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Model Dependence of the Electromagnetic Corrections to Lepton Pair Production in Electron-Positron Collisions [1]

M. Böhm
Physikalisches Institut der Universität, D-8700 Würzburg, Federal Republic of Germany

W. Hollik
II. Institut für Theoretische Physik der Universität, D-2000 Hamburg, Federal Republic of Germany

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Abstract. The electromagnetic radiative corrections to e^+e^--annihilation into lepton pairs are calculated including hard photon bremsstrahlung in the standard and extended electroweak models. For $\sqrt{s} = 38$ GeV we find for the standard model, that the radiative corrections to Z-exchange reduce the charge asymmetry up to 1% which is mainly caused by the virtual and soft photon parts of the corrections. For models with a higher forward-backward asymmetry the situation is found to be quite similar.

1. Introduction

The measurements of a clear asymmetry in the angular distribution in $e^+e^- \to \mu^+\mu^-$ by the JADE, MARK-J and TASSO groups at PETRA [1] strongly support the existence of a neutral weak boson as predicted by the electroweak standard gauge model [2]. The low energy limit of this model has been tested very stringently by combining the experiments of neutrino scattering [3] and electron-deuteron scattering [4]. The e^+e^- experiments have now achieved an energy and an accuracy where not only deviations from the pointlike 4-fermion interaction, but also the effects of additional neutral currents can be determined. Such alternative models with more than one neutral boson have been proposed by several authors: "Conservative" extensions according to $SU(2) \times U(1) \times G$ [5-7] and left-right symmetric models with $SU(2)_L \times SU(2)_R \times U(1)$ [8]. A common feature of these models is the possibility of a Z_1 boson $\lesssim 90$ GeV (together with a Z_2 above) and correspondingly a higher forward-backward asymmetry. Limits on the Z boson masses in 2-Z-models of the first type have been deduced by the PETRA groups [1] and for left-right symmetric models by Barger et al. [9].

The presently applied methods to determine the electro-weak parameters in single as well as in more boson models are based on the Born approximation for the $\gamma - Z_{(k)}$ interference and the pure $Z_{(k)}$ exchange contributions or on an effective neutral current Hamiltonian valid for $q^2 \ll M^2_{1,2}$. Radiative corrections from higher order QED are applied only to the pure γ exchange part in the cross section. They are model independent and give rise to a positive QED charge asymmetry, whereas the electroweak charge symmetry is negative and model dependent.

A complete treatment of the electromagnetic radiative corrections includes real and virtual photon contributions also to the $\gamma - Z_{(k)}$ interference and $Z_{(k)}$ exchange terms in the differential cross section. These should be respected when comparing the experimental data with the model predictions for several reasons:

- They are expected to become visible in the charge asymmetry and, for alternative models, also in the integrated cross section at PETRA energies;
- like the pure QED corrections they include bremsstrahlung and therefore depend on the specific experimental set up (e.g. energy and/or accollinearity cuts to the outgoing leptons);
- together with the pure QED corrections they are expected to be at present energies the most essential part of the complete electroweak corrections (for the standard model see [10-12, 17]).

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The present paper contains an investigation of the effects of the complete electromagnetic corrections in $e^+e^- \to \mu^+\mu^-(\gamma)$. In particular their influence on cross sections and charge asymmetries at PETRA is worked out. Special care was taken to exhibit the rôle that hard photons play in an additional reduction of the electroweak charge asymmetry for μ pairs.

Section 2 contains the virtual and soft photon corrections of order α^3 to $e^+e^- \to \gamma, Z_1, Z_2, \ldots \to \mu^+\mu^-$ without a restriction on the number of gauge bosons. In Sect. 3 the differential cross section for hard photon emission is given. A numerical evaluation and discussion is performed in Sect. 4 for the standard model and for 2-boson models with 2 different values (M_1, M_2) for the boson masses in order to estimate the model dependence of the corrections.

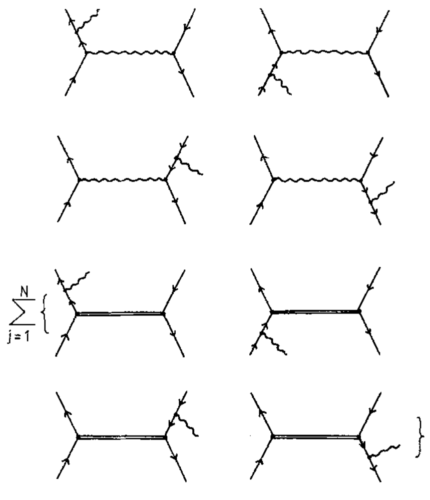

Fig. 2. Feynman diagrams for bremsstrahlung corrections to $e^+e^- \to f\bar{f}$

2. Virtual and Soft Photon Corrections

For energies large compared to the fermion masses $(s \gg m_f^2)$ the cross section of e^+e^--annihilation into $\mu^+\mu^-$ or $\tau^+\tau^-$ can be decomposed into the photon exchange contribution σ^γ, the photon-Z_j-boson interference terms $\sigma^{\gamma Z_j}$ and the diagonal σ^{Z_j} and non-diagonal $\sigma^{Z_j Z_k}$ contributions of the heavy vector bosons in the following way:

$$\frac{4s}{\alpha^2}\cdot\frac{d\sigma}{d\Omega} = \sigma^\gamma + \sum_{j=1}^{N} (\sigma^{\gamma Z_j} + \sigma^{Z_j}) + \sum_{k=2}^{N}\sum_{j=1}^{k-1} \sigma^{Z_j Z_k}. \tag{2.1}$$

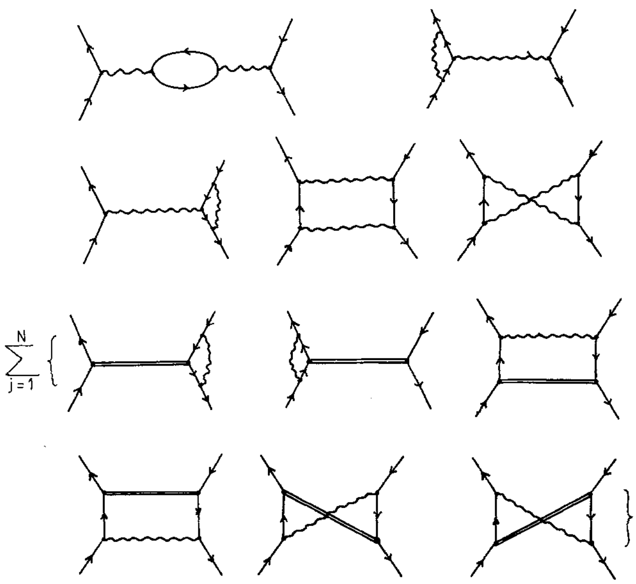

Fig. 1. Feynman diagrams for the virtual radiative corrections to $e^+e^- \to f\bar{f}$ ($\sim\!\sim = \gamma$-exchange, $=\!=\; Z_j$ exchange)

The virtual and soft photon contributions of the diagram of Figs. 1 and 2 are summed up to infrared C_{IR} and finite C_F corrections to the lowest order expressions σ_0:

$$\begin{aligned}
\sigma^\gamma &= \sigma_0^\gamma(1 + C_{IR}^\gamma + C_F^\gamma)\\
\sigma^{\gamma Z_j} &= \sigma_0^{\gamma Z_j}(1 + C_{IR}^{\gamma Z_j} + C_F^{\gamma Z_j})\\
\sigma^{Z_j} &= \sigma_0^{Z_j}(1 + C_{IR}^{Z_j} + C_F^{Z_j})\\
\sigma^{Z_j Z_k} &= \sigma_0^{Z_j Z_k}(1 + C_{IR}^{Z_j Z_k} + C_F^{Z_j Z_k}).
\end{aligned} \tag{2.2}$$

Assuming universality the lowest order cross sections are built from the vector and axial vector coupling constants v_j, a_j of the leptons to the Z_j's normalized to the electric charge and the reduced propagators of the Z_j-bosons with masses M_j and total widths Γ_j:

$$\chi_j(s) = s/(s - s_j) \quad \text{with } s_j = M_j^2 - iM_j\Gamma_j \tag{2.3}$$

in the following way [13]:

$$\begin{aligned}
\sigma_0^\gamma &= 1 + c^2\\
\sigma_0^{\gamma Z_j} &= 2\,\mathrm{Re}\,\chi_j[v_j^2\cdot(1+c^2) + a_j^2\cdot 2c]\\
\sigma_0^{Z_j} &= |\chi_j|^2[(v_j^2+a_j^2)^2\cdot(1+c^2) + 4v_j^2a_j^2\cdot 2c]\\
\sigma_0^{Z_jZ_k} &= 2\,\mathrm{Re}(\chi_j\chi_k^*)\,[(v_jv_k + a_ja_k)^2\cdot(1+c^2)\\
&\quad + (v_ja_k + v_ka_j)^2\cdot 2c],
\end{aligned} \tag{2.4}$$

where $c = \cos\theta$ with θ the angle between the incoming electron and the final μ^- or τ^-.

The corrections C_{IR} and C_F, including soft photon emission up to a maximum photon energy $\omega_1 \ll E = \sqrt{s}/2$, for models with one intermediate boson have been derived in [11, 12, 15]. Therefore we present here without entering again into details their generalisation to the multi-boson case:

$$C_{IR}^{\gamma}=\frac{2\alpha}{\pi}(\beta_e+\beta_f+\beta_{\rm int})\ln\frac{\omega_1}{E}$$

$$C_{IR}^{\gamma Z_j}=\frac{2\alpha}{\pi}\,\mathrm{Re}\left\{\left(1+i\frac{M_j\Gamma_j}{M_j^2-s}\right)\left[\beta_e\ln\left(\frac{\omega_1}{E}\cdot\frac{s_j-s}{s_j-s+2\omega_1\sqrt{s}}\right)\right.\right.$$
$$\left.\left.+\beta_f\ln\frac{\omega_1}{E}+\frac{1}{2}\beta_{\rm int}\ln\left(\left(\frac{\omega_1}{E}\right)^2\cdot\frac{s}{s_j-s+2\omega_1\sqrt{s}}\right)\right]\right\}$$

$$C_{IR}^{Z_j}=\frac{2\alpha}{\pi}\left\{\beta_e\ln\left|\frac{\omega_1}{E}\cdot\frac{s_j-s}{s_j-s+2\omega_1\sqrt{s}}\right|\right.$$
$$+\beta_e\frac{s-M_j^2}{M_j\Gamma_j}(\phi_j-\phi_j^0)$$
$$\left.+\beta_f\ln\frac{\omega_1}{E}+\beta_{\rm int}\ln\left|\frac{\omega_1}{E}\cdot\frac{s}{s_j-s+2\omega_1\sqrt{s}}\right|\right\}$$

$$C_{IR}^{Z_jZ_k}=\frac{2\alpha}{\pi}\,\mathrm{Re}(1+i\rho_{jk})$$
$$\cdot\left\{\beta_e\ln\frac{(s_j-s)(s_k^*-s)}{(s_j-s+2\omega_1\sqrt{s})(s_k^*-s+2\omega_1\sqrt{s})}\right.$$
$$+\beta_f\ln\frac{\omega_1}{E}+\frac{1}{2}\beta_{\rm int}\ln\left(\left(\frac{\omega_1}{E}\right)^2\right.$$
$$\left.\left.\cdot\frac{s^2}{(s_j-s+2\omega_1\sqrt{s})(s_k^*-s+2\omega_1\sqrt{s})}\right)\right\}, \tag{2.5}$$

$$C_F^{\gamma}=C_s+2\,\mathrm{Re}\,\Pi^{\gamma}+\frac{\alpha}{\pi}(X+V_1+A_1\cdot 2c/(1+c^2))$$

$$C^{\gamma Z_j}=C_s+\mathrm{Re}\,\Pi^{\gamma}+\frac{\alpha}{\pi}(X+\tfrac{1}{2}V_1+\tfrac{1}{2}V_1^j)$$
$$-\alpha\frac{M_j\Gamma_j}{s-M_j^2}(V_2-V_2^j-\tfrac{1}{3}R)$$
$$+\left[\frac{\alpha}{2\pi}(A_1+A_1^j)-\alpha\frac{M_j\Gamma_j}{s-M_j^2}(A_2-A_2^j)\right]\sigma_5^{\gamma Z_j}/\sigma_0^{\gamma Z_j}$$

$$C_F^{Z_j}=C_s+\frac{\alpha}{\pi}(X+V_1^j+A_1^j\sigma_5^{Z_j}/\sigma_0^{Z_j})$$

$$C_F^{Z_jZ_k}=C_s+\frac{\alpha}{\pi}(X+\tfrac{1}{2}V_1^j+\tfrac{1}{2}V_1^k)-\alpha\rho_{jk}(V_2^j-V_2^k)$$
$$+\left[\frac{\alpha}{2\pi}(A_1^j+A_1^k)-\alpha\rho_{jk}(A_2^j-A_2^k)\right]\sigma_5^{Z_jZ_k}/\sigma_0^{Z_jZ_k}. \tag{2.6}$$

Here we have used the following abbreviations ($f=\mu/\tau$):

$$\beta_e=\ln\frac{s}{m_e^2}-1,\quad \beta_f=\ln\frac{s}{m_f^2}-1,\quad \beta_{\rm int}=2\ln\frac{1-c}{1+c},$$
$$C_s=\frac{3\alpha}{2\pi}(\beta_e+\beta_f)+\frac{2\alpha}{\pi}(\tfrac{1}{3}\pi^2-\tfrac{1}{2}) \tag{2.7}$$

$$\phi_j^0=\arctan\frac{M_j^2-s}{M_j\Gamma_j},\quad \phi_j=\arctan\frac{M_j^2-s+2\omega_1\sqrt{s}}{M_j\Gamma_j}$$
$$\rho_{jk}=\frac{-M_j\Gamma_j(s-M_k^2)+M_k\Gamma_k(s-M_j^2)}{(s-M_j^2)(s-M_k^2)+M_j\Gamma_jM_k\Gamma_k}. \tag{2.8}$$

The formulas (2.6) contain the photon self energy:

$$\Pi^{\gamma}=\frac{\alpha}{3\pi}\sum_{\substack{\text{Lept}\\\text{Quarks}}}Q_F^2(\beta_F-\tfrac{2}{3})-i\frac{\alpha}{3}R;\qquad R=\sum_{\substack{\text{Lept}\\\text{Quarks}}}Q_F^2, \tag{2.9}$$

the finite antisymmetric bremsstrahlung term:

$$X(c)=\left(\ln\frac{1-c}{2}\right)^2-\left(\ln\frac{1+c}{2}\right)^2-2\,\mathrm{Sp}\left(\frac{1-c}{2}\right)$$
$$+2Sp\left(\frac{1+c}{2}\right)^2 \tag{2.10}$$

with the Spence function: $\mathrm{Sp}(z)=-\int_0^1 dt\frac{1}{t}\ln(1-zt)$ and the finite parts of the $\gamma\gamma$- and γZ_j-box diagrams $V_1,\ldots,A_2^j$. Their explicit form is given in the appendix. The expressions $\sigma_5^{\gamma},\ldots,\sigma_5^{Z_jZ_k}$ result from $\sigma_0^{\gamma},\ldots,\sigma_0^{Z_jZ_k}$, (2.4), by the substitution $1+c^2\leftrightarrow 2c$.

3. Hard Photon Corrections

We expect the expressions (2.2)-(2.6) for the radiative corrections to be a reasonable approximation for photon energies of the order $\omega_1\approx 0.1\,E$. For smaller values of the experimental energy resolution multiphoton emission becomes important and should be respected by suitable exponentiation [14, 15]. On the other hand, for higher values of the energy of the emitted quantum, the hard photon corrections have to be taken into account. This amounts to the use of the exact single photon emission matrix element together with the complete treatment of the kinematics and phase space of the 3-particle final state:

$$e^+(q)+e^-(p)\to l^+(q')+l^-(p')+\gamma(k). \tag{3.1}$$

Following the notations of Berends et al. [12] we introduce the invariants:

$$\begin{aligned} s&=(p+q)^2, & t&=(q-q')^2, & u&=(q-p')^2,\\ s'&=(p'+q')^2, & t'&=(p-p')^2, & u'&=(p-q')^2. \end{aligned} \tag{3.2}$$

The cross section $d^5\sigma$ which is differential with respect to the fermion solid angle $d\Omega$, the photon energy dk_0 and the photon solid angle $d\Omega_\gamma$ ($\theta_\gamma,\phi_\gamma$ refer to polar coordinates with $\mathbf{p}'$ as polar axis) emerges as:

$$\frac{4s}{\alpha^2}\cdot\frac{d^5\sigma}{d\Omega d\Omega_\gamma dk_0}=\frac{2\alpha}{\pi^2}\cdot\frac{k_0p_0'}{2E-k_0+k_0\cos\theta_\gamma}(X_i+X_f+X_{\rm int}) \tag{3.3}$$

with:

$$X_i = -\frac{m_e^2}{2s'^2}\left[A(s')\left(\frac{t^2}{(kp)^2}+\frac{t'^2}{(kq)^2}\right) + B(s')\left(\frac{u^2}{(kp)^2}+\frac{u'^2}{(kq)^2}\right)\right] + \frac{A(s')(t^2+t'^2)+B(s')(u^2+u'^2)}{4s'(kp)(kq)};$$

$$X_f = -\frac{m_f^2}{2s^2}\left[A(s)\left(\frac{t^2}{(kp')^2}+\frac{t'^2}{(kq')^2}\right) + B(s)\left(\frac{u'^2}{(kp')^2}+\frac{u^2}{(kq')^2}\right)\right] + \frac{A(s)(t^2+t'^2)+B(s)(u^2+u'^2)}{4s(kp')(kq')};$$

$$X_{\text{int}} = \frac{C(s,s')(t^2+t'^2)+D(s,s')(u^2+u'^2)}{4ss'} \cdot \left(\frac{u}{(kp')(kq)}+\frac{u'}{(kp)(kq')} - \frac{t}{(kq)(kq')}-\frac{t'}{(kp)(kp')}\right) + \frac{(u^2-u'^2)(s-s')F(s,s')}{2ss'(kp)(kq)(kp')(kq')}\varepsilon_{\mu\nu\rho\sigma}q^\mu p^\nu q'^\rho p'^\sigma. \tag{3.4}$$

The indices i, f, int denote initial, final state emission and the interference between them. The double poles in the square brackets of X_i, X_f contain mass singularities which are compensated by the factors m_e^2, m_f^2. In this sense (3.3) is valid for $s \gg m_e^2$, m_f^2 up to $O\left(\frac{m_e^2}{s}, \frac{m_f^2}{s}\right)$. The electroweak parameters like coupling constants, masses and width of the Z_j-bosons determine the functions $A, \ldots, F$ with the result

$$\begin{aligned}
A(s) &= 1+\sum_j (v_j^2-a_j^2)\cdot 2\,\mathrm{Re}\,\chi_j(s) + \sum_j (v_j^2-a_j^2)^2|\chi_j(s)|^2 \\
&\quad + \sum_{j<k}(v_j^2-a_j^2)(v_k^2-a_k^2)\,2\,\mathrm{Re}(\chi_j(s)\chi_k^*(s)),\\
B(s) &= 1+\sum_j (v_j^2+a_j^2)\,2\,\mathrm{Re}\,\chi_j(s) + \sum_j [(v_j^2+a_j^2)^2 + 4v_j^2a_j^2]|\chi_j(s)|^2 + \sum_{j<k}[(v_j^2+a_j^2)(v_k^2+a_k^2) \\
&\quad + 4v_ja_jv_ka_k]\,2\,\mathrm{Re}(\chi_j(s)\chi_k^*(s)),\\
C(s,s') &= 1+\sum_j (v_j^2-a_j^2)\,\mathrm{Re}(\chi_j(s)+\chi_j(s')) \\
&\quad + \sum_j (v_j^2-a_j^2)^2\,\mathrm{Re}(\chi_j(s)\chi_j^*(s')) \\
&\quad + \sum_{j<k}(v_j^2-a_j^2)(v_k^2-a_k^2)\,\mathrm{Re}(\chi_j(s')\chi_k^*(s) \\
&\quad + \chi_k(s')\chi_j^*(s)),\\
D(s,s') &= 1+\sum_j (v_j^2+a_j^2)\,\mathrm{Re}(\chi_j(s)+\chi_j(s')) \\
&\quad + \sum_j [(v_j^2+a_j^2)^2+4v_j^2a_j^2]\,\mathrm{Re}(\chi_j(s)\chi_j^*(s')) \\
&\quad + \sum_{j<k}[(v_j^2+a_j^2)(v_k^2+a_k^2) \\
&\quad + 4v_ja_jv_ka_k]\,\mathrm{Re}(\chi_j(s')\chi_k^*(s)+\chi_k(s')\chi_j^*(s)),\\
F(s,s') &= \sum_j v_ja_j\,\mathrm{Im}(\chi_j(s')-\chi_j(s)) \\
&\quad + \sum_j 2v_ja_j(v_j^2+a_j^2)\,\mathrm{Im}(\chi_j(s')\chi_j^*(s)) \\
&\quad + \sum_{j<k}(v_ja_k+v_ka_j)(v_jv_k+a_ja_k) \\
&\quad \cdot \mathrm{Im}(\chi_j(s')\chi_k^*(s)+\chi_k(s')\chi_j^*(s)).
\end{aligned} \tag{3.5}$$

The summations are to be understood in the sense of (2.1). Equations (2.1)-(3.5) represent a complete set of formulas for the calculation of virtual and real photon corrections to $e^+e^- \to l^+l^-$ annihilation in the frame of electroweak models with one (standard model) or several neutral intermediate bosons Z_j. Purely weak higher order contributions are not included.

4. Numerical Results and Discussion

The hard bremsstrahlung contribution (3.3) to the cross section for lepton pair production is differential with respect to the angles and energies of the emitted photons. Integration over the photon phase space has to be performed respecting the specific experimental conditions. A commonly accepted method is to restrict the accollinearity angle $\delta = \sphericalangle(\mathbf{p}', -\mathbf{q}')$. A maximum photon energy ΔE allows final leptonic energies between $\sqrt{s}-\Delta E$ and $\sqrt{s}$. We perform analytically the integration over the azimuth ϕ_γ and part of the θ_γ and k_0 integrations. Special care was taken to guarantee the required accuracy in those regions where the cross sections are large. For numerical evaluations we choose $\delta = 10°$ with $\Delta E = 0.5\,E$ $(E=\sqrt{s}/2)$ and $\Delta E = 0.1\,E$ with no accollinearity cut. The virtual and soft photon parts of the cross section are calculated from the equations of Sect. 2.

4.1. Standard Model Results

Let us first apply our calculations to the standard model with one Z boson. We choose $M_Z = 89$ GeV and $\Gamma_Z = 2.5$ GeV for the mass and width of the Z boson. The coupling constants are calculated with the Weinberg angle $\sin^2\theta_W = 0.23$. In order to get an impression of the magnitude of the radiative corrections to the Z-exchange contribution we discuss the charge asymmetry for μ pair production

$$A_c = \frac{d\sigma(\theta)-d\sigma(\pi-\theta)}{d\sigma(\theta)+d\sigma(\pi-\theta)} \tag{4.1}$$

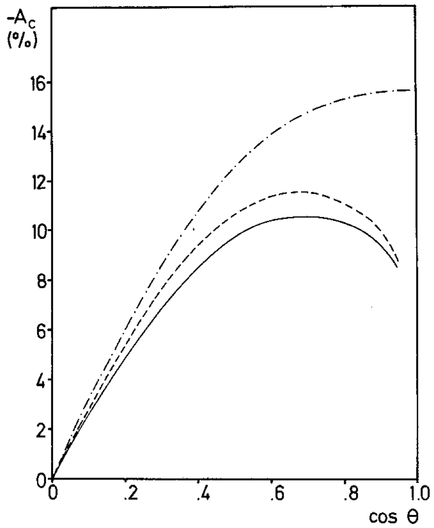

Fig. 3. The standard model charge asymmetry A_c at $\sqrt{s}=38$ GeV ($\Delta E=0{,}5E$, $\delta=10°$) in lowest order (-·-·-), including pure QED corrections (- - -) and including also the radiative corrections to Z exchange (——)

at $\sqrt{s}=38$ GeV. In Fig. 3 we show besides

(i) the lowest order results

also

(ii) the results including radiative corrections to the QED part σ^{γ} in (2.1), but not to the Z and γZ interference parts $\sigma^{\gamma Z}$ and σ^{Z} (reduced QED corrections),

and

(iii) including radiative corrections also to the weak contribution $\sigma^{\gamma Z}$ and σ^{Z} (full QED corrections).

A comparison between the curves for the cases (iii) and (ii) shows that the charge asymmetry is reduced by the radiative corrections to $\sigma^{\gamma Z}$ and σ^{Z} up to 1%. The values of the forward-backward asymmetry integrated over θ in the interval $|\cos\theta| \leqq 0.8$ in $e^+e^- \to \mu^+\mu^-$ are presented in the following table:

	(i)	(ii)	(iii)
$A_{FB}(\%)$	-10.38	-8.52	-7.78

The numerical error of the integration in A_{FB} is estimated to be smaller than 0.04%. The difference between A_{FB} (ii) and (iii) shows clearly that radiative corrections to $\sigma^{\gamma Z}$ and σ^{Z} become important already at present energies.

For the integrated cross section, on the other hand, we do not get a measurable change by the additional radiative corrections.

It should be emphasized that the reduction of the charge asymmetry is not an effect due to the emission of hard photons. In order to clarify this point we have evaluated the influence of the radiative corrections for the situation of a maximum photon energy $\Delta E=0.1\,E$, where the soft photon approximation of Sect. 2 ($\omega_1=\Delta E$) becomes reasonably good. Figure 4 contains the percentage corrections to the lowest order differential electroweak cross section for the two cases:

(i) Radiative corrections only to γ exchange

(ii) radiative corrections also to γ and Z^0 exchange, $\sigma^{\gamma Z}$ and σ^{Z}.

Besides the calculations including hard photons also the corresponding soft photon approximations are displayed (upper curves). The difference between the soft photon approximation and the exact treatment is $\lesssim 0.5\%$ in the case (i) and $\leqq 0.7\%$ for (ii).

On the other hand, however, the differences between (ii) and the approximation (i) amount to $\pm(3-4)\%$ and are therefore more important than the differences between soft and hard photons. The difference between (i) and (ii) is amplified in the forward-backward asymmetry since the corrections in (ii) are more negative in the backward and less negative in the forward direction than those of the approximation (i).

The differential charge asymmetry (4.1) is shown in Fig. 5. The differences between (i) and (ii) are even more pronounced than in the case discussed above, where also hard photons play an important role.

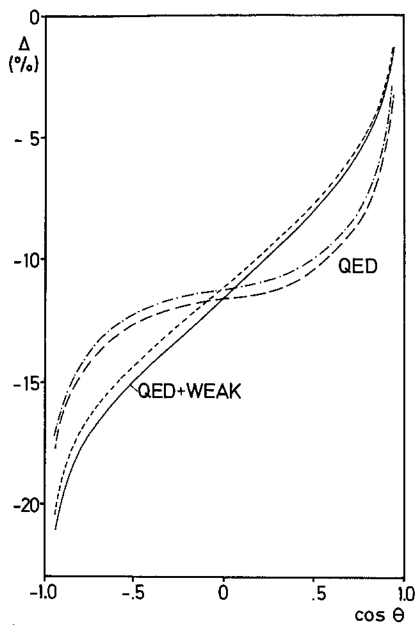

Fig. 4. Percentage corrections to the lowest order standard model differential cross section to the photon exchange only (QED) and to photon and Z exchange (QED+WEAK) in the soft photon approximation (-·-, - - -) and with hard photons (—, —— —) ($\sqrt{s}=38$ GeV, $\Delta E=0.1\,E$)

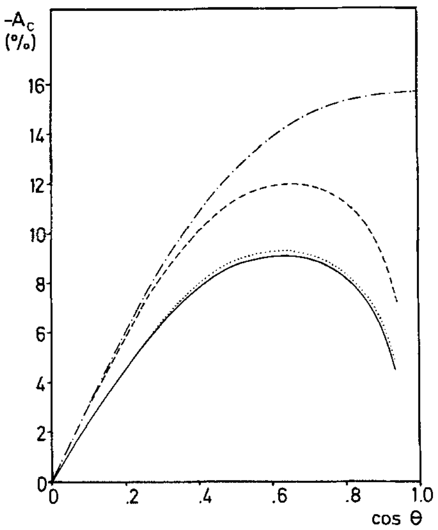

Fig. 5. The charge asymmetry A_c ($\sqrt{s}=38$ GeV, $\Delta E=0.1E$) in lowest order (-·-), with radiative corrections to photon exchange (---) and the complete calculations with soft (...) and hard photons (——)

Furthermore, the hard photon contribution to the charge asymmetry is very small ($\lesssim 0.25\,\%$), as is also shown in Fig. 5, where the soft photon approximation (dotted curve) is very close to the prediction with the complete bremsstrahlung calculation.

The additional reduction of A_c by the radiative corrections to Z^0 exchange is therefore clearly an effect which has to be assigned to the virtual and soft photon corrections.

4.2. *More Boson Models*

If we abandon the restriction of a single neutral weak boson we have to admit the class of left-right symmetric models as well as the "conservative" extensions of the standard group, $SU(2)\times U(1)\times G$.

Since our approach is restricted to the photonic corrections, which at least at present energies should be the most essential ones, the only input in the calculation are the masses of the neutral bosons. To these the coupling constants are related in a model dependent way. According to (2.5–6), (3.5), the corrections then become also model dependent. For a precise determination of the boundaries for the model parameters (coupling constants) from experimental data the corrected rather than the Born expressions should be used. The correct treatment of the radiative corrections will become essential in particular at LEP/SLC energies for an analysis in the frame of e.g. left-right symmetric models or to separate them from the GSW model, because extended gauge models may also simulate the GSW features in the unpolarized sector [16].

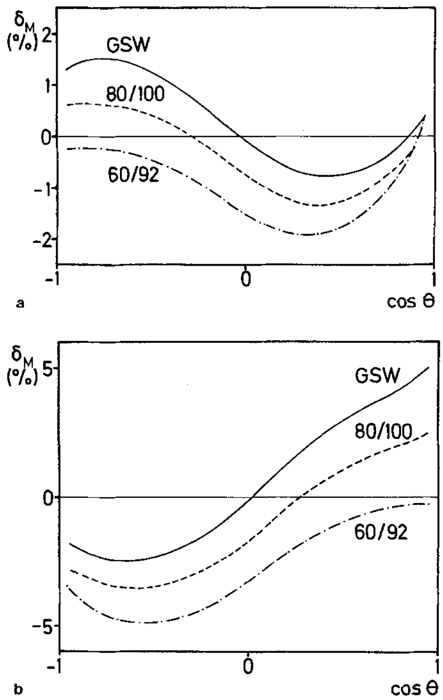

Fig. 6a and b. δ_M (4.2) for different (GSW, 80/100, 69/92) gauge boson masses. $\sqrt{s}=38$ GeV. **a** $\delta=10°$, $\Delta E=0.5E$, **b** $\Delta E=0.1E$, no acollinearity cut

The great variety of different models makes it necessary to restrict our discussion to specific examples. We choose the most conservative extension of the standard model with 2 neutral bosons according to the group $SU(2)\times U(1)\times \tilde{U}(1)$ [6] and no change in the charged sector. In Fig. 6 we have plotted the difference

$$\delta_M=(d\sigma^{(i)}-d\sigma^{(ii)}/d\sigma^{\mathrm{QED}}\star \tag{4.2}$$

for two pairs of Z_1, Z_2 masses and the standard model (GSW) as a measure of the model dependence at presently accessible energies. As in (4.1) $d\sigma^{(i)}$ includes corrections only to γ-exchange, $d\sigma^{(ii)}$ also to boson exchanges and interference parts.

The dependence of δ_M on the specific model is quantitatively enhanced if the photon energy is constrained to lower values of ΔE, i.e. if the photons become softer. In the GSW model δ_M changes its sign at $\theta=90°$ with the consequence that the integrated cross section is nearly independent of corrections to Z exchange. This is no longer true for the case of a light Z_1 boson, where δ_M is negative at all values of θ and therefore gives an additional negative correction also to the integrated cross section.

$\star$ $d\sigma^{\mathrm{QED}}$ includes α^3 corrections

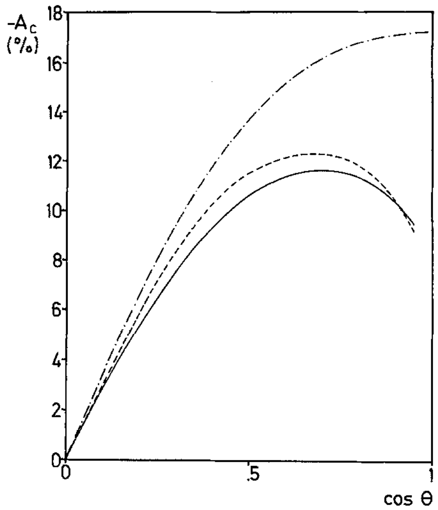

Fig. 7. The charge asymmetry A_c (4.1) for $M_1=60$ GeV, $M_2=92$ GeV in Born approximation (–·–·–), with radiative corrections to the QED part (– – –) and radiative corrections to all Born diagrams (——) ($\delta=10°$, $\Delta E=0.5E$). $\sqrt{s}=38$ GeV

On the other hand, if we exclude the small angle range ($|\cos\theta|\geqq 0.8$) the additional corrections to the charge asymmetry (4.1) are approximately the same for all the models under consideration, since the δ_M curves are nearly parallel.

Figure 7 explicitly shows A_c for a light Z_1 boson which predicts at 1.3% higher forward-backward asymmetry than the GSW model in lowest order. Comparing with Fig. 3 one finds that the further reduction of A_c in the 2 boson case is slightly smaller than in the GSW model. At small angles both curves nearly coincide. This means that the difference in A_c occuring at the Born level essentially survives the application of radiative corrections with the tendency to become even higher.

Appendix

The functions $V_{1,2}$, $A_{1,2}$ and $V^j_{1,2}$, $A^j_{1,2}$, which appear in (2.6) are the real and imaginary parts of the following expressions:

$$V=V_1+2\pi i V_2=M(s,t)-M(s,u)$$
$$A=A_1+2\pi i A_2=M(s,t)+M(s,u)$$
$$V^j=V^j_1+2\pi i V^j_2=M^j_V(s,t)-M^j_V(s,u)$$
$$A^j=A^j_1+2\pi i A^j_2=M^j_A(s,t)+M^j_A(s,u)$$

with $t=-\frac{1}{2}s(1-\cos\theta)$, $u=-\frac{1}{2}s(1+\cos\theta)$ and

$$M(s,t)=\frac{s}{2(s+t)}\ln t/(s+i\varepsilon) -\frac{s(s+2t)}{4(s+t)^2}\left[\ln^2 t/(s+i\varepsilon)+\pi^2\right],$$

$$\begin{aligned}M^j_V(s,t)=&2\,\mathrm{Sp}\left(-\frac{t}{s}\right)+2\ln\left(-\frac{t}{s}\right)\ln\left(\frac{s+t}{-t}\right)\\&-2\ln\left(\frac{s}{s_j}\right)\ln\left(-\frac{t}{s}\right)\\&-2\ln\left(\frac{s_j+t}{s}\right)\ln\left(\frac{s+t}{-t}\right)\\&+2\ln\left(\frac{s_j-s}{s}\right)\ln\left(\frac{s+t}{s}\right)\\&-\ln\left(\frac{s_j-s}{s_j+t}\right)\left[4\pi i+2\ln\left(\frac{s_j-s}{s+t}\right)\right]\\&+\ln^2\left(\frac{s-s_j}{s+t}\right)-\ln^2\left(\frac{-s_j-t}{s+t}\right)\\&-2\,\mathrm{Sp}\left(\frac{t}{t+s_j}\right)+2\,\mathrm{Sp}\left(\frac{t}{t+s}\right)\\&-2\,\mathrm{Sp}\left(\frac{s+t}{s-s_j}\right)-2\,\mathrm{Sp}\left(\frac{s_j+t}{s_j-s}\right)\\&-\frac{s-s_j}{s}\left\{\frac{s_j}{s+t}\ln\left(\frac{s_j}{-t}\right)+\frac{s_j-s}{s+t}\ln\left(\frac{-t}{s_j-s}\right)\right.\\&+\left[\frac{3s(s-s_j)+2s(t+s_j)}{(s+t)^2}\right.\\&\left.-\frac{4s}{s+t}\right]\left[\mathrm{Sp}\left(\frac{s}{-t-i\varepsilon}\right)+\mathrm{Sp}\left(\frac{s_j+t}{s_j-s}\right)\right.\\&\left.\left.-\mathrm{Sp}\left(\frac{s}{t}\cdot\frac{t+s_j}{s-s_j}\right)-\frac{\pi^2}{6}\right]\right\}\end{aligned}$$

$$\begin{aligned}M^j_A(s,t)=&\frac{s-s_j}{s}\left\{-\frac{s_j}{s+t}\ln\left(\frac{s_j}{-t}\right)+\frac{s-s_j}{s+t}\ln\left(\frac{-t}{s_j-s}\right)\right.\\&+\frac{s(s-s_j)+2s(s_j+t)}{(s+t)^2}\left[\mathrm{Sp}\left(\frac{s}{-t+i\varepsilon}\right)\right.\\&\left.\left.+\mathrm{Sp}\left(\frac{s_j+t}{s_j-s}\right)-\mathrm{Sp}\left(\frac{s}{t}\cdot\frac{t+s_j}{s-s_j}\right)-\frac{\pi^2}{6}\right]\right\}\end{aligned}$$

s_j means: $s_j=M_j^2-iM_j\Gamma_j$ for the j-th neutral boson.

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