = 17.8\ (\text{GeV})^2$ instead. Finally the dashed line with no structure corresponds to the form factor determined by using the parametrizations of ref. [10] with the values of the parameters corresponding to the fit shown in fig. 1. These three predictions for F_π coincide up to $|t| = 6\ (\text{GeV})^2$ where they are in agreement with the data from electroproduction [12]. In the work of ref. [9] the derivatives of the parametrization of the data are not continuous. This parametrization is in conflict with analyticity and the resulting F_π is also structureless.

There are some theoretical arguments [13] in favour of the existence of a zero in hadron form factors, but so far no experimental evidence. This is an important challenge which requires new data at large momentum transfer. It is worth noting that many nuclei have zeroes in their charge form factor [14]. If these zeroes are also present in the case of hadrons they will certainly represent an important information for our understanding of their composite nature.

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