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Remarks about the Counting of $\bar{\nu}\nu$ Generations Using the $e^+e^- \to \gamma Z^0$ Process

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Abstract. In the present note we further explore the possibility of measuring the number of $\bar{\nu}\nu$ generations by means of $Z^0 \to \bar{\nu}\nu$ decays. We investigate in particular the case where the radiative transitions $e^+e^- \to \gamma Z^0$ are studied on the Z^0 resonance itself. The various background reactions are discussed as well as the statistical significance that can be obtained for an additional $\bar{\nu}\nu$ generation.

I. Introduction

The suggestion to determine the number of existing neutrino generations using the $Z^0 \to \bar{\nu}\nu$ decays from the $e^+e^- \to \gamma Z^0$ transitions has been widely discussed [1–3]. Let us recall that the width Γ_z of the Z^0 gauge boson can be expressed in terms of the number of fundamental fermions. In the standard model [4] with massless neutrinos it is given by:

$$\Gamma_z = \frac{G_F M_z^3}{24\sqrt{2}\pi}\{2N_\nu + [1+(1-4\sin^2\theta_W)^2]N_l + 3[1+(1-\tfrac{8}{3}\sin^2\theta_W)^2]N_{2/3} + 3[1+(1-\tfrac{4}{3}\sin^2\theta_W)^2]N_{-1/3}\}$$

where G_F is the Fermi coupling constant, M_z is the Z^0 mass and θ_W is the Weinberg angle [4]. Here, N_ν is the number of neutrinos, N_l is the number of leptons and $N_{2/3}(N_{-1/3})$ the number of quarks of charge $\frac{2}{3}(-\frac{1}{3})$. Clearly only fermions having masses smaller than $M_z/2$ contribute to the above expressions in which phase space effects have been ignored. For three generations and $\sin^2\theta_W = 0.23$ one obtains $\Gamma_z = 2.6$ GeV whereas additional fermions will lead to an increase of:

0.08 GeV for an additional lepton

0.16 GeV for an additional neutrino

0.27 GeV for an additional charge $\frac{2}{3}$ quark

0.35 GeV for an additional charge $\frac{1}{3}$ quark.

Radiative corrections increase these values and the Z^0 mass by a few percent. These radiative corrections are not considered here.

As already pointed out [2], the detection of an additional (fourth) neutrino as a three standard deviation effect would require the measurement of Γ_z with an accuracy of about 2%. Such a precision is not easy to obtain as one expects to measure the Z^0 excitation curve with a point to point uncertainty of 5 to 10%, yielding thus an error of typically 6% on the determination of Γ_z [5]. In the following we will consider the values of 5% and 7%.

The method of counting the number of neutrino generations by means of the radiative transition $e^+e^- \to \gamma Z^0$ has also been envisaged [1–3]. The idea is to sit at a center-of-mass energy well above the Z^0 mass and to search for $e^+e^- \to \gamma Z^0$ events where $Z^0 \to \bar{\nu}\nu$. Initially it was proposed to carry out this study at a c.m. energy of about 14 GeV above the Z^0 mass [2]. Integrated over the photon momentum spectrum in the range of 12 to 16.5 GeV, the cross section for the $e^+e^- \to \gamma\bar{\nu}\nu$ process was found to be $\sigma \simeq 0.025$ nb. Apart from the inconvenience of dealing with such a small cross section, the running at 14 GeV above the Z^0 does not allow the simultaneous study of the gross features of the Z^0 boson. It was therefore suggested [3,6] to run at the Z^0 itself and to detect the $e^+e^- \to \gamma\bar{\nu}\nu$ processes with photons of low momenta, i.e. $k \lesssim 3$ GeV. In fact benefiting from the $1/k$ behavior of the bremsstrahlung spectrum a

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*** On leave from: Harvard University, Cambridge, Mass., USA. Work supported in part by the U.S. Department of Energy, Contract No. DE-AS02-76ER03227 counting rate about 10 times higher than in the former case can be obtained.

The aim of this note is to further explore the possibility of neutrino-counting while carrying out the experiment at a c.m. energy corresponding to the Z^0 mass. We will in particular discuss the feasibility of such an experiment taking into account the various backgrounds as well as the errors made in the determination of the Z^0 width. The background reactions which will be considered are $e^+e^- \to \gamma\mu^+\mu^-$, γe^+e^-, $\gamma\gamma\gamma$ where one photon is recorded in the detector and where the other outgoing particles escape detection.

2. The $e^+e^- \to \gamma\bar{\nu}\nu$ Process

In order to calculate the $e^+e^- \to \gamma\bar{\nu}\nu$ cross section [$\sigma(\gamma\bar{\nu}\nu)$] we used the differential cross section derived in [1] in the framework of the standard model, i.e.

$$\frac{d^2\sigma(\gamma\bar{\nu}\nu)}{dx\,d\cos\theta_\gamma} = \frac{G_F^2\alpha s(1-x)[(1-x/2)^2 + x^2\cos^2\theta_\gamma/4]}{6\pi^2 x\sin^2\theta_\gamma}\cdot F(x)$$

with

$$F(x) = \frac{N_\nu(g_v^2+g_A^2) + 2(g_v+g_A)[1-s(1-x)/M_z^2]}{[1-s(1-x)/M_z^2]^2 + \Gamma_z^2/M_z^2} + 2.$$

Here $\sqrt{s}$ is the c.m. energy, α is the fine structure constant, g_A and g_v are the axial and vector-coupling constants given by:

$$g_A = -\tfrac{1}{2}$$
$$g_v = -\tfrac{1}{2} + 2\sin^2\theta_W$$

θ_γ is the angle between the outgoing photon and the incident beam direction, and x is the ratio of the photon energy to that of the beam. Taking for the Z^0 mass $M_z = 89$ GeV we integrated the above expression in the domain defined by $|\cos\theta_\gamma| < 0.7$ and $0.25 < k < (\sqrt{s} - 89 + 2.45)$ GeV using $\sqrt{s} = 89$ and 90 GeV. The $|\cos\theta_\gamma|$ range was chosen to allow a good detection efficiency of the photon in any detector whereas the highest $(\sqrt{s} - 89 + 2.45)$ GeV limit is such that the endpoint energy of the γ spectrum corresponds always to the same position on the Z^0 resonance curve (left tail). The calculated cross sections are plotted in Fig. 1 as a function of Γ_z for $N_\nu = 3, 4$. One sees from these plots that $\sigma(\gamma\bar{\nu}\nu)$ is strongly dependent on Γ_z. By comparing the minimum value of the cross section for $N_\nu = 4$ within one standard deviation of Γ_z and the maximum value for $N_\nu = 3$, we obtain Table 1, which gives the precision required in the measurement of $\sigma(\gamma\bar{\nu}\nu)$ in order to detect an additional neutrino generation as a 3 or 5 standard deviation (s.d.) effect. To this end we use errors in Γ_z of 5 and 7%. In both cases the high precision required for the measurement

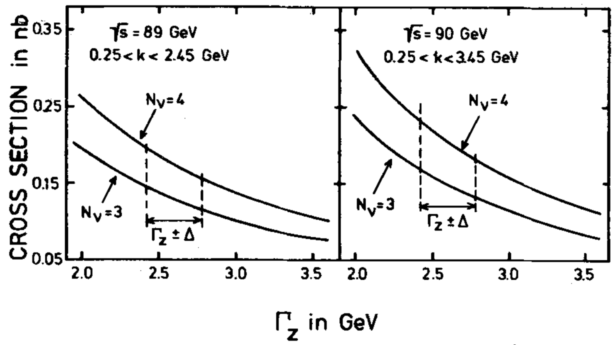

Fig. 1. The $\sigma(\gamma\bar{\nu}\nu)$ cross section as a function of the Z^0 width Γ_z for c.m. energies of $\sqrt{s} = 89, 90$ GeV. Here, N_ν represents the number of neutrinos and Δ is the error on Γ_z(7%) at the value of $\Gamma_z = 2.6$ GeV

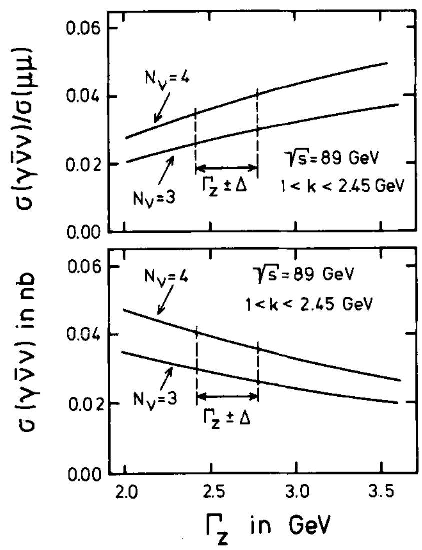

Fig. 2. The $\sigma(\gamma\bar{\nu}\nu)$ and $R_\nu = \sigma(\gamma\bar{\nu}\nu)/\sigma(\mu\mu)$ quantities as a function of Γ_z for a γ reduced momentum spectrum (1.0–2.45 GeV)

of the absolute $\sigma(\gamma\bar{\nu}\nu)$ cross section makes the neutrino-counting difficult if the luminosity is only known within a point to point uncertainty of 5 to 10%. For comparison we also give in Fig. 2 the variation of $\sigma(\gamma\bar{\nu}\nu)$ as a function of Γ_z for $1 < k < 2.45$ GeV (see also Table 1).

To diminish the influence of the luminosity it is desirable to use the cross section ratio

$$R_\nu = \frac{\sigma(\gamma\bar{\nu}\nu)}{\sigma(\mu\mu)}$$

instead of $\sigma(\gamma\bar{\nu}\nu)$ itself. Here the luminosity will only enter in the background subtraction (see below). The $e^+e^- \to \mu^+\mu^-$ cross section $\sigma(\mu\mu)$ is simply obtained by integrating in the $|\cos\theta_\mu| < 0.7$ range the following differential cross section [7]:

$$\frac{d\sigma(\mu\mu)}{d\Omega} = \frac{\alpha^2}{4s}(1+\cos^2\theta_\mu)$$
$$\cdot(1 + 2g_v\mathrm{Re}(X) + (g_v^2+g_A^2)|X|^2)$$

Table 1. Precision (%) with which $\sigma(\gamma\bar{\nu}\nu)$ or $R_\nu = \sigma(\gamma\bar{\nu}\nu)/\sigma(\mu\mu)$ have to be measured in order to detect an extra neutrino as a 3 or 5 standard deviation effect. The calculations were made by assuming two values for the errors on Γ_z, namely 5 and 7%

measured quantity	$\sqrt{s}$ (GeV)	$\sigma(\gamma\bar{\nu}\nu)$ (nb)	momentum limits (GeV)	$\Delta\Gamma_z/\Gamma_z = 0.07$ 3-σ eff. %	$\Delta\Gamma_z/\Gamma_z = 0.07$ 5-σ eff. %	$\Delta\Gamma_z/\Gamma_z = 0.05$ 3-σ eff. %	$\Delta\Gamma_z/\Gamma_z = 0.05$ 5-σ effect %
$\sigma(\gamma\bar{\nu}\nu)$	89	0.130	0.25 – 2.45	3.1	1.8	5.1	3.1
	90	0.150	0.25 – 3.45	2.4	1.5	4.9	2.9
	89	0.028	1.00 – 2.45	7.1	4.3	8.3	5.0
	89.5	0.163	0.25 – 2.95	1.8	1.1	2.5	1.5
$\frac{\sigma(\gamma\bar{\nu}\nu)}{\sigma(\mu\mu)}$	89		0.25 – 2.45	8.7	5.2	9.7	5.8
	90		0.25 – 3.45	9.2	5.5	9.5	5.7
	89		1.00 – 2.45	6.0	3.6	7.7	4.6
	89.5		0.25 – 2.95	11.9	7.1	11.9	7.1

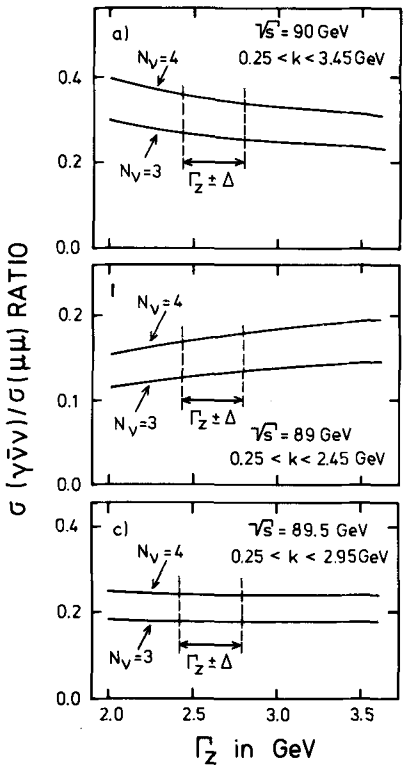

Fig. 3. For $N_\nu = 3,4$ the cross section ratio $R_\nu = \sigma(\gamma\bar{\nu}\nu)/\sigma(\mu\mu)$ as a function of Γ_z for various c.m. energies

where

$$X = \frac{G_E M_z^2}{2\sqrt{2}\pi\alpha}\left[\frac{s}{s - M_z^2 + iM_z\Gamma_z}\right]$$

The term odd in $\cos\theta_\mu$ has been omitted as it does not contribute to the integrated cross section (θ_μ is the angle of the outgoing muon with respect to the beam direction). The ratio R_ν is less dependent on Γ_z (Figs. 2, 3) than $\sigma(\gamma\bar{\nu}\nu)$. This latter fact in conjunction with the smaller influence of the luminosity measurement makes the R_ν ratio very convenient for the determination of the number of neutrinos. Fig. 3 also shows that, for $\sqrt{s} = 89$ GeV, R_ν increases slightly with Γ_z, whereas at $\sqrt{s} = 90$ GeV the situation is opposite. This indicates that one may find a $\sqrt{s}$ value where R_ν ($N_\nu = 3,4$) does not depend on Γ_z at all. For the $0.25 < k < 2.45$ GeV range this occurs at $\sqrt{s} = 89.5$ GeV (Fig. 3c). In Table 1 we also indicate the precision which should be obtained on R_ν in order to detect an additional neutrino. The errors tolerated are now higher than in the case of the $\sigma(\gamma\bar{\nu}\nu)$ measurement.

3. The Background Reactions

We now examine the importance of the various background reactions. Clearly the $e^+e^- \to \gamma e^+e^-, \gamma\gamma\gamma$ reactions may simulate the process under study if the final state is such that only one photon in the required momentum range is detected. Moreover at the center of mass energy corresponding to the Z^0 mass one has also to consider the background induced by the $e^+e^- \to \gamma\mu^+\mu^-$ reaction which has a sizeable cross section (see below). The above reactions are considered as contributing to the background if they also fulfill the following conditions:

$$\left[\begin{array}{ll} e^+e^- \to \gamma\mu^+\mu^- & \\ |\cos\theta_\gamma| < 0.7 & \theta_\mu^\pm < \theta_{\max} \end{array}\right.$$

$$\left[\begin{array}{ll} e^+e^- \to \gamma e^+e^- & \\ |\cos\theta_\gamma| < 0.7 & \theta_e^\pm < \theta_{\max} \end{array}\right.$$

$e^+e^- \to \gamma\gamma\gamma$
One photon having its momentum in the chosen momentum range and being emitted with

$|\cos\theta_i| < 0.7$ whereas the other ones have an emission angle $\theta_{j \neq i} < \theta_{max}$ (θ_i is the angle between the outgoing particle i and the beam direction).

For the veto angle limit θ_{max} we considered two values $\theta_{max} = 20°$ and $30°$, beyond which the outgoing particles can be detected easily. In practice, θ_{max} can be made much smaller by the addition of veto counters.

As already stated above, the $|\cos\theta_\gamma| < 0.7$ limit was chosen to allow a good detection efficiency for the $e^+e^- \to \gamma\bar{\nu}\nu$ reaction. Furthermore the background reactions decrease more rapidly than the $e^+e^- \to \gamma\bar{\nu}\nu$ reaction for increasing θ_γ [8].

In order to estimate the background cross sections we used existing Monte Carlo programs [9]. For the $e^+e^- \to \gamma\mu^+\mu^-$ case the contribution of the Z^0 was taken into account, while for $e^+e^- \to \gamma\gamma\gamma$, the diagram of Fig. 4 was not included as its contribution was found to be small. For radiative Bhabha, to the best of our knowledge, no Monte Carlo program exists yet that includes the Z^0 contribution. However, most of the e^+e^- background events are expected to arise from the t-exchange diagram contributing to the $e^+e^- \to \gamma e^+e^-$ reaction (no Z^0 influence apart from interference terms). The utilisation of the $e^+e^- \to e^+e^-\gamma$ Berends/Kleiss Monte Carlo program requires, however, a great deal of computer time. We have therefore only estimated the inclusive photon cross section by using the Williams–Weizsäcker approximation*. The background cross sections obtained from the $e^+e^- \to \gamma e^+e^-$ reaction (Table 2) are thus overestimated as no tagging condition has been applied to the outgoing $e^\pm$.

All the background cross sections used for the cases which will be considered below are given in Table 2. Because of the uncertainties and to be on the conservative side, we will use in the following a total background cross section (σ_b) which is twice as much as that given in Table 2.

4. Discussion and Conclusions

In order to investigate the feasibility of the neutrino counting experiment at the Z^0 we will consider two limiting cases concerning the luminosity ($\mathcal{L}$) and the running period (T):

$\mathcal{L} = 10^{30}\,\mathrm{cm}^{-2}\,\mathrm{s}^{-1}$, $T = 6$ months,
i.e. $\int \mathcal{L}\,dt \simeq 15.6\,\mathrm{pb}^{-1}$
$\mathcal{L} = 10^{29}\,\mathrm{cm}^{-2}\,\mathrm{s}^{-1}$, $T = 1$ year,
i.e. $\int \mathcal{L}\,dt \simeq 3.1\,\mathrm{pb}^{-1}$

* T. Walsh, private communication: The photon inclusive cross section in this approximation is given by:

$$\sigma = \frac{48}{\pi}\alpha\sigma_0 \ln\left(\frac{E_B}{m_e}\right)\ln\left(\frac{k_{max}}{k_{min}}\right)\frac{|\cos\theta_\gamma|}{1-\cos^2\theta_\gamma}$$

where $k_{max}(k_{min})$ is the maximum (minimum) photon momentum, E_B is the beam momentum and m_e the mass of the electron. Here, σ_0 is the point like $e^+e^- \to \mu^+\mu^-$ cross section at the considered c.m. energy

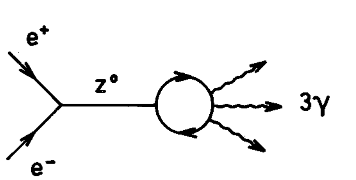

Fig. 4. Diagram which was not taken into account in the $e^+e^- \to 3\gamma$ background calculation. Here the 3 photons are coupled to the Z^0 via a quark or lepton loop

We consider these cases where the luminosity is known with a precision of 2.5%, 5% and 10%. We study here the measurement of the R_ν ratio taking into account the various backgrounds as well as the errors made on the luminosity and on the determination of the width Γ_z($\Delta\Gamma_z/\Gamma_z = 5\%$, 7%).

We give in Tables 3 and 4 the number of standard deviations (s.d.) with which a fourth neutrino will be observed for the various cases. One sees from these tables that at $\sqrt{s} = 89\,\mathrm{GeV}$ an additional generation can be easily detected with the method outlined if $0.25 < k < 2.45\,\mathrm{GeV}$ for the detected photon. This remains true if the background cross sections are twice those given in Table 2 even in the case of the low luminosity ($3.1\,\mathrm{pb}^{-1}$). For the same γ momentum range the number of standard deviations is significantly increased when the c.m. energy is 0.5 GeV above the Z^0 mass. In the framework of our assumptions this corresponds to the position where the results do not depend on the precision reached in the Γ_z measurement. By carrying out an experiment at this c.m. energy one has also the opportunity to study the properties of the Z^0 boson (for $\Gamma_z = 2.6\,\mathrm{GeV}$, the ratio of the peak cross section $\sigma(\sqrt{s} = M_z)$ to that obtained at $\sqrt{s} = M_z + 0.5\,\mathrm{GeV}$ is about 1/0.87).

In order to extract from the experimental R_ν value the number of neutrinos one has also to take into account the c.m. energy dispersion of the e^+e^- collider. In other words, the experimental R_ν values have to be compared with the theoretical ones folded with the energy resolution of the machine. This is a rather trivial procedure as long as the dispersion σ_E (in fact the energy resolution function which we assume to be gaussian) is known. An additional error will of course occur if σ_E itself is only known within a given precision. In order to appreciate the importance of this effect let us consider the case where σ_E is known to within 10%. Even for a large energy dispersion [3] of $\sigma_E = 0.04\sqrt{s}$, the theoretical R_ν value at $\sqrt{s} = M_z$ will suffer an uncertainty of only 0.7%. This uncertainty (which is almost entirely due to the $\sigma(\mu\mu)$ cross section) will not change significantly the number of standard deviations given in Table 3 and 4. For instance, in Table 3 the value of 5.9 s.d. corresponding to $\sqrt{s} = 89\,\mathrm{GeV}$, $\sigma_b = 59\,\mathrm{pb}$, $\Delta\Gamma_z/\Gamma_z = 0.05$, $\Delta\mathcal{L}/\mathcal{L} = 0.05$ and $0.25 < k < 2.45\,\mathrm{GeV}$ will be reduced to 5.2 s.d. The present example shows that the knowledge of σ_E within

Table 2. The background cross sections for $\theta_{max} = 20°$, $30°$ (obtained with the help of the Monte Carlo programs) which will be used below

background	$\sqrt{s} = 89$ GeV $0.25 < k < 2.45$ GeV		$\sqrt{s} = 89$ GeV $1 < k < 2.45$ GeV		$\sqrt{s} = 89.5$ GeV $0.25 < k < 2.95$ GeV	
	$\theta_{max} = 20°$	$\theta_{max} = 30°$	$\theta_{max} = 20°$	$\theta_{max} = 30°$	$\theta_{max} = 20°$	$\theta_{max} = 30°$
$e^+e^- \to \gamma\mu^+\mu^-$	4.2 pb	10.0 pb	1.3 pb	3.5 pb	4.8 pb	10.4 pb
$e^+e^- \to \gamma\gamma\gamma$	6.0 pb	6.2 pb	2.2 pb	2.2 pb	7.5 pb	8.5 pb
$e^+e^- \to \gamma e^+e^-$	43 pb	43 pb	17 pb	17 pb	47 pb	47 pb
Total	53.2 pb	59.2 pb	20.5 pb	22.7 pb	59.3 pb	65.9 pb

Table 3. Statistical significance (in standard deviations) with which a fourth ν will be observed for various cases. The values were obtained with an integrated luminosity of $\int \mathscr{L}\, dt = 15.6\,\mathrm{pb}^{-1}$ corresponding to a six month running period with $\mathscr{L} = 10^{30}\,\mathrm{cm}^{-2}\,\mathrm{s}^{-1}$. Note that with our assumptions the calculated values do not depend on Γ_z at the c.m. energy of $\sqrt{s} = 89.5$ GeV

$\sqrt{s}$ (GeV)	γ spectrum range (GeV)	$\sigma(\gamma\bar{\nu}\nu)$ (nb)	σ_b (pb)	$\Delta\Gamma_z/\Gamma_z$	Statistical significance for a 4th $\bar{\nu}\nu$ generation (s.d.) $\frac{\Delta\mathscr{L}}{\mathscr{L}} = 0.10$	$\frac{\Delta\mathscr{L}}{\mathscr{L}} = 0.05$	$\frac{\Delta\mathscr{L}}{\mathscr{L}} = 0.025$
89	0.25 – 2.45	0.13	53.2	0.07	3.9	5.6	7.2
				0.05	4.3	6.2	8.0
89	0.25 – 2.45	0.13	59.2	0.07	3.6	5.3	6.9
				0.05	4.0	5.9	7.7
89	0.25 – 2.45	0.13	2 × 53.2	0.07	2.3	3.7	5.2
				0.05	2.6	4.1	5.8
89	0.25 – 2.45	0.13	2 × 59.2	0.07	2.2	3.4	4.9
				0.05	2.4	3.8	5.5
89.5	0.25 – 2.45	0.163	59.3		6.0	8.6	11.0
89.5	0.25 – 2.45	0.163	65.9		5.6	8.1	10.6
89.5	0.25 – 2.45	0.163	2 × 59.3		3.6	5.7	8.0
89.5	0.25 – 2.45	0.163	2 × 65.9		3.3	5.3	7.6
89	1.0 – 2.45	0.028	20.5	0.07	1.3	1.8	2.2
				0.05	1.7	2.3	2.9
89	1.0 – 2.45	0.028	22.7	0.07	1.2	1.7	2.1
				0.05	1.6	2.2	2.7

10% appears to be acceptable. In any case in order to determine the number of $\bar{\nu}\nu$ generations one has to know the energy resolution function of the collider and eventually its evolution as a function of time.

To summarize we have shown that the measurement of the number of neutrinos can be carried out by means of the $e^+e^- \to \gamma Z^0$ transitions studied on the Z^0 resonance itself. We used rather pessimistic estimates for the background induced by the reactions $e^+e^- \to \gamma\mu^+\mu^-$, γe^+e^-, $\gamma\gamma\gamma$. A more accurate estimation of the back-ground cross sections will be one of the most crucial problems to be solved. This could be done for instance by using the theoretical work of [11] as the starting point.

The present note has also shown that one has to detect photons of low momentum, preferably in the 0.25 – 2.45 GeV region. The extraction of the number of neutrinos from the data is based on the comparison

Table 4. Same as in Table 3 but with a one year running period using a luminosity of $\mathscr{L} = 10^{29}\,\mathrm{cm}^{-2}\,\mathrm{s}^{-1}$ corresponding to $\int \mathscr{L}\,dt \simeq 3.1\,\mathrm{pb}^{-1}$

$\sqrt{s}$ (GeV)	γ spectrum range (GeV)	$\sigma(\gamma\bar{\nu}\nu)$ (nb)	σ_b (pb)	$\Delta\Gamma_z/\Gamma_z$	Statistical significance for a 4th $\bar{\nu}\nu$ generation (s.d.) $\frac{\Delta\mathscr{L}}{\mathscr{L}}=0.10$	$\frac{\Delta\mathscr{L}}{\mathscr{L}}=0.05$	$\frac{\Delta\mathscr{L}}{\mathscr{L}}=0.025$
89	0.25 – 2.45	0.13	53.2	0.07	2.6	3.3	3.8
				0.05	2.9	3.7	4.2
89	0.25 – 2.45	0.13	59.2	0.07	2.5	3.2	3.7
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89	0.25 – 2.45	0.13	2 × 53.2	0.07	1.8	2.4	3.0
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89.5	0.25 – 2.45	0.163	65.9		3.8	4.9	5.7
89.5	0.25 – 2.45	0.163	2 × 59.3		2.7	3.8	4.7
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				0.05	1.1	1.3	1.5
89	1.0 – 2.45	0.028	22.7	0.07	0.8	1.0	1.1
				0.05	1.0	1.3	1.4

of the experimental $R_\nu = \sigma(\gamma\bar{\nu}\nu)/\sigma(\mu\mu)$ value with the theoretical predictions. As a further cross check it is also essential to study the photon spectrum shape and to compare it with the theoretical prediction in order to verify the validity of the underlying assumptions. The detector chosen for a neutrino counting experiment should thus have a good energy resolution for low momentum photons.

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