A DIRAC–KÄHLER APPROACH TO THE TWO DIMENSIONAL WESS–ZUMINO $N = 2$ MODEL ON THE LATTICE

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We apply the Dirac–Kähler formalism to the two dimensional Wess–Zumino $N = 2$ model. On the hamiltonian lattice the model exhibits invariance under a lattice subalgebra of supersymmetry. We also describe the euclidean space–time lattice version of this model with the action invariant under the supersymmetry transformations which have the correct continuum limit.

The problem of constructing a supersymmetric theory on the lattice has been the subject of some recent articles [1–4] [‡1,2] with the special attention to the two dimensional Wess–Zumino theory. The authors of refs. [2–4] make use of the Nicolai mapping as a tool for the formulation of supersymmetric euclidean actions on the space–time lattice, while in refs. [1,4] use is made of geometric fermions, in the sense of Dirac–Kähler formalism, in order to obtain hamiltonians on the space lattice.

The purpose of the present letter is to write down an explicit Dirac–Kähler (D–K) model for the supersymmetric $N = 2$ Wess–Zumino (W–Z) two dimensional theory. We shall write down the lagrangian and the corresponding differential equations in differential form. From these we obtain the conserved supersymmetric currents and extend these considerations to the space–time lattice. For this purpose we shall use extensively the differential approach to the Dirac equations developed in ref. [5].

We have found that, in our D–K model, it is possible to have an action in the space–time lattice which is invariant under the supersymmetric transformations (as defined below) with some harmless additional lattice terms. In the continuum limit the action reproduces the correct continuum action. The role of the additional terms is merely restricted to preserve the invariance under supersymmetry transformations without violating the Osterwalder Schrader positivity condition.

In the hamiltonian approach (with time continuous) the hamiltonian operator can be introduced as the square of the discretized supercharges. This leads however to the additional, surface lattice terms which become total derivatives in the naive continuum limit. This feature is typical for the so-called lattice sub-algebra of the supersymmetry algebra [1,2].

We shall firstly formulate our model in the two dimensional Minkowski space with the mapping [5]

$$\gamma^{\mu} \leftrightarrow \mathrm{d}x^{\mu} \vee \qquad (1)$$

($\vee$ = Clifford product), where we choose $\gamma^2 = \sigma_2$ and

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[‡1] For the earlier attempts see the references quoted in ref. [1].
[‡2] Ref. [2] has a rather complete list of references.

$\gamma^1 = i\sigma_1$ (σ_1, σ_2 are the Pauli matrices; our time index is denoted by 2). We will assume that in our D–X model the fermions are described by the differential forms:

$$\Psi = f_0 + (f_{\mu\nu}/2!)\,dx^\mu \wedge dx^\nu = f_0 + f_{12}\,dx^{12}\,, \quad (2)$$

$$\Psi^* = f_0^* + (f_{\mu\nu}^*/2!)\,dx^\mu \wedge dx^\nu = f_0^* + f_{12}^*\,dx^{12}\,, \quad (3)$$

with $\mu, \nu = 1, 2$, $dx^{12} = dx^1 \wedge dx^2$ and f_0^*, f_{12}^* denoting the complex conjugates of the Grassmann variables f_0, f_{12}. We will impose the following equal time anticomutation relations:

$$\{f_0^*(x), f_0(y)\} = \{f_{12}^*(x), f_{12}(y)\} = \tfrac{1}{2}\delta(x-y)\,,$$

$$\{f_0, f_{12}\} = \{f_0^*, f_{12}\} = \{f_0, f_{12}^*\} = \{f_0^*, f_{12}^*\} = 0\,. \quad (4)$$

The boson field is assumed to be described by the differential forms:

$$\Phi = \varphi_\mu dx^\mu\,, \quad \Phi^* = \varphi_\mu^* dx^\mu \quad (5)$$

where

$$\varphi_\mu = \partial_\mu \varphi\,, \quad \varphi_\mu^* = \partial_\mu \varphi^* \quad (6)$$

and φ^* is the complex conjugate of the complex scalar field.

We shall propose the following lagrangian density:

$$\mathcal{L} = \mathcal{P}_0[\Phi^* \vee \Phi + 2i\Psi^* \vee (d-\delta)\,dx^2 \vee \Psi - W'^* W'$$
$$- i\Psi^* \vee dx^2 \vee \Psi \vee (dx^{12}+1) \vee dx^2 W''$$
$$- i\Psi^* \vee dx^2 \vee \Psi \vee (dx^{12}-1) \vee dx^2 W''^*]\,, \quad (7)$$

where W depends only on the scalar field φ and W^* only on φ^*.

$\mathcal{P}_0$ projects zero form components from the above Clifford products, and $W' = dW/d\varphi$, $W'' = d^2W/d\varphi^2$, $W'^* = dW^*/d\varphi^*$, $W''^* = d^2W^*/d\varphi^{*2}$ etc.

It is easy to see that, by making the identification $\chi_1 = f_0 + f_{12}$, $\chi_2 = f_0 - f_{12}$, which are the components of a spinor in two dimensions, our lagrangian (7) corresponds exactly to the usual $N = 2$ Wess–Zumino model in two dimensions. Let us observe that these two components, in the notation of ref. [5], have different flavours.

From (7) we obtain the following D–K equations:

$$(d-\delta)\Phi = -W''^* W'$$
$$- i\mathcal{P}_0[\Psi^* \vee dx^2 \vee \Psi \vee (dx^{12}-1) \vee dx^2]\,W'''^*\,, \quad (8)$$

$$(d-\delta)\Psi' = \tfrac{1}{2}dx^2 \vee \Psi \vee (dx^{12}+1) W''$$
$$+ \tfrac{1}{2}dx^2 \vee \Psi \vee (dx^{12}-1) W''^*\,, \quad (9)$$

where we have introduced $\Psi' = dx^2 \vee \Psi \vee dx^2 = \mathcal{B}\Psi = f_0 - f_{12}\,dx^{12}$, $\mathcal{B}$ being the antiautomorphism defined in ref. [5]. In components, eq. (9) write as:

$$(\partial_1 + \partial_2)(f_0 + f_{12}) = -(f_0 - f_{12})W''^*\,,$$

$$(\partial_1 - \partial_2)(f_0 - f_{12}) = -(f_0 + f_{12})W''\,. \quad (10)$$

Consider now the 1 scalar product of Kähler [5], constructed from the boson and fermion differential form $(\Phi, \Psi')_1$. By making use of the identity [5]:

$$d(\Psi, \Psi')_1 = [(d-\delta)\Phi, \Psi']_0 + [\Phi, (d-\delta)\Psi']_0 \quad (11)$$

it follows after use of eqs. (8) and (9):

$$\tfrac{1}{2}d[(\Phi, \Psi')_1 + (\Psi', \Phi)_1]$$
$$= [-W''^* W' f_0 + \tfrac{1}{2}(\varphi_1 + \varphi_2)(f_0 + f_{12})W''$$
$$+ \tfrac{1}{2}(\varphi_1 - \varphi_2)(f_0 - f_{12})W''^*]\,\epsilon\,, \quad (12)$$

where $\epsilon = \frac{1}{2}\epsilon_{\mu\nu} dx^\mu \wedge dx^\nu$ denotes the volume element (our convention is $\epsilon_{12} = -\epsilon_{21} = 1$).

Now our lagrangian (7), or the corresponding equations (8) and (9) are invariant under the "dual transformations":

$$\Psi \to \tilde{\Psi} = \Psi \vee dx^2 \quad (\text{or } f_0 \leftrightarrow f_{12})\,,$$

$$W'' \to -W''\,, \quad W''^* \to -W''^*\,, \quad \varphi \to -\varphi\,, \quad \varphi^* \to -\varphi^*\,,$$

$$W' \to W'\,, \quad W'^* \to W'^*\,. \quad (13)$$

Therefore we obtain the dual relation of (12) by making the substitutions (13):

$$\tfrac{1}{2}[\widetilde{(\Phi, \Psi')_1} + \widetilde{(\Psi', \Phi)_1}]$$
$$= [W''^* W' f_{12} + \tfrac{1}{2}(\varphi_1 + \varphi_2)(f_0 + f_{12})W''$$
$$- \tfrac{1}{2}(\varphi_1 - \varphi_2)(f_0 - f_{12})W''^*]\,\epsilon\,. \quad (14)$$

Adding expressions (12) and (14) we obtain:

$$d[\tfrac{1}{2}(\Phi,\Psi')_1+\tfrac{1}{2}(\Psi',\Phi)_1+\tfrac{1}{2}(\widetilde{\Phi,\Psi'})_1+\tfrac{1}{2}(\widetilde{\Psi',\Phi})$$

$$-W'(\Psi'-\widetilde{\Psi}')\vee dx^2]=0\,, \qquad (15)$$

where use was made of the relation:

$$d(W'\Psi'\vee dx^2)-d(W'\widetilde{\Psi}'\vee dx^2)$$

$$=(f_0+f_{12})(\partial_1+\partial_2)W'-W'W''^*(f_0-f_{12})\,, \qquad (16)$$

which follows from the equation of motion (10).

From the expression (15) it follows that we have a conserved supercurrent with components:

$$j_1=(\varphi_1+\varphi_2)(f_0-f_{12})+(f_0+f_{12})W'\,,$$

$$j_2=(\varphi_1+\varphi_2)(f_0-f_{12})-(f_0+f_{12})W'\,. \qquad (17)$$

If instead of Φ in expression (15) we use Φ^*, we shall obtain in a similar way a conserved supercurrent with components:

$$j_1'=(\varphi_1^*-\varphi_2^*)(f_0+f_{12})+(f_0-f_{12})W'^*\,,$$

$$j_2'=-(\varphi_1^*-\varphi_2^*)(f_0+f_{12})+(f_0-f_{12})W'^*\,. \qquad (18)$$

The complex conjugates of (17) and (18) also give conserved supercurrent with components j_1^*, j_2^* and $j_1'^*, j_2'^*$. Integrating over the space the time components of these currents we obtain the corresponding charges Q, Q', Q^* and Q'^*, which will induce on the field components (after the use of expressions (4) and the corresponding equal time commutators for the boson field) the following transformations:

By Q:

$$\delta\varphi=0\,,\quad \delta\varphi^*=-i(f_0-f_{12})\,,$$

$$\delta f_0=0\,,\quad \delta f_0^*=\tfrac{1}{2}(\varphi_1+\varphi_2)-\tfrac{1}{2}W'\,,$$

$$\delta f_{12}=0\,,\quad \delta f_{12}^*=-\tfrac{1}{2}(\varphi_1+\varphi_2)-\tfrac{1}{2}W'\,. \qquad (19)$$

By Q':

$$\delta'\varphi=-i(f_0+f_{12})\,,\quad \delta'\varphi^*=0\,,$$

$$\delta' f_0=0\,,\quad \delta' f_0^*=-\tfrac{1}{2}(\varphi_1^*-\varphi_2^*)+\tfrac{1}{2}W'^*\,,$$

$$\delta' f_{12}=0\,,\quad \delta' f_{12}^*=-\tfrac{1}{2}(\varphi_1^*-\varphi_2^*)-\tfrac{1}{2}W'^*\,. \qquad (20)$$

By Q^*:

$$\delta^*\varphi=-i(f_0^*-f_{12}^*)\,,\quad \delta^*\varphi^*=0\,,$$

$$\delta^* f_0=\tfrac{1}{2}(\varphi_1^*+\varphi_2^*)-\tfrac{1}{2}W'^*\,,\quad \delta^* f_0^*=0\,,$$

$$\delta^* f_{12}=-\tfrac{1}{2}(\varphi_1^*+\varphi_2^*)-\tfrac{1}{2}W'^*\,,\quad \delta^* f_{12}^*=0\,. \qquad (21)$$

By Q'^*:

$$\delta'^*\varphi=0\,,\quad \delta'^*\varphi^*=-i(f_0^*+f_{12}^*)\,,$$

$$\delta'^* f_0=-\tfrac{1}{2}(\varphi_1-\varphi_2)+\tfrac{1}{2}W'\,,\quad \delta'^* f_0^*=0\,,$$

$$\delta'^* f_{12}=-\tfrac{1}{2}(\varphi_1-\varphi_2)-\tfrac{1}{2}W'\,,\quad \delta'^* f_{12}^*=0 \qquad (22)$$

$\delta\varphi=[Q,\varphi]\,, \delta f_0=\{Q,f_0\}$, etc.

Now let us go to the euclidean formulation. Corresponding to (7) we find the following euclidean lagrangian density:

$$\mathcal{L}_E=\mathcal{P}_0[\Phi^*\vee\Phi+W'^*W'+2\Psi^*\vee(d-\delta)dx^2\vee\Psi$$

$$+\Psi^*\vee dx^2\vee\Psi\vee(1-i dx'^2)\vee dx^2 W''$$

$$+\Psi^*\vee dx^2\vee\Psi\vee(1+i dx'^2)\vee dx^2 W''^*]\,. \qquad (23)$$

Here the mapping (1) corresponds to the following choice of the hermitian euclidean matrices: $\gamma^0=\sigma_2$, $\gamma'=\sigma_1$. Now we can write the euclidean action defined on a space–time discrete euclidean lattice:

$$S_E=\sum_{x_1,x_2}\{\Delta_\mu^+\varphi^*\Delta_\mu^+\varphi+W'^*W'$$

$$+2f_0^*(-\Delta_1^-f_{12}+\Delta_2^-f_0)+2f_{12}^*(\Delta_1^+f_0+\Delta_2^+f_{12})$$

$$+[f_0^*f_0-f_{12}^*f_{12}+if_0^*f_{12}+if_{12}^*f_0]W''$$

$$+[f_0^*f_0-f_{12}^*f_{12}-if_0^*f_{12}-if_{12}^*f_0]W''^*\}\,. \qquad (24)$$

This expression was obtained from (23) after the substitution $d\to\Delta^+, \delta\to\Delta^-$, where Δ^+ corresponds to the boundary operator $\check{\Delta}$ and Δ^- to the coboundary operator $\check{\nabla}$ of ref. [5].

The equations of motion which follow from (24) are

$$\Delta_2^-f_0-\Delta_1^-f_{12}=-\tfrac{1}{2}[(f_0+if_{12})W''+(f_2-if_{12})W''^*] \qquad (25a)$$

$$\Delta_1^+f_0+\Delta_2^+f_{12}=-\tfrac{1}{2}i[(f_0+if_{12})W''-(f_0-if_{12})W''^*]\,, \qquad (25b)$$

$$\Delta_\mu^- \Delta_\mu^+ \varphi^* = W'^* W'' + [\ ] W''' , \tag{26a}$$

$$\Delta_\mu^- \Delta_\mu^+ \varphi = W''^* W' + [\ ] W'''^* . \tag{26b}$$

Where the [ ] which multiplies W''' in (26a) is the same which multiplies W'' in expression (24). Correspondingly the [ ] which multiplies W'''^* in (26b) is the same which multiplies W''^* in (24).

These equations of motion are free from the problem of energy doubling.

In order to obtain the euclidean lattice version of supersymmetry transformations (19)–(22) we first transcribe them to euclidean metric by making the substitution $\Delta_2 \to -\mathrm{i}\Delta_2, f_{12} \to \mathrm{i}f_{12}$,

$$W' \to \mathrm{i}W' , \quad f_{12}^* \to -\mathrm{i}f_{12}^* , \quad W'^* \to -\mathrm{i}W'^* , \tag{27}$$

and then discretize by requiring the kinetic part of S_E to be invariant. This procedure leads uniquely to the following supersymmetry transformations

$$\delta\varphi^* = \mathrm{i}(f_0 - \mathrm{i}f_{12}) , \quad \delta f_0^* = \tfrac{1}{2}(\Delta_1^- - \mathrm{i}\Delta_2^-)\varphi - \tfrac{1}{2}\mathrm{i}W' ,$$

$$\delta f_{12}^* = -\tfrac{1}{2}\mathrm{i}(\Delta_1^+ - \mathrm{i}\Delta_2^+)\varphi + \tfrac{1}{2}W' ; \tag{19'}$$

$$\delta'\varphi = \mathrm{i}(f_0 + \mathrm{i}f_{12}), \quad \delta' f_0^* = -\tfrac{1}{2}(\Delta_1^- + \mathrm{i}\Delta_2^-)\varphi^* - \tfrac{1}{2}\mathrm{i}W'^* ,$$

$$\delta' f_{12}^* = -\tfrac{1}{2}\mathrm{i}(\Delta_1^+ + \mathrm{i}\Delta_2^+)\varphi^* - \tfrac{1}{2}W'^* ; \tag{20'}$$

$$\delta^*\varphi = \mathrm{i}(f_0^* + \mathrm{i}f_{12}^*), \quad \delta^* f_0 = \tfrac{1}{2}(\Delta_1^- - \mathrm{i}\Delta_2^+)\varphi^* + \tfrac{1}{2}\mathrm{i}W'^2 ,$$

$$\delta^* f_{12} = \tfrac{1}{2}\mathrm{i}(\Delta_1^+ - \mathrm{i}\Delta_2^-)\varphi^* + \tfrac{1}{2}W'^* ; \tag{21'}$$

$$\delta'^*\varphi^* = \mathrm{i}(f_0^* - \mathrm{i}f_{12}^*), \quad \delta'^* f_0 = -\tfrac{1}{2}(\Delta_1^- + \mathrm{i}\Delta_2^+)\varphi + \tfrac{1}{2}\mathrm{i}W' ,$$

$$\delta'^* f_{12} = \tfrac{1}{2}\mathrm{i}(\Delta_1^+ + \mathrm{i}\Delta_2^-)\varphi - \tfrac{1}{2}W' ; \tag{22'}$$

where only non-zero terms were written. It turns out that the new euclidean action defined as

$$S_\mathrm{E} + \sum_{x_1, x_2} [\mathrm{i}W'^*(-\Delta_1^\mathrm{A} + \mathrm{i}\Delta_2^\mathrm{A})\varphi + \mathrm{i}W'(\Delta_1^\mathrm{A} + \mathrm{i}\Delta_2^\mathrm{A})\varphi^*] , \tag{28}$$

where S_E is given by (24) and $\Delta_1^\mathrm{A} = \frac{1}{2}(\Delta_1^+ - \Delta_1^-) = a\,\frac{1}{2}\Delta_1^+\Delta_1^-$ (a lattice spacing), is left invariant under all four supersymmetry transformations (19′)–(22′). Clearly in the limit $a \to 0$ the action reduces to the correct continuum action for the $N = 2$ Wess–Zumino model.

Moreover (28) possesses invariance under the parity and time reflection and therefore satisfies the Osterwalder–Schrader positivity condition [6].

We note, that for the continuous time and discrete space the supersymmetry transformations (19)–(22) with $\varphi_1 = \Delta_1^- \varphi$ when multiplied by f_0 and $\varphi_1 = \Delta_1^+\varphi$ when multiplied by f_{12}, will follow from the form of the charges Q, Q', Q^* and Q'^* and the equal time canonical commutation relations for the boson field and the relations (4) for the fermion field. In this case it is possible to define a hamiltonian corresponding to the lagrangian density (7) given by:

$$H = \sum_x \mathcal{H} = \sum_x \mathcal{H}_\mathrm{B} + \sum_x \mathcal{H}_\mathrm{F} , \tag{29}$$

where the sum extends over the lattice points and where

$$\mathcal{H}_\mathrm{B} = (\Delta_1^+ A)^2 + (\partial_2 A)^2 + (\Delta_1^+ B)^2 + (\partial_2 B)^2 + U^2 + V^2 , \tag{30a}$$

$$\mathcal{H}_\mathrm{F} = -4\mathrm{i}\alpha_0\Delta_1^-\alpha_{12} - 4\mathrm{i}\beta_0\Delta_1^-\beta_{12} + 4\mathrm{i}\frac{\delta U}{\delta A}\alpha_0\alpha_{12} - 4\mathrm{i}\frac{\delta V}{\delta A}\alpha_0\beta_0 + 4\mathrm{i}\frac{\delta U}{\delta B}\beta_{12}\alpha_{12} - 4\mathrm{i}\frac{\delta V}{\delta B}\beta_{12}\beta_0 , \tag{30b}$$

where we introduced the definitions: $\varphi = A + \mathrm{i}B, f_0 = \alpha_0 + \mathrm{i}\beta_0, f_{12} = \alpha_{12} + \mathrm{i}\beta_{12}, W' = U + \mathrm{i}V$. The hamiltonian (30) is not any more invariant under the supersymmetric transformations (19)–(22) on the space lattice, giving rise to the different lattice surface terms. Separating the supercharges Q, Q', Q^*, Q'^* into its real and imaginary parts $Q = Q_1 + \mathrm{i}Q_2, Q' = Q_1' + \mathrm{i}Q_2'$, and defining $q_1 = Q_1 + Q_1', q_2 = Q_2 + Q_2', q_3 = Q_1 - Q_1', q_4 = Q_2 - Q_2'$, it is possible to show that

$$2q_1^2 = 2q_2^2 = \sum_x (\mathcal{H} + U\Delta_1^+ A - V\Delta_1^- B) , \tag{31a}$$

$$2q_3^2 = 2q_4^2 = \sum_x (\mathcal{H} - U\Delta_1^- A + V\Delta_1^+ B) . \tag{31b}$$

We see therefore that in expressions (31a) and (31b) we have two different classes of hamiltonian on the space lattice, differing in the surface lattice terms. Going to the space continuum these hamiltonians coincide.

In the space lattice it holds:

$$\{q_1, q_4\} = -\{q_2, q_3\}$$

$$= \sum_x [V(x)\Delta_1^s A(x) + U(x)\Delta_1^s B(x)]$$

$$= -\tfrac{1}{2}\mathrm{i} \sum_x (W'\Delta_1^s \varphi - W'^* \Delta_1^s \varphi^*) , \qquad (32)$$

with $\Delta_1^s = \frac{1}{2}(\Delta_1^+ + \Delta_1^-)$ which goes to zero in the continuum limit. In this limit we also have

$$\{q_1, q_3\} = \{q_2, q_4\} = P \qquad (33)$$

where P is the momentum operator of the system. Introducing now $Q_1^1 = q_1 - q_3, Q_2^1 = q_1 + q_3, Q_1^2 = q_2 - q_4, Q_2^2 = q_2 + q_4$, in the continuum space–time, (31a), (31b), (32) and (33) can be written as:

$$\{Q_\alpha^a, Q_\beta^b\} = 2\delta^{ab}(\delta_{\alpha\beta}H - (\sigma_3)_{\alpha\beta}P) , \qquad (34)$$

$a, b = 1, 2$ and $\alpha, \beta = 1, 2$. Finally, in a way similar to ref. [2], we can consider the operator

$$T = \sum_x [\alpha_0(x)\beta_0(x) + \alpha_{12}(x)\beta_{12}(x)] .$$

It is simple to see that $[T, q_1] = -q_2$, $[T, q_2] = q_1$, and therefore T is the generator of an O_2 group in the space (q_1, q_2). Similarly T is also a generator of an O_2 group in the space (q_3, q_4), since $[T, q_3] = -q_4$ and $[T, q_4] = q_3$. Therefore the two classes (q_1, q_2) and (q_3, q_4) are on the same footing, although on the space lattice we cannot consider them simultaneously because of expression (32) and (33). Since T makes the changes $\alpha_0 \to \beta_0, \beta_0 \to -\alpha_0, \alpha_{12} \to \beta_{12}$ and $\beta_{12} \to -\alpha_{12}$, we see that it leaves the lattice hamiltonians (30a) and (30b) invariant.

In conclusion we have succeeded in constructing a D–K model, for the two dimensional Wess–Zumino $N = 2$ lagrangian, which when extended to the space–time lattice presents some nice features. Evidently other D–K models can be constructed for the $N = 2$ two dimensional Wess–Zumino lagrangian and which will present similar interesting properties.

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