MONTE CARLO CALCULATION OF SU(2) GLUEBALL STATES WITH SYMANZIK's TREE-LEVEL IMPROVED ACTION

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With the tree-level improved 4d SU(2) lattice gauge theory we carry out a Monte Carlo calculation of the spectrum. We find a scaling window for the 0^+ state, leading to $m(0^+) \approx 53\,\Lambda_L^{TI}$. Results for the 2^+ state are somewhat inconclusive but also consistent with scaling.

Let us consider 4d SU(2) lattice gauge theory. In the continuum limit each physical quantity is proportional to an appropriate power of the correlation length (inverse mass gap) ξ with an universal coefficient. For non-zero lattice spacing $a \neq 0$ this "scaling" is violated by non-universal terms of order $(a/\xi)^2 \times \ln(\xi/a)$. Symanzik [1] proposed to reduce these violations to order $(a/\xi)^4 \ln(\xi/a)$ by including in the lattice action irrelevant terms, which can be determined in perturbation theory, in $1/N$ expansion or, in principle, also by Monte Carlo checks of scaling. For 4d lattice gauge theories the tree-level improved (TI) action is known due to work by Weisz [2] and Curci et al. [3]. The latter authors showed that Symanzik's program, to all leading logarithms, can be accomplished by adding to the Wilson lagrangian the simplest term of dimension 6, i.e. the rectangle.

As in the Monte Carlo (MC) calculation of ref. [4] we use the choice

$$S^{TI} = -\frac{4}{g^2}\left(\frac{5}{3}\sum_{\square}\mathrm{Re}\,(\mathrm{tr}\,\square) - \frac{1}{12}\sum_{\square\!\square}\mathrm{Re}\,(\mathrm{tr}\,\square\!\square)\right) . \tag{1}$$

Here $\square\!\square$ represents the rectangular double plaquettes of size 1×2.

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Using Wilson's [5] action Creutz [6] carried out a well-known MC calculation of the SU(2) and SU(3) string tension, and established a scaling window. With some reservations in mind this allows to extrapolate the continuum limit. A reasonable estimate of the SU(2) string tension (close to that of refs. [6,7]) is

$$(K^W)^{1/2} = (79 \pm 12)\Lambda_L^W . \tag{2}$$

For the SU(2) TI action (1) a MC calculation of the string tension was carried out in ref. [4]. A scaling window is found, which leads to the estimate

$$(K^{TI})^{1/2} = (17.9 \pm 1.0)\,\Lambda_L^{TI} . \tag{3}$$

Combining eqs. (2) and (3) yields

$$\Lambda_L^{TI}/\Lambda_L^W = 4.4 \pm 0.9 , \tag{4}$$

in good agreement with the perturbative claim about $\Lambda_L^{TI}/\Lambda_L^W$ in ref. [3]. The very low statistics of ref. [4] prevent, however, conclusions about improvements. Also the definition of the string tension was not taken TI, but the numerical relevance of this can argued to be minor. We like to emphasize that improvement in the sense of Symanzik [1] is relevant for mass ratios.

For the SU(2) 0^+ glueball (mass gap) a scaling window was first obtained in ref. [8]. Wilson's action was used and the final estimate reads

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$$m(0^+)^{\mathrm{W}} = (170 \pm 30)\Lambda_{\mathrm{L}}^{\mathrm{W}} . \qquad (5)$$

Excited SU(2) glueball states were also studied in the literature [9–12], but the situation is unsatisfactory, because no convincing scaling windows could be established.

In this letter we report first results of a MC calculation of the SU(2) spectrum with the TI action (1). We use a variant of the MC variational method, which was pioneered in refs. [8–10]. Our lattice size is $5^3 \cdot 8$ (5^3 is the spacelike box and 8 is the extension in time direction). In view of the double plaquettes in the action (1) 5^3 seems to be the smallest feasible spacelike extent. As in ref. [13] our calculation is based on 21 different Wilson loops W_i ($i = 1, ..., 21$) of length $\leqslant 8$. We perform always two MC sweeps before measuring the 21 loops. Each of our considered β-values is based on 20000 such double sweeps and measurements, 1200 sweeps without measurements were always done first for reaching equilibrium.

In view of our previous experience [8,13] we decided to restrict minimization to on-diagonal correlations

$$C_i(t) = \langle W_i(t)\, W_i(0)\rangle - \langle W_i(0)\rangle^2 . \qquad (6)$$

More precisely: We consider the glueball mass definitions [8]

$$m_i(t) = -t^{-1} \ln [C_i(t)/C_i(0)] , \qquad (7a)$$

and

$$\hat{m}_i(t) = -\ln [C_i(t)/C_i(t-1)] . \qquad (7b)$$

For $t = 1$ we pick out the best = lowest values $m_i(1) = \hat{m}_i(1)$, and verify that they give compatible results for $m_i(2)$ and $\hat{m}_i(2)$.

The best results for the 0^+ state (i.e. the A_1^+ representation [13]) are always obtained with the operators of fig. 1. Their $m_i(t)$ values are collected in table 1. The values $\beta = 1.5, 1.6$ and 1.7 indicate a scaling curve, which should be improved. From fig. 2 we obtain the preliminary estimate

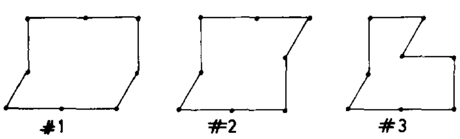

Fig. 1. Best trial operators for the 0^+ state.

Table 1
0^+ estimate from the operators of fig. 1.

β	OP	$m(1)$	$m(2)$	$\hat{m}(2)$	Used
1.5	1	2.07±0.03	2.15±0.13	2.23±0.28	–
1.5	2	2.07±0.03	2.19±0.14	2.31±0.30	–
1.5	3	2.03±0.03	2.12±0.11	2.22±0.24	3
1.6	1	1.81±0.03	1.80±0.08	1.79±0.16	1
1.6	2	1.82±0.03	1.79±0.08	1.77±0.16	–
1.6	3	1.83±0.02	1.80±0.07	1.77±0.16	–
1.7	1	1.75±0.03	1.59±0.06	1.43±0.12	–
1.7	2	1.75±0.03	1.56±0.06	1.38±0.12	2
1.7	3	1.81±0.03	1.63±0.06	1.45±0.12	–
1.8	1	2.03±0.04	1.91±0.09	1.78±0.21	–
1.8	2	2.01±0.03	1.89±0.09	1.76±0.21	2
1.8	3	2.08±0.03	1.92±0.09	1.77±0.20	–
1.9	1	2.26±0.03	2.04±0.31	1.82±0.30	1
1.9	2	2.23±0.04	2.02±0.13	1.82±0.27	–
1.9	3	2.32±0.04	2.11±0.16	1.90±0.34	–

$$m(0^+) = (53 \pm 8)\Lambda_{\mathrm{L}}^{\mathrm{TI}} . \qquad (8)$$

From all considered 21 operators we have in fig. 2 always used the one (as indicated in table 1) which gives the lowest value for $m_i(1)$.

Scaling with respect to the Λ-parameter (asymptotic scaling in the notation of ref. [14] seems not be improved. Symanzik's improvement program is made for mass ratios and not for asymptotic scaling, therefore the reader should not get confused from the "experimental" MC results as obtained in the 2d O(3) σ-model [15,16,14].

Further we note from fig. 2 that $a/\xi > 1$ (as for the Wilson action [8]) in the presently accessible

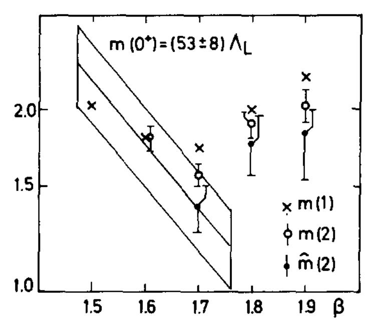

Fig. 2. $m(0^+)$ estimate.

scaling window. One may wonder how Symanzik's improvement program could work at all in this range of ξ. This is indeed a critical point. To sharpen its understanding, let us expand the ratios of two masses in the following way:

$$m_1/m_2 = c_0 + \sum_{n=1}^{\infty} c_n (a/\xi_r)^{2n} \ln(a/\xi_r) ,$$

with

$$\xi_r = \alpha\xi .$$

In ref. [17] ξ_r is called "relevant range of interaction". The parameter α should be choosen to make numerically $c_n \approx 1$ for the next (unknown) correction. No cogent arguments for the often assumed choice $\alpha = 1$ exist, as has been stressed in ref. [17]. On the contrary: MC investigations and heuristic arguments indicate $\alpha \approx 4$ for 4d SU(2) [and SU(3)] lattice gauge theories.

As mass ratios are T-improved, we are particularly interested in excited glueball states. Following the classification of ref. [13] we have considered 0^-, 1^+ and 2^+ states (A_1^-, T_1^+ and E^+ representations of the cubic group. There is an improvement in the following sense: In contrary to results with the standard action [12,18] and with Manton's action [13] a region of β is found, where mass ratios from values for $t = 1$ become constant within statistical errors. The relevant results are collected in table 2. For $\beta = 1.55-1.70$ all mass ratios fit into the following rather narrow bounds:

$$m(2^+) = (1.67 \pm 0.05) m(0^+) , \tag{9a}$$

$$m(0^-) = (2.27 \pm 0.07) m(0^+) , \tag{9b}$$

$$m(1^+) = (2.67 \pm 0.10) m(0^+) . \tag{9c}$$

Table 2
Mass ratios from $t = 1$ correlations [a)]

β	$m(2^+)/m(0^+)$	$m(0^-)/m(0^+)$	$m(1^+)/m(0^+)$
1.50	1.56±0.04	2.13±0.14	2.5 ±0.1
1.55	1.65±0.03	2.23±0.08	2.7 ±0.2
1.60	1.65±0.04	2.29±0.07	2.7 ±0.2
1.65	1.70±0.03	2.27±0.03	2.86±0.11
1.70	1.68±0.03	2.28±0.04	2.77±0.07
1.75	1.51±0.06	2.00±0.07	2.36±0.13

a) This table was added in the revised version and relies on an extended statistics.

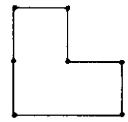

Fig. 3. Best trial operator for the 2^+ state.

It is of course unsatisfactory to rely on correlations taken at distance $t = 1$, and the relevance for the continuum limit remains obscure. It must, however, be emphasized that for 0^- and 1^+ only this correlation is out of the statistical noise, such that conclusions become possible at all.

For the 2^{++} state, which is our lowest excited state, we obtain also some signals at distance $t = 2$, but noise is still seen to be a severe problem. With the exception $\beta = 1.9$ the lowest value for $m_i^{E^+}(1)$ is always obtained from the operator of fig. 3. Some other operators give slightly higher results and the $m_i^{E^+}(2)$, $\hat{m}_i^{E^+}(2)$ results of all these operators are compatible. The 2^+ values of fig. 4 are based on the operator of fig. 3. We recognize that the results are consistent with scaling (in the same region where 0^+ scales), albeit rather inconclusive. A very tentative continuum estimate is

$$m(2^+) = (75 \pm 13)\Lambda_L^{TI} . \tag{9d}$$

An analysis with much higher statistics may confirm or contradict this estimate.

In conclusion: The TI action (1) has given a 0^+ es-

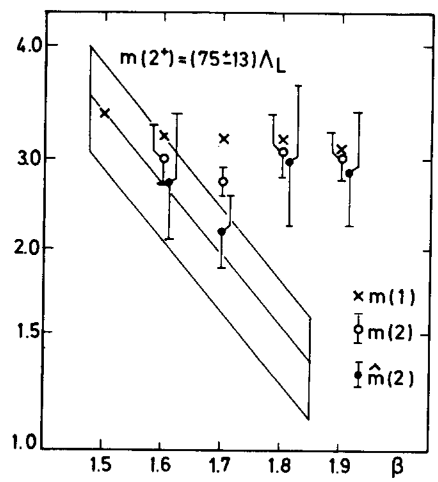

Fig. 4. $m(2^+)$ estimate.

timate (8), which is consistent with the result (5) for Wilson's action. Combining eqs. (5) and (8) gives

$$\Lambda_L^{TI}/\Lambda_L^W = 3.6 \pm 0.8 \, , \tag{10}$$

and should be compared with eq. (4). For the 2^+ state the TI action seems to allow more definite results than previous investigations with other actions. A very high MC statistic is, however, required and the final results could still be in contradiction with scaling.

Symanzik's improvement program was first investigated within the 2d O(3) non-linear σ-model [14–16]. TI action and one-loop improved action significantly give different results. We like to argue that 4d gauge theories are more well-behaved, but MC calculations with a one-loop improved SU(2) action are certainly an important next step and consistency check.

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