Nuclear Physics B237 (1984) 237–248

MESON SPECTRUM IN QUENCHED QCD ON A 16^4 LATTICE

P. HASENFRATZ

CERN, Geneva, Switzerland

I. MONTVAY*

II. Institut für Theoretische Physik der Universität Hamburg, Fed. Rep. Germany

Received 11 August 1983
(Revised 24 November 1983)

The masses of pseudoscalar and vector mesons and their radial excitations are measured in the quenched approximation to QCD by a 32nd-order numerical hopping-parameter expansion on a 16^4 lattice. Scaling behaviour, finite-size effects and the influence of statistics are studied.

1. Introduction

The numerical evaluation of lattice QCD [1] offers the exciting possibility to do, for the first time, a calculation of the hadron spectrum from first principles [2–8]. Despite questions on the implementation of chiral symmetry on the lattice and on the suppression of the effects of virtual quark pairs ("quenched approximation"), this is a qualitatively new situation compared to earlier phenomenological hadron models. Of course, MC calculations include some approximation: the relatively small number of lattice points by which the infinite-volume space-time continuum is approximated and the limited statistics of Monte Carlo measurements result in a moderate accuracy. However, in this case, these approximations can be checked – and hopefully controlled – in a systematic way.

In a previous paper [9] we argued that in the first hadron-spectrum calculations, the physical lattice volume was unacceptably small. Apart from some simple plausibility arguments and a comparison with a discretized, non-relativistic quantum mechanical example, our conclusions were based on a calculation at $\beta = 6/g^2 = 6.0$ on an 8^4 lattice. Subsequent spectrum calculations revealed similar problems [10–14], indicating that for a serious investigation the lattice volume should be increased significantly.

In order to cure the finite-size problem we decided to do a calculation on a lattice with considerably larger physical size: we took a 16^4 lattice and lowered the β-value

* Supported by Bundesministerium für Forschung und Technologie, Bonn, Germany.

to $\beta = 5.7$ and 5.4. The results of this calculation on the pseudoscalar and vector meson spectrum are presented in this paper. We hope to complete the measurement of the simplest baryon masses in the near future.

In sect. 2 a brief description of the numerical procedure is given. Those readers who are less interested in these details might skip this part and start with sect. 3, where the results are summarized. The conclusion is presented in sect. 4.

2. Numerical procedure

The calculation is based on the hopping-parameter expansion [15,5] for Wilson fermions [1,15]. The fermionic part of the action can be written as

$$S_{\mathrm{f}} = \sum_{x,y} \bar{\psi}_x Q_{xy} \psi_y , \tag{1}$$

where the quark matrix Q is defined as

$$\begin{aligned} Q &= 1 - KM(U), \\ M(U)_{x_1,x_2} &= \sum_{x,\mu} (r+\gamma_\mu) U(x,\mu) \delta_{x_1,x} \delta_{x+\hat{\mu},x_2} . \end{aligned} \tag{2}$$

Here x denotes the points of a hypercubic, euclidean lattice, $\mu = \pm 1, \pm 2, \pm 3, \pm 4$ are the directions, $\gamma_\mu = -\gamma_{-\mu}$ is a hermitian Dirac-matrix and $U(x,\mu)$ is the SU(3) gauge field variable on the link $x \to x + \hat{\mu}$. The parameter $0 < r \leqslant 1$ removes the superfluous fermion states in the continuum limit $a \to 0$, and (for the case $r = 1$, which we shall consider)

$$K = \left(8 + 2am_{\mathrm{q}}\right)^{-1}, \tag{3}$$

is the "hopping parameter" describing the dependence on the bare quark mass m_{q}.

The hadron propagators are constructed from the matrix elements of the quark propagator Q^{-1}, which can be expanded in powers of K as

$$Q^{-1} = \sum_{n=0}^{\infty} K^n M(U)^n . \tag{4}$$

Taking some initial quark state vector $|\mathrm{i}\rangle$, we have the simple iterative relation

$$\begin{aligned} |v_n\rangle &= M(U)|v_{n-1}\rangle, \qquad n = 1,2,\dots, \\ |v_0\rangle &\equiv |\mathrm{i}\rangle . \end{aligned} \tag{5}$$

This relation can be used for the computation of the hopping-parameter expansion coefficients of the quark propagator matrix element $\langle \mathrm{f} | Q^{-1} | \mathrm{i} \rangle$.

In the "quenched approximation" the effect of virtual quark-antiquark pairs is neglected, therefore the fluctuation of the gauge field is determined by the pure gauge field action (we use the standard Wilson action [1]).

The first step in the calculation is to produce a sequence of gauge field configurations via a Monte Carlo procedure. We have chosen the coupling constant values $\beta \equiv 6/g^2 = 5.4$ and 5.7. On the 16^4 lattice the starting configuration was obtained by copying previously equilibrated 8^4 lattices 16 times over the large lattice. At $\beta = 5.7$ we created a sequence from a cold start (unit gauge matrices) too. After 500–800 Metropolis sweeps* with 6–10 hits per link, we started to collect the configurations on magnetic tapes.

In order to fix the physical scale we measured the string tension on these configurations in collaboration with Gutbrod and Kunszt [16]. The largest loop we were able to measure was 3×3 and 4×4 at $\beta = 5.4$ and 5.7, respectively, therefore our result is presumably an upper bound for the corresponding lattice units [16]

$$a \leqslant (1.68 \pm 0.03)\ \mathrm{GeV}^{-1}, \qquad \beta = 5.40,$$

$$a \leqslant (1.05 \pm 0.08)\ \mathrm{GeV}^{-1}, \qquad \beta = 5.70. \tag{6}$$

These numbers give an estimate for the physical size of our 16^4 lattice: ~ 5.3 and ~ 3.3 fm at $\beta = 5.4$ and 5.7, respectively.

At $\beta = 5.40$ the background fields over which the quark propagator has been calculated, were separated by ~ 20 sweeps. A 32nd-order expansion was performed on 19 separated configurations. In order to increase the statistics of the lower-order coefficients an additional 12th-order expansion was performed on 50 configurations separated by ~ 10 sweeps in average.

At $\beta = 5.70$ the configurations were separated by 100 sweeps, but then a 32nd order and (from 12 randomly chosen initial points) 24th-order expansions were performed on the same configuration. Altogether 12 propagators of 32nd order and 144 propagators of 24th order were obtained.

For the gauge fields we used periodic boundary conditions, while the quarks were not constrained in their propagation over the copied background field: "copied gauge field method" [5, 9].

The advantage of the "copied gauge field method" is, that the quark propagators are defined for continuous four-momenta (unlike in a periodic box, where the euclidean momenta have discrete values). Therefore, it is possible to analytically continue the propagators to real energies. This allows us to look directly for the particle singularities (in the hopping-parameter variable) by a Padé-approximant

* It took a longer time to equilibrate the cold lattice.

technique. In the "periodic box" method of the hopping-parameter expansion [9] or in the iterative techniques [2–4] the hadron masses are identified by observing the exponential fall-off of the propagators in configuration space, therefore no direct information is obtained on the nature of the singularity and the separation of multiple poles (e.g. due to radial excitations) is more difficult.

Our results for the quark propagator matrix elements were recorded on magnetic tapes. From these numbers it is possible to build up the Green functions for any multi-quark configuration. For the 0^- and 1^- mesons we used the local quark operators $\bar{\psi}_x\gamma_5\psi_x$ and $\bar{\psi}_x\gamma_1\psi_x$. Here we are concerned only with mesons built up from equal mass quarks.

Having the expansion coefficients of the meson propagators, one has to look for the poles corresponding to meson bound states. Following ref. [5], we first Laplace-transformed the amplitudes and then searched for the poles in the Padé approximants to the series at different fixed meson masses. The series for mesons have only even powers of K, therefore the highest Padé approximants are $15/1, 14/2, \ldots, 0/16$. These extensive Padé tables are stable down to masses $am \simeq 0.3$ for 0^- and $am \simeq 0.7$ for 1^- states. The pole position was estimated by taking the average of all Padé's of order 14, 15 and 16.

We had to invest a considerable amount of effort to bring our computer program to a form, where the limitation in going to still larger lattices and higher orders in the hopping parameter is mainly due to lack of CPU time. An additional problem, especially in high orders, is the large amount of data produced by calculating the quark propagators.

The total amount of CPU time was ~ 250 hours on the Siemens 7.882 computer at the computer center of the University of Hamburg and ~ 300 hours zero-priority time on the CERN IBM machines.

3. Discussion of the results

Before turning to the quantitative results let us discuss a few general points.

In our previous work [9] we raised arguments that the small lattices used in earlier calculations are unacceptably small for the purpose of hadron spectroscopy at $\beta = 6.0$. Among the arguments we refer to two results obtained in a 24th-order hopping-parameter expansion on an 8^4 lattice at $\beta = 6.0$.

First, we compared the expansion coefficients obtained in a periodic 8^4 box with those obtained on an ∞^4 lattice, where 8^4 gauge field configurations were copied. The higher-order expansion coefficients were different (sometimes by orders of magnitude) in the two cases, showing that the periodically closed Wilson loops, present in the first case, dominate the higher orders of the expansion. It was expected [17] that these "fake loops" are mainly responsible for the large fluctuations found on small periodic lattices [6, 7]. This expectation was confirmed later by detailed studies on the rôle of polarized, periodically closed Wilson loops [11, 12].

We did not carry out a similar analysis on the 16^4 lattice at $\beta = 5.4$ and 5.7 since no similar problem is expected to occur. Even at $\beta = 5.7$, our lattice is much larger than the critical size (6^4) for non-zero expectation value of the Wilson lines closed by periodicity*.

Second, we tried to check the overall strength of the singularity in the hopping-parameter plane. We found that for small meson masses the series was similar to that of a free field theory showing a very weak (like $x \ln x$ or weaker) singularity. This we interpreted as due to the tunneling of the quarks through the periodic potential acting between a quark and an antiquark in a periodic gauge field background.

The ratio test of our new series at $(m_\pi a)^2 = 1.2$ obtained on the 16^4 lattice at $\beta = 5.40$ is consistent with an overall crude fit $(K^2 - K_0^2)^{-\gamma}$ where $\gamma \simeq 0.6$–0.7. On the other hand at $\beta = 5.7$, $(m_\pi a)^2 = 1.0$, the ratio test gives $\gamma \simeq 0.1$ at $\leqslant 32$nd order, and γ is even smaller at $\leqslant 24$th order. These singularities are substantially stronger than the ones observed at $\beta = 6.0$ on the 8^4 lattice, nevertheless they are still far from a pure-pole behaviour. On our present lattices the tunneling is probably weak, the expectation value of the periodically closed loops is zero, therefore the result of the ratio test is most probably due to the fact that our series are still not long enough to identify the nature of the singularities. (This holds, of course, even more for ref. [9].) The situation is made more difficult by the fact that in all channels more physical poles are expected to occur due to the radial excitations. The Padé analysis based on any finite series can never exactly decide between a series of poles and a cut. It can only give some hints, according to which we think that one can actually interpret the singularity structure on our *present* lattice as due to two stable poles corresponding to the ground state and the first radial excitation (see below).

Let us turn now to the quantitative results. The nearest poles of the Padé approximants are identified with the lowest-lying pseudoscalar ("π") and vector ("ρ") mesons. The results are shown in figs. 1 and 2 for $\beta = 5.4$ and 5.7, respectively. The dimensionless mass ma is given for the ρ-meson, while $(m_\pi a)^2$ is plotted for the pion. The values represent the average of the pole positions in the 28th, 30th and 32nd order Padé approximants. The indicated error is a sum of the uncertainty coming from the Padé table and of the statistical error. The statistical error was estimated by dividing the configurations into four subsets and considering the four different mass values as independent measurements.

As the figures show, for small mass values ($m_\pi a \leqslant 0.8$) the pion mass-squared $(m_\pi a)^2$ is – in a very good approximation – a linear function of $K_{cr} - K$ (K_{cr} is the hopping-parameter value corresponding to a massless pion). Additionally, the Padé table is reasonably stable even at $(m_\pi a)^2 = 0.1$, which leads to a rather precise

* Note that the critical size depends on the shape: for a lattice much larger in three directions than in the fourth one the critical temperature size for $\beta = 5.7$ is $N_\beta \sim 4$. This shape dependence was first noticed by E. Kovacs in the SU(2) lattice gauge theory [20]. The above estimate for the critical size is based on our own measurement on an 8^4 lattice, where the critical β-value turns out to be $\beta \simeq 6.0$.

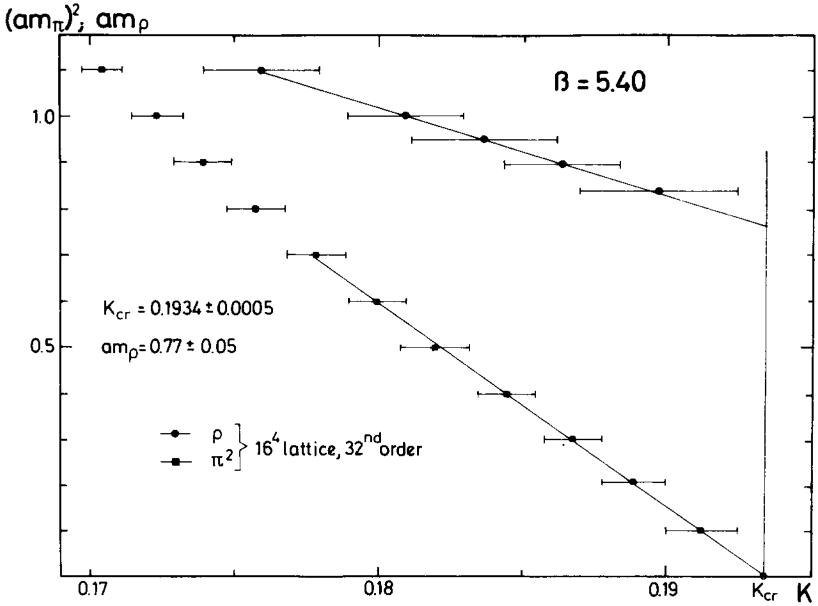

Fig. 1. The mass-squared in the pseudoscalar channel and the mass in the vector-meson channel for $\beta = 5.4$ near the critical value of the hopping parameter (K_{cr}), where the pion mass vanishes. Horizontal errorbars represent the estimated statistical errors and the errors coming from the Padé analysis.

estimate for K_{cr}:

$$K_{cr}\,(\beta = 5.4) = 0.1934 \pm 0.0005\,,$$
$$K_{cr}\,(\beta = 5.7) = 0.1690 \pm 0.0005\,. \tag{7}$$

At the critical value $K = K_{cr}$, we obtain for the ρ-meson mass:

$$m_\rho a = 0.77 \pm 0.05\,, \qquad \beta = 5.4\,,$$
$$m_\rho a = 0.58^{+0.12}_{-0.06}\,, \qquad \beta = 5.7\,. \tag{8}$$

The asymmetric error on $m_\rho a$ at $\beta = 5.7$ is due to the last point ($am = 0.7$) where the ρ-pole can still be determined. This point has, in our data, a peculiar behaviour: there seems to be a systematic shift upwards if the statistics is increased. This is best seen in the central part of the Padé table for the highest orders, where both nominator and denominator are higher than, say, 6th order. The shift in K can be as much as $\Delta K \sim 0.002$, but decreasing the statistics leads to an increase in the error of the Padé analysis, therefore we cannot really tell, whether this is a real effect or it is just an accident. No such effect is seen for the pion pole, and at $\beta = 5.4$ the effect is much weaker (if present at all). Since the physical value of $(am_\pi)^2$ is small, $K_{u,d}$ is very close to K_{cr}. Therefore $m_\rho a$ is essentially the same at $K=K_{u,d}$ as at $K=K_{cr}$. Using the experimental value $m_\rho = 0.78$ GeV we obtain for the lattice spacing:

$$a=(0.99\pm0.06)\ \mathrm{GeV}^{-1}\simeq(0.2\pm0.01)\ \mathrm{fm},\qquad \beta=5.4,$$
$$a=\left(0.74^{+0.16}_{-0.08}\right)\mathrm{GeV}^{-1}\simeq\left(0.15^{+0.03}_{-0.02}\right)\mathrm{fm},\qquad \beta=5.7. \tag{9}$$

The change in the lattice unit $a(5.4)=(1.34^{+0.26}_{-0.34})a(5.7)$ is consistent with the one-loop renormalization group requirement $a(5.4)=1.4a(5.7)$, but the errors are too large to draw a conclusion. The uncertainty comes mainly from the ρ channel, where the statistical and Padé errors are larger than that for the pion. On the other hand, the quark mass can be extracted using the pion curve only, leading to much smaller errors for this quantity. According to figs. 1 and 2, near $K=K_{cr}$ we have

$$(am_\pi)^2\sim45\,(K_{cr}-K),\qquad \beta=5.4,$$
$$(am_\pi)^2\sim56\,(K_{cr}-K),\qquad \beta=5.7. \tag{10}$$

From the relation $am_q\sim(K_{cr}-K)/2K_{cr}^2$ we obtain

$$\frac{a(5.4)}{a(5.7)}\sim\frac{45}{56}\,\frac{K_{cr}(5.4)^2}{K_{cr}(5.7)^2}\,\frac{m_q(5.4)}{m_q(5.7)}\sim1.05\frac{m_q(5.4)}{m_q(5.7)}=1.07. \tag{11}$$

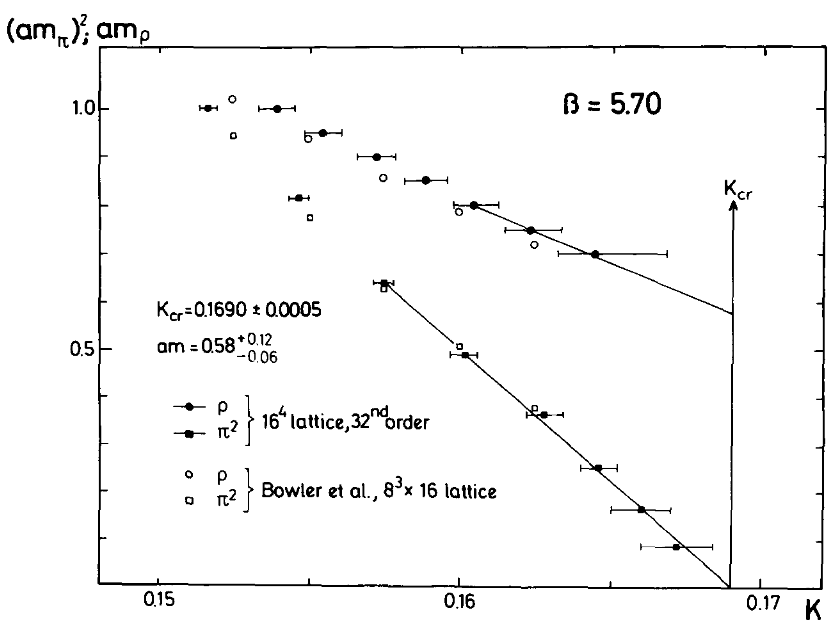

Fig. 2. The same as fig. 1 for $\beta=5.7$. For comparison, the results of Bowler et al. [14] obtained on an $8^3\times16$ lattice are also shown.

Here, in the last step, we used that $\beta^{4/11}m_q(\beta)$ is a renormalization group invariant quantity, which is a one-loop perturbative result [18]. The ratio between the lattice units obtained this way is considerably less than the required factor 1.4. The dimensionless quark mass seems to change too slowly as the function of the coupling. A similar conclusion was obtained by Fukugita et al. [13] in studying meson spectroscopy in SU(2) on smaller lattices. There is, however, a possibility that the rather ad hoc definition of the quark mass used above does not lead to a well-defined physical quantity.

The lattice distance as given in eq. (9) is smaller than that obtained by measuring the string tension on the same gauge field configurations, eq. (6), [16]. On the other hand eq. (9) is consistent (via the one-loop RG formula) with the result $a(\beta = 6.0) \sim 1/2.1\ \mathrm{GeV}^{-1}$ obtained by Lipps et al. [19] in a recent work on a $10^3 \times 20$ lattice. Clearly, the question of scaling and the relation between the pure gauge theory results and the quenched approximation requires further careful study.

For comparison, the results obtained by Bowler et al. [14] on an $8^3 \times 16$ lattice at $\beta = 5.7$ are also given in fig. 2. There is a perfect agreement in the pion channel, while the ρ masses are shifted to somewhat smaller values on the smaller lattice. A similar effect is observed by comparing our hopping-parameter results on 8^4 and 16^4 lattices at $\beta = 5.7$ (fig. 3): no difference can be seen in the pion channel, while the ρ masses move downwards when the lattice size is decreased.

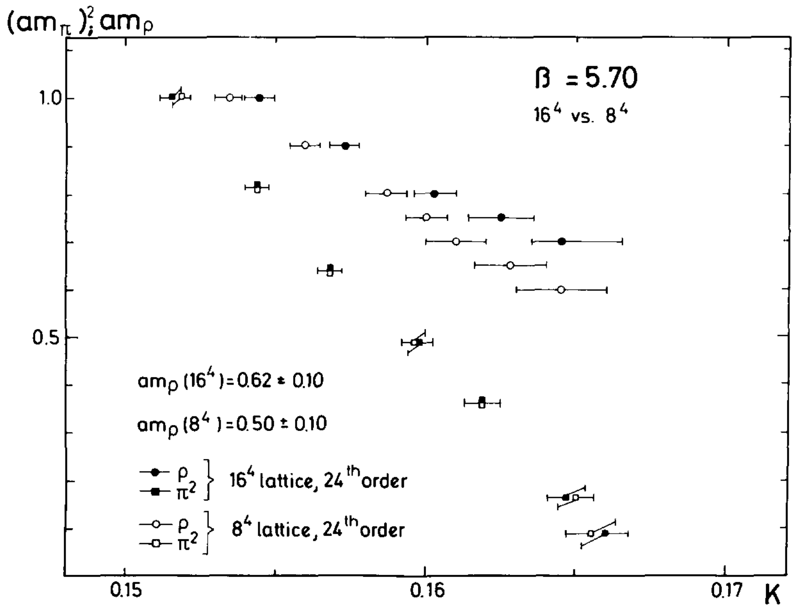

Fig. 3. The size dependence of pseudoscalar- and vector-meson masses between 8^4 and 16^4 lattices for $\beta = 5.7$ obtained in a 24th-order hopping-parameter expansion.

The good agreement of the pion curve with ref. [14] (where the quark propagators were determined by a different, iterative method) implies that our 32nd-order series at $\beta = 5.7$ is long enough to determine the meson masses with good accuracy (the situation is, of course, even better at $\beta = 5.4$). The numerical hopping-parameter expansion is competitive also from the practical point of view: a 32nd-order expansion on a 16^4 lattice takes at least one order of magnitude less computer time than the usual (e.g. Gauss-Seidel) numerical methods. A particularly nice feature of the hopping-parameter expansion method is that the complete hopping-parameter dependence (that is also flavour dependence) of the quark propagator is determined in one calculation. The price one has to pay for this is the large amount of data (few hundred Mbytes), which one has to move in building up the hadron propagators from quark propagators.

Most of the Padé entries have also a second pole in both the pion and the rho channels. Especially at $\beta = 5.7$ these poles show a consistent pattern and their position is satisfactorily stable under changing the analysis (like leaving out a few terms at the beginning or at the end of the series). In particular, the zero in the Padé table between the two poles is roughly in the middle, showing that the two poles have roughly the same strength. These features are rather different from the situation observed at $\beta = 6.0$ in 24th order on the 8^4 lattice in ref. [9]. There the position of the second pole is changing if the first few coefficients are omitted from the series and, in addition, the zero is at least 10 times closer to the first pole than to the second one (i.e. the second "pole" is 10 times stronger). On the basis of these qualitative differences we think that the best interpretation of the singularity structure seen in [9] is indeed a cut, but on our present lattices it is tempting to interpret the two stable poles as describing the ground states and first radial excitations. The result is shown in fig. 4. Using eqs. (7, 8) one obtains for ρ'

$$am_{\rho'} = 1.2^{+0.2}_{-0.1},$$

$$m_{\rho'}/m_\rho = 2.07 \pm 0.45, \qquad \text{(experiment: 2.05)}. \tag{12}$$

Knowing the value of a, it is possible to determine the hopping-parameter values for strange and charmed quarks from the masses of the ϕ and J/ψ mesons, respectively:

$$K_s = 0.163^{+0.002}_{-0.006}, \qquad K_c = 0.113^{+0.008}_{-0.016}. \tag{13}$$

Comparing the positions of the first and second poles in the 1^- channel this gives

$$m_{\phi'}/m_\phi = 1.85 \pm 0.15, \qquad \text{(experiment: 1.65 or 1.81)},$$

$$m_{\psi'}/m_\psi = 1.21 \pm 0.05, \qquad \text{(experiment: 1.22)}. \tag{14}$$

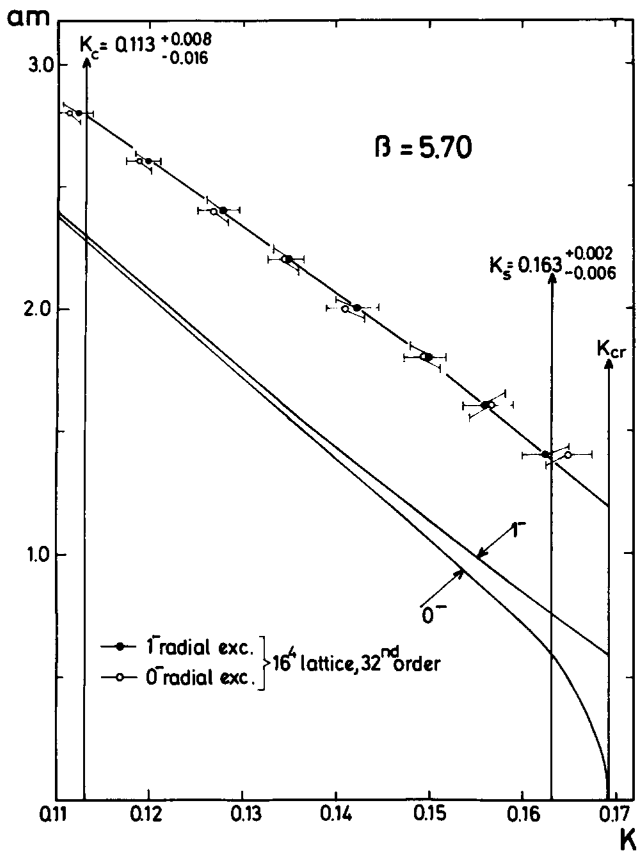

Fig. 4. The ground states and first radial excitations in the 0^-- and 1^--channels at $\beta = 5.7$. The hopping-parameter values $K_{s(c)}$ for strange (charmed) quarks are shown by vertical lines.

Note that in fig. 4 the first radial excitations in the vector and pseudoscalar channels nearly coincide. This may be a property of the quenched approximation.

4. Conclusions

As is described in this paper, by using a well-organized input-output procedure, a high-order numerical hopping-parameter expansion can be obtained on large (16^4 or larger) gauge field configurations. For a 32nd-order expansion the required computer time is much less (~ by an order of magnitude) than that required by direct numerical inversion methods. At the expense of CPU time, the order of the expansion can be increased, since the time increases as a power of the order and not exponentially*. At $\beta = 5.7$ the 32nd-order series is long enough for the accurate determination of meson masses. It may, however, be that at larger β (e.g. $\beta = 6.0$) still higher orders are required.

We have studied the meson spectrum at $\beta = 5.4$ and 5.7 on a 16^4 lattice. As the physical size of the lattice is large, no significant finite size effects are expected in this case.

In the pion channel the errors (coming from the Padé analysis and from the statistical fluctuations) are small and the critical hopping parameter is determined with a good precision. In the ρ-meson channel the errors are larger which is reflected in the uncertainty of the prediction of the lattice unit a at $\beta = 5.4$ and 5.7. This prediction is consistent with the one-loop renormalization group formula, but the errors are large and the values are smaller than the string tension predictions.

At $\beta = 5.7$ we observed some systematic shift in the central value of the ρ mass if the statistics was increased. We do not understand this phenomenon, it might be a numerical accident.

At $\beta = 5.7$ we compared our results with those obtained on smaller (8^4 or $8^3 \times 16$) lattices. The pion does not show finite-size effects, while the ρ mass values are shifted to somewhat smaller values as the lattice is halved.

A second stable pole showed up clearly in many Padé's and we attempted to determine the first radial excitations. The numbers are better than expected.

We thank F. Gutbrod, Z. Kunszt, G. Martinelli and R. Petronzio for discussions. We are grateful to CERN, DESY and in particular to the Computer Centre of the University of Hamburg for supporting us with computing facilities.

References

[1] K.G. Wilson, Phys. Rev. D10 (1974) 2445
[2] H. Hamber and G. Parisi, Phys. Rev. Lett. 47 (1981) 1792; Phys. Rev. D27 (1983) 208
[3] E. Marinari, G. Parisi and C. Rebbi, Phys. Rev. Lett. 47 (1981) 1795
[4] D.H. Weingarten, Phys. Lett. B109 (1982) 57; Nucl. Phys. B215[FS7] (1983) 1
[5] A. Hasenfratz, P. Hasenfratz, Z. Kunszt and C.B. Lang, Phys. Lett. B110 (1982) 282; B117 (1982) 81
[6] F. Fucito, G. Martinelli, C. Omero, G. Parisi, R. Petronzio and F. Rapuano, Nucl. Phys. B210[FS6] (1982) 407
[7] G. Martinelli, C. Omero, G. Parisi and R. Petronzio, Phys. Lett. B117 (1982) 434
[8] K.C. Bowler, E. Marinari, G.S. Pawley, F. Rapuano and D.J. Wallace, Nucl. Phys. B220[FS8] (1983) 137
[9] P. Hasenfratz and I. Montvay, Phys. Rev. Lett. 50 (1983) 309
[10] C. Bernard, T. Draper and K. Olynyk, Phys. Rev. D27 (1983) 227
[11] R. Gupta and A. Patel, Phys. Lett. 124B (1983) 94
[12] G. Martinelli, G. Parisi, R. Petronzio and F. Rapuano, Phys. Lett. B122 (1983) 283
[13] M. Fukugita, T. Kaneko and A. Ukawa, KEK preprint TH 58 (1982)

* The power depends on whether a periodic box or copied gauge fields are used. In the first case it also depends on the ratio of the maximum order to the box size.

[14] K.C. Bowler, G.S. Pawley and D.J. Wallace, Proc. of HEP83, Brighton, 1983 (Rutherford Lab, Chilton, 1984) p. 15
[15] K.G. Wilson, in New phenomena in subnuclear physics, Erice 1975, ed. A. Zichichi (Plenum, New York, 1977)
[16] F. Gutbrod, P. Hasenfratz, Z. Kunszt and I. Montvay, Phys. Lett. 128B (1983) 415
[17] P. Hasenfratz, Proc. Johns Hopkins Workshop, Florence 1982
[18] E. de Rafael, in Lecture notes in physics, eds. J.L. Alonso and R. Tarrach (Springer, 1980) vol. 118
[19] H. Lipps, G. Martinelli, R. Petronzio and F. Rapuano, Phys. Lett. 126B (1983) 250
[20] E. Kovacs, Phys. Lett. B118 (1982) 125