Photon in U(1) Lattice Gauge Theory

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A Monte Carlo calculation of the spectrum of four-dimensional U(1) lattice gauge theory has been carried out. In the scaling limit $\beta \to \beta_c$ massive 0^{++}, 1^{+-}, and 2^{++} states are indicated. On the critical line $\beta > \beta_c$ striking evidence is found for a massless photon, and no signal is found for other states.

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In the strong-coupling (SC) region (β small) Abelian as well as non-Abelian lattice gauge theories (LGT) are in the confinement phase. The famous confinement problem for non-Abelian LGT consists in proving that this phase extends to the continuum limit: lattice constant $a \to 0$, $\beta \to \infty$. On the other hand we would like to recover a free field theory of massless photons from the four-dimensional Abelian U(1) LGT. In a fundamental paper on LGT Wilson[1] therefore conjectured that the U(1) LGT undergoes a phase transition as the coupling constant β is varied, with a nonconfining phase at weak coupling.

Later the existence of these two phases has been rigorously proven by Guth[2] and the result has been generalized by Fröhlich and Spencer.[3] Monte Carlo calculations[4] indicate a second-order phase transition at $\beta \approx 1.0$. For large enough β perturbation theory becomes applicable and the existence of a zero-mass state has been proven.[3] No rigorous results exist for the whole region $\beta > \beta_c$. In analogy to the two-dimensional X-Y model one expects a critical line of mass-zero field theories. In this Letter we demonstrate by a Monte Carlo (MC) simulation that this picture is correct. Our massless excitation has the quantum numbers 1^{+-} of an axial vector and provides direct evidence for the existence of a massless state with the quantum numbers of the photon. Let us consider in free-field theory a 1^{--} photon vector state $|p, s\rangle$ with p momentum and s helicity. The combination

$$|\hat{\vec{p}}\rangle = \tfrac{1}{2}\sqrt{2}\,(|\vec{p}, s\rangle - |\vec{p}, -s\rangle)$$

has parity $P = +1$. With use of free fields it is easily checked that in the naive continuum limit this state has an overlap with our 1^{+-} state.

We consider U(1) LGT with the Wilson[1] action. At each link b of a hypercubic four-dimensional lattice there is an element $U(b) = \exp(i\theta_b) \in U(1)$, and averages are calculated with the partition function

$$Z = \int_{-\pi}^{\pi} \prod_b d\theta_b \exp[\beta \operatorname{Re} \textstyle\sum_p U(\dot{p})]. \qquad (1)$$

For each plaquette p, $U(\dot{p})$ is the ordered product of the four link matrices surrounding the plaquette.

For our MC calculation we use as in Ref. 4 the Metropolis method and approximate U(1) by $Z(1000)$. Most of our calculations are carried out on an $4^3 \times 8$ lattice with cyclic boundary conditions. 4^3 is the spacelike box and 8 is the extension in Euclidean "time" direction. At $\beta = 1.3$ some finite-size consistency checks are carried out on an 8^4 lattice.

Our results are based on (diagonal) correlations between Wilson loops up to length 6, as depicted in Fig. 1. In the work of Kogut, Sinclair, and Susskind[5] and of Berg and Billoire[6] the irreducible representations of the cubic group on these were constructed. We wish to remind the reader that there exist five irreducible repre-

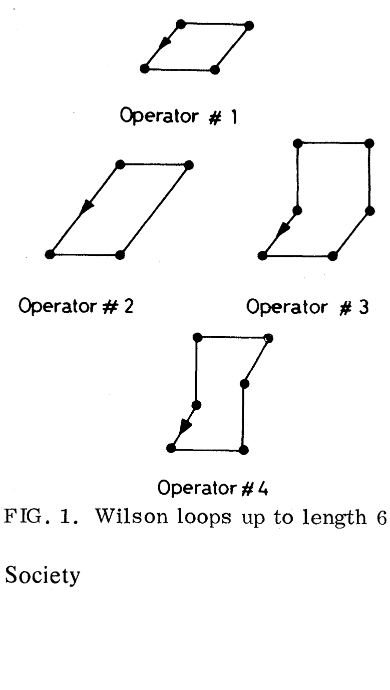

FIG. 1. Wilson loops up to length 6.

sentations of the cubic group. In the standard notation for point groups A_1 and A_2 are the one-dimensional representations, E is the two-dimensional representation, T_1 and T_2 are the three-dimensional representations. Under certain assumptions[6] we have the following correspondence in the continuum limit of a LGT:

$$A_1^{PC} \to 0^{PC}, \quad T_1^{PC} \to 1^{PC}, \quad E^{PC} \to 2^{PC}, \quad T_2^{PC} \to 2^{PC}, \quad \text{and} \quad A_2^{PC} \to 3^{PC} \tag{2}$$

(P denotes parity, C denotes charge-conjugation parity).

In the present paper we consider for some of the irreducible representations states of momentum $\vec{k} = (k_\perp, k_2, k_3)$, $k_i = 2\pi n/L$ ($n = 0, \pm 1, \ldots, \pm[L/2]$); L is the spacelike lattice size. Problems with phases are avoided by the trick of Kimura and Ukawa[7]: We first construct the irreducible representation in question on a spacelike cube and then we perform the Fourier transformation for the cube operators $C_j(\vec{x}, t)$:

$$\tilde{C}_j(\vec{k}, t) = \sum_{\vec{x}} e^{i\vec{k}\cdot\vec{x}} C_j(\vec{x}, t). \tag{3}$$

Here $\vec{x}$ is the position of the center of the cube. The index $j = (\mathrm{OP}, R)$ labels the operators OP and representations $R = A_1^{++}$, T_1^{+-}, E^{++}, etc. Of course a Wilson loop may contribute to several cubes. In Ref. 7 this construction was carried out for the one-plaquette operator in the A_1^{++} representation. The generalization to other representations and operators is, however, straightforward.

We will calculate correlation functions

$$\rho_j(\vec{k}, t) = \mathrm{Re}\langle 0 | \tilde{C}_j^*(\vec{k}, t) \tilde{C}_j(\vec{k}, 0) | 0 \rangle \tag{4}$$

and define corresponding energies by means of

$$E_j(|\vec{k}|, t) = -(1/t)\ln[\rho_j(\vec{k}, t)/\rho_j(\vec{k}, 0)]. \tag{5}$$

These energies are upper bounds for the energy $E_R(|\vec{k}|)$ of the lowest state, which couples to the irreducible representation R in question. According to Eq. (2) we now abbreviate these states by 0^{++}, 1^{+-}, 2^{++}, etc. If the relativistic energy-momentum dispersion is restored, we expect

$$E_R(|\vec{k}|) = (m_R^2 + \vec{k}^2)^{1/2}. \tag{6}$$

In the practical MC calculation statistical noise limits us to rather short distances: $t = 0$, 1, and only in some cases are reasonable results also obtained for $t = 2$. If the state $\tilde{C}_j(\vec{k}, t)|0\rangle$ is a good approximation to the wave function of the lowest state in question, already $E_j(|\vec{k}|, 1)$ may be a rather close bound to the energy $E_j(|\vec{k}|)$.

We now present our results. At each considered β value on the $4^3 \times 8$ lattice we have performed about 10 000 double sweeps and we did measurements after each double sweep. We have used random upgrading[6] and a sweep is defined by upgrading each link variable once in the mean. At each β value, between 1200 and 1800 sweeps without measurements were done for reaching equilibrium.

Let us first consider momentum $\vec{k} = 0$ states. In Fig. 2 our distance $t = 1$ energy results for the lowest-lying states 0^{++}, 1^{+-}, and 2^{++} are given. For guiding the eyes MC points of the same state are connected with straight lines. The bounds $E(\vec{k} = 0, t = 1)$ on the energies (= masses) decrease as one approaches the critical point from below: $\beta \to \beta_c \approx 1.0$ ($\beta < \beta_c$). The energy results from distance $t = 2$ are of course better (= lower) bounds, but as in non-Abelian gauge theories[6] reliable results can hardly be obtained if $E(|\vec{k}| = 0, t = 1) \gtrsim 2$. For the states 0^{++} and 1^{+-} the distance $t = 2$ results are given in Table I. We use always the operator which gives the lowest result also used at distance $t = 1$. For the other considered representations distance $t = 1$ energies are higher (see Table II).

From Fig. 2 we note a clear difference between U(1) and non-Abelian gauge theories: the relative lightness of the 1^{+-} state. To summarize: In the scaling limit $\beta \to \beta_c$ a spectrum of massive

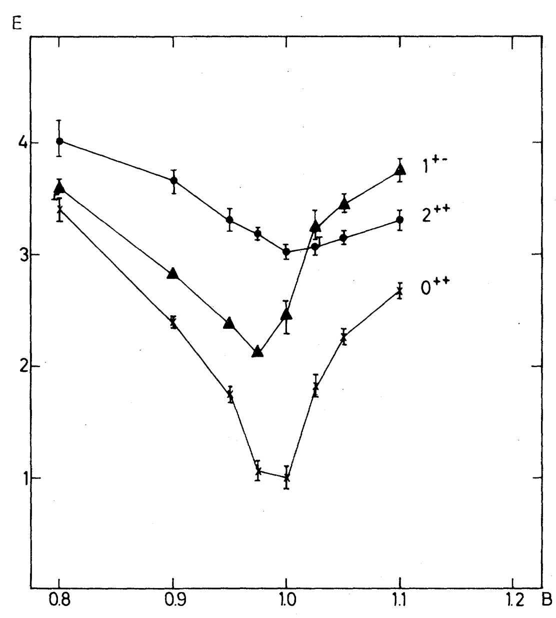

FIG. 2. $E(|\vec{k}| = 0, t = 1)$ for the three lowest-lying states.

TABLE I. $E(k=0,t=2)$ results for 0^{++} and 1^{+-} states. Because of the limited statistics the given error bars are not always reliable. In brackets the used operator as explained in the text is indicated.

β	$E_{A_1^{++}}(0,2)$, (OP)	$E_{T_1^{+-}}(0,2)$, (OP)
0.90	2.4 ± 0.3, (3)	2.9 ± 0.3, (3)
0.95	1.82 ± 0.10, (3)	2.25 ± 0.10, (3)
0.975	0.94 ± 0.10, (2)	1.99 ± 0.06, (3)
1.0	0.83 ± 0.05, (2)	2.27 ± 0.15, (3)
1.025	1.58 ± 0.10, (3)	...
1.05	2.04 ± 0.11, (2)	...
1.1	2.55 ± 0.25, (4)	...

0^{++}, 1^{+-}, 2^{++} (and eventually other) states is indicated with

$$m(0^{++})< m(1^{+-})< m(2^{++}). \tag{7}$$

As in non-Abelian gauge theories it would be pointless to estimate precise mass ratios with the present method. As a result of bad wave functions, ratios at distance $t=1$ are not stable and at distance $t=2$ statistical noise is a severe problem. For small values of β ($\beta=0.8$) our results for 0^{++} are, within statistical errors, in agreement with existing strong-coupling expansion results.[8] Qualitatively our results below the critical point b_c are in agreement with a scenario of a spectrum of massive magnetic monopoles.[9]

Above or near the critical point β_c the short-distance energy definitions $E_j(0,t)$ begin to approach their spin-wave ($\beta\to\infty$) limits. For $t=1$, 2 the values are presumably high. Some leading order ($\beta\to\infty$) calculations were carried out in Refs. 6 and 10. In the present case on a $4^3\times8$ lattice these results read $E_{(1,A_1^{++})}(0,1)\approx3.96$ and $E_{(1,E^{++})}(0,1)\approx3.83$.

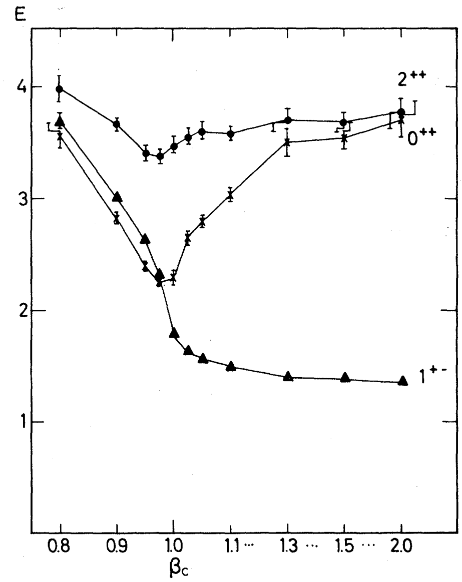

FIG. 3. $E(|\vec{k}|=2\pi/4,t=1)$ for the three lowest-lying states.

Our final Fig. 3 represents results from the one-plaquette operator for the 0^{++}, 1^{+-}, and 2^{++} states with momentum

$$|\vec{k}|=2\pi/4. \tag{8}$$

For the T_1^{+-} representation (this means 1^{+-} axial vector), a dramatic change (as compared with Fig. 2) is observed. For $\beta\to\beta_c$ the T_1^{+-} energy values start to undershoot the A_1^{++} and E^{++} ener-

TABLE II. $E(|\vec{k}|=0,t=1)$ results for A_2^{++}, A_2^{+-}, A_1^{--}, E^{--}, T_2^{++}, T_2^{+-}, T_1^{-+}, T_2^{-+}, and T_2^{--} states.

	$\beta=0.9$	$\beta=1.0$	$\beta=1.1$
A_2^{++}, (OP)	$4.83^{+0.83}_{-0.45}$, (2)	$4.84^{+0.74}_{-0.43}$, (2)	3.95 ± 0.20, (2)
A_2^{+-}, (OP)	>5.48, (4)	>4.85, (4)	$5.68^{+0.87}_{-0.46}$, (4)
A_1^{--}, (OP)	$4.92^{+0.60}_{-0.38}$, (3)	$5.80^{+2.40}_{-0.66}$, (3)	..., (3)
E^{--}, (OP)	$5.29^{+0.67}_{-0.40}$, (3)	$5.02^{+0.58}_{-0.39}$, (3)	..., (3)
T_2^{++}, (OP)	4.58 ± 0.20, (3)	3.76 ± 0.10, (3)	3.77 ± 0.08, (4)
T_2^{+-}, (OP)	$4.98^{+0.46}_{-0.31}$, (2)	$5.38^{+0.67}_{-0.41}$, (2)	4.66 ± 0.25, (2)
T_1^{-+}, (OP)	>5.45, (3)	>5.25, (3)	$5.60^{+1.21}_{-0.54}$, (3)
T_2^{-+}, (OP)	$5.40^{+0.48}_{-0.32}$, (3)	4.25 ± 0.15, (3)	4.23 ± 0.18, (3)
T_2^{--}, (OP)	>6.26, (3)	$5.66^{+0.73}_{-0.43}$, (3)	$5.36^{+0.67}_{-0.39}$, (3)

TABLE III. $E(|\vec{k}| = 2\pi/4, t = 2)$ results for the photon 1^{+-}.

β	$E_{(1,\, T_1^{+-})}(2\pi/4,\, 2)$
0.90	...
0.95	2.70 ± 0.20
0.975	2.22 ± 0.09
1.0	1.63 ± 0.03
1.025	1.51 ± 0.02
1.05	1.44 ± 0.02
1.1	1.41 ± 0.02
1.3	1.37 ± 0.02
1.5	1.36 ± 0.02

gies, and for $\beta > \beta_c$, $\beta \to \infty$ we find

$$E_{(1,T_1^{+-})}(2\pi/4, 1) \to \text{const} \approx 1.38. \tag{9}$$

From the relativistic dispersion law (6) of a free photon we get $[(2\pi/4)^2]^{1/2} = \pi/2 \approx 1.57$, and the discrepancy with (9) is argued to be due to our small spacelike lattice. Indeed replacing equation (6) by $E_R(|\vec{k}|) = [m_R^2 + \sum_i (2 - 2\cos k_i)]^{1/2}$ yields $[2 - 2\cos(2\pi/4)]^{1/2} = \sqrt{2} \approx 1.41$ in good agreement with (9). Distance $t = 2$ results are similar; they are collected in Table III. Furthermore we did a finite-size check at $\beta = 1.3$ on an 8^4 lattice. We carried out 3000 double sweeps with measurement (186 sweeps for equilibrium). The results for the T_1^{+-} state and lowest momentum $|\vec{k}| > 0$ are

$$\begin{aligned} E(2\pi/8, 1) &= 0.870 \pm 0.036, \\ E(2\pi/8, 2) &= 0.780 \pm 0.033, \end{aligned} \tag{10}$$

in good agreement with the relativistic dispersion $[(2\pi/8)^2]^{1/2} = \pi/4 \approx 0.785$.

We interpret the result as clear evidence for a massless photon on the critical line $\beta > \beta_c$. It is amazing that the photon can be detected at short distances in a MC calculation on a finite lattice, although the correlation length is infinite. For momentum $\vec{k} = 0$ the power-law behavior of the correlation function leads at short distances to spin-wave results, which prevent us from seeing massless excitations. By giving a small momentum $\vec{k}$ to our considered states we can, however, clearly project a massless 1^{+-} axial vector. This implies that other excitations in the T_1^{+-} channel have a mass much higher than $2\pi/4$ or decouple from the one-plaquette operator. Otherwise we would not get a good projection onto the energy $E_{\vec{k}} = |\vec{k}|$ from considering correlations at such a short distance like $t = 1$. There is no contradiction between the 1^{+-} behavior in Figs. 2 and 3, because for momentum $\vec{k} = 0$ a power-law behavior is expected. Applying then definition (5) reflects only the short-distance power law and does not give any information about the real mass of the state.

The interested reader may think about doing some further checks. For instance analytic (spin-wave) calculations can be carried out, and one can also consider directly a 1^{--} vector state in a MC simulation, as it follows from the classification of Ref. 6 that there are several length-8 Wilson loops which have an irreducible T_1^{--} representation. Finally the outlined MC procedure may also be useful for detecting massless excitations in other lattice theories, for instance phonons in solid-state physics, and considering momentum eigenstates will certainly also be useful in glueball calculations for non-Abelian lattice gauge theories.

In conclusion we have recovered the massless photon from four-dimensional U(1) lattice gauge theory by means of a MC simulation. This is a nice example for the possible power of MC techniques and a sensitive distinction between Abelian and non-Abelian lattice gauge theories.

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[1]K. G. Wilson, Phys. Rev. D 10, 2445 (1974).
[2]A. H. Guth, Phys. Rev. D 21, 2291 (1980).
[3]J. Fröhlich and T. Spencer, Commun. Math. Phys. 81, 411 (1982).
[4]M. Creutz, L. Jacobs, and C. Rebbi, Phys. Rev. D 20, 1915 (1979).
[5]J. Kogut, D. K. Sinclair, and L. Susskind, Nucl. Phys. B114, 199 (1976).
[6]B. Berg and A. Billoire, Phys. Lett. 114B, 324 (1982), and Nucl. Phys. B221, 109 (1983).
[7]N. Kimura and A. Ukawa, Nucl. Phys. B205 [FS5], 637 (1982).
[8]G. Münster, Nucl. Phys. B190 [FS3], 439 (1981), and 200 [FS4], 536(E) (1982), and 205 [FS5], 648(E) (1982).
[9]See, for instance, H. Kleinert, in "Gauge Interactions: Theory and Experiment," Proceedings of Course 20, International School of Subnuclear Physics, Erice, Trapani, Italy, 1982, edited by A. Zichichi (Plenum, New York, to be published).
[10]B. Berg, A. Billoire, and C. Rebbi, Ann. Phys. (N.Y.) 142, 185 (1982), and 146, 476 (1983).