

Proton Capture by Magnetic Monopoles

K. Olaussen

CERN, CH-1211 Genève 23, Switzerland, and Institute of Theoretical Physics, University of Trondheim, Norges Tekniske Høgskole, N-7034 Trondheim, Norway^(a)

and

H. A. Olsen

Institute of Physics, University of Trondheim, Norges Lærerhøgskole, N-7055 Dragvoll, Norway

and

P. Osland

Deutsches Elektronen-Synchrotron DESY, D-2000 Hamburg 52, Germany

and

I. Øverbø

Institute of Physics, University of Trondheim, Norges Lærerhøgskole, N-7055 Dragvoll, Norway
(Received 3 October 1983)

The cross section for radiative capture of protons by monopoles is calculated by the use of bound-state and scattering wave functions obtained with the Kazama-Yang Hamiltonian, i.e., with a pointlike proton. For proton velocities $\beta=10^{-5}$ – 10^{-3} , the cross sections for capture into the lowest bound states, with binding energies of 938 MeV, 263 keV, and 105 eV, are found to be of the order of 10^{-28} – 10^{-26} cm². For the state with binding energy of 263 keV, the capture length in water is found to be $[170(\beta/10^{-4})^{0.48}]$ m. Observation of photons from the capture process would indicate the presence of monopoles.

PACS numbers: 14.80.Hv, 25.90.+k, 36.10.-k

The Kazama-Yang Hamiltonian¹ leads to the prediction of bound states consisting of a monopole and a fermion. While this Hamiltonian requires a nonzero anomalous magnetic moment, it does not account for other structure of finite-size fermions like the proton, and applies only to Dirac monopoles. There are two reasons why the Kazama-Yang Hamiltonian gives only an approximate description of the monopole-proton system: (i) The monopoles of grand unification² also have an SU(3) color magnetic charge. This charge is believed to be screened by the vacuum at distances much larger than $\Lambda_{\text{QCD}}^{-1} \approx 1$ fm. Since the rms radius of the first excited ($n=1$) bound state is about 10 fm, and larger for the more weakly bound states, and since the proton is a color singlet, we believe the presence of a color magnetic charge to have little influence on the system. (ii) The proton has a finite size. Thus, at short distances the anomalous magnetic moment should be described by a finite distribution taking into account the granular structure and magnetic polarizability. Since such finite-size effects will fall off exponentially as the proton-monopole distance increases, they will lead to only a small perturbation of the ($n \geq 1$) bound-state wave functions in the region where they are large.³ Therefore, our calculation of capture

cross sections should not be very sensitive to finite-size effects. The binding energies are more sensitive to the behavior at short distances, because of the singular nature of the $1/r^2$ potential.

In the case of the states of lowest angular momentum, $j = |eg| - \frac{1}{2}$, $eg = \pm \frac{1}{2}, \pm 1, \dots$, the bound-state spectrum has been investigated by Kazama and Yang,¹ and by the present authors.³ For protons, with anomalous magnetic moment $\kappa = 1.79$, and monopoles of minimal magnetic charge, $|eg| = \frac{1}{2}$, some properties of the lowest bound states are given in Table I.

With analytic (albeit approximate) results for the bound-state wave function,³ it is straightforward to calculate the cross sections for ra-

TABLE I. The lowest monopole-proton bound states, characterized by binding energy (E_B), size (r_{rms}), and wave length (λ) of the photon emitted in radiative capture from an initial state of zero kinetic energy.

n	E_B	r_{rms}	λ
3	0.04 eV	0.23 Å	3×10^5 Å
2	105 eV	460 fm	120 Å
1	263 keV	9 fm	0.048 Å
0	938 MeV		

diative capture,

monopole + free proton \rightarrow monopole + bound proton + photon.

The proton anomalous magnetic moment is not large enough for bound states of higher angular momenta to exist.⁴ The bound states will thus have angular momentum $j=0$, and the capture will predominantly take place from initial states of $j=1$.

As opposed to the case of mesonic and other "exotic" (Coulombic) atoms, there will be no cascade since the monopole-proton bound states all have $j=0$. Following capture to an excited state, the system can only relax via collisional deexcitation or two-photon emission. What one might hope to detect, therefore, would be just single photons.

For nonrelativistic monopoles of velocity β , the photon energies would be given by

$$\omega = E_n + \frac{1}{2}M\beta^2, \quad (1)$$

with E_n the binding energy as given in the table, and M the proton mass. If the monopole velocity is low, $\beta \lesssim 10^{-3}$, capture to the lowest states ($n \leq 1$) would yield monochromatic and characteristic photons.

The capture cross section is given by

$$d\sigma = (2\pi/\beta) |M_{fi}|^2 \delta(\frac{1}{2}M\beta^2 + E_n - \omega) d^3k / (2\pi)^3 2\omega, \quad (2)$$

with

$$M_{fi} = e \int d^3r \psi_{\text{bound}}^\dagger(\vec{r}) (\vec{\alpha} \cdot \vec{e}^*) e^{-i\vec{k} \cdot \vec{r}} \psi_{\text{in}}(\vec{r}). \quad (3)$$

The zero-energy ($n=0$) bound-state wave function has been given in Ref. 1, and approximate results for the excited-state ($n \geq 1$) wave functions have been given in Ref. 3. While the latter are not exact, they are highly accurate in the present case of weak binding, $E_n \ll M$, and have been adopted for our calculation.

For the initial states we have considered three approximations: (i) plane waves, (ii) the Kazama-Yang-Goldhaber wave functions,⁵ and (iii) a modified version of the Kazama-Yang-Goldhaber wave functions, which takes into account the pro-

ton anomalous magnetic moment. We shall shortly return to a description of these.

In the nonrelativistic limit, $\beta \ll 1$, incident plane waves lead to a matrix element proportional to β , and hence to a cross section $\sigma \sim \beta$. This plane-wave approximation turns out to be inadequate; the distortion of the incident wave due to the interaction of the monopole with the magnetic moment is very strong, and results in a quite different β dependence for the cross section.

The modified version of the Kazama-Yang-Goldhaber wave functions that we have used for the cross sections presented in Fig. 1 is obtained by solving the radial equations¹ in the exterior region, i.e., for $r \gg 1/M$, for $\kappa \neq 0$. We find, for $1 \ll Mr \ll 1/\beta$,

$$\psi_{jm} = \frac{1}{pr} \left[\begin{array}{l} f(r) [\cos(\varphi) \xi_{jm}^{(1)} - \sin(\varphi) \xi_{jm}^{(2)}] \\ -i [g_+(r) \sin(\varphi) \xi_{jm}^{(1)} - g_-(r) \cos(\varphi) \xi_{jm}^{(2)}] \end{array} \right], \quad (4)$$

with

$$f(r) = (pr)^{1/2} J_\nu(pr), \quad g_\pm(r) = \frac{\beta}{8\nu_-} [1 + 2(\nu_+ \mu)] (pr)^{1/2} J_{\nu_- - 1}(pr), \quad (5)$$

$$\nu_- = [\mu^2 + \frac{1}{4} - (\mu^2 + q^2 \kappa^2)^{1/2}]^{1/2}, \quad (6)$$

$$\tan \varphi = [(\mu^2 + q^2 \kappa^2)^{1/2} - \mu] / \kappa q. \quad (7)$$

Here $\mu = [(j + \frac{1}{2})^2 - q^2]^{1/2}$, $q = eg$, and $\xi^{(1)}$ and $\xi^{(2)}$ are the two-component eigensections introduced in Ref. 5. For a monopole of minimal magnetic charge, one has $q = \pm \frac{1}{2}$.

The full solution in the exterior region, $r \gg 1/M$, also contains terms given by Bessel functions of the order $\nu_- + 1$, and of the orders ν_+ , $\nu_+ - 1$, and $\nu_+ + 1$ {where $\nu_+ = [\mu^2 + \frac{1}{4} + (\mu^2 + q^2 \kappa^2)^{1/2}]^{1/2}$ }, as well as of the corresponding negative orders. Since we are here only interested in small velocities β , the latter are unimportant. The matching with the interior region⁶ takes place at such a small value of $pr = \beta Mr$ that the negative-order Bessel functions are already large and therefore acquire very small coefficients.⁷ The remaining (positive-order) Bessel functions contribute to higher order in βMr and

therefore are negligible in the region $1 \ll Mr \ll 1/\beta$.

In the limit

$$\beta \ll (2\epsilon_n)^{1/2}, \quad (8)$$

which corresponds to the incoming-proton kinetic energy $p^2/2M$ (in the monopole rest system) being much smaller than the binding energy, $\epsilon_n M = E_n$, the matrix elements can be evaluated analytically. The cross section is given in this limit by

$$\sigma_n(\beta) = (\alpha/M^2)(2\epsilon_n/\beta^2)^{1-\nu} C_q, \quad n \geq 1, \quad (9)$$

where (for $q = \frac{1}{2}$)

$$C_{1/2} = \frac{2\pi^2}{3} \frac{\sinh(\pi\bar{\beta})}{\pi\bar{\beta}} \left[\frac{1}{\Gamma(\nu_- + 1)} \left| \Gamma\left(\frac{3+\nu_- + i\bar{\beta}}{2}\right) \right|^2 \right]^2 (1 + \sin 2\varphi) \quad (10)$$

and the bound state is characterized by³

$$\bar{\beta} = \frac{1}{2}(2\kappa - 1)^{1/2}. \quad (11)$$

With $\kappa = 1.79$, we find

$$\sigma_n = 0.628(2\epsilon_n/\beta^2)^{0.24} \times 10^{-28} \text{ cm}^2; \quad n \geq 1. \quad (12)$$

For capture to the lowest excited state, with $\epsilon_1 = 2.808 \times 10^{-4}$ (see Ref. 3), we get

$$\sigma_1 = 8.75(10^{-4}/\beta)^{0.48} \times 10^{-28} \text{ cm}^2. \quad (13)$$

For capture to the zero-energy state,⁸ we shall here only quote an estimate (for $\beta \ll 1$), obtained with the wave function of Eq. (4), which can at best be a rough approximation for the small distances involved here:

$$\sigma_0 \approx 6(10^{-4}/\beta)^{0.48} \times 10^{-27} \text{ cm}^2. \quad (14)$$

Results of a numerical evaluation of the cross section are given in Fig. 1 for the capture to the lowest states, for velocities in the range $10^{-5} \leq \beta \leq 10^{-3}$.

In addition to the two-body bound states considered in Refs. 1 and 3, it has also been shown that there exist three-body bound states consisting of a monopole, a proton, and an electron.¹⁰ The latter states have atomic dimensions and, if the monopole velocity is sufficiently low, capture cross sections are presumably of the order of atomic capture cross sections. These monopole molecules¹⁰ have a binding energy of the order of 1 eV, which is small compared to the kinetic energy of protons incident on the monopole at relative velocities $\beta \geq 10^{-4}$. We therefore believe that the formation of monopole molecules does not significantly impede the capture to the more strongly bound states discussed above, unless $\beta^2 \lesssim 2\epsilon_{\text{molecule}}$, where $\epsilon_{\text{molecule}} \approx 1 \text{ eV}/938 \text{ MeV} \approx 10^{-9}$.

Let us now consider more specifically the $n = 1$ state ($E_B = 263 \text{ keV}$).⁸ At a velocity $\beta = 10^{-4}$, the cross section for capture to this state is 8.75

$\times 10^{-28} \text{ cm}^2$, and the capture length in water is $L = [2\sigma_{\text{capt}} n_{\text{H}_2\text{O}}]^{-1} \approx 171 \text{ m}$. Even if the monopole has previously captured a hydrogen atom or a proton in a higher excited state, those binding energies are so small that the probability that an $n = 1$ state is formed is essentially as if the monopole were free. We therefore expect that after penetrating an average depth of $\sim 170 \text{ m}$ of water, an $n = 1$ state is formed, with the emission of a photon. Observation of these monochromatic pho-

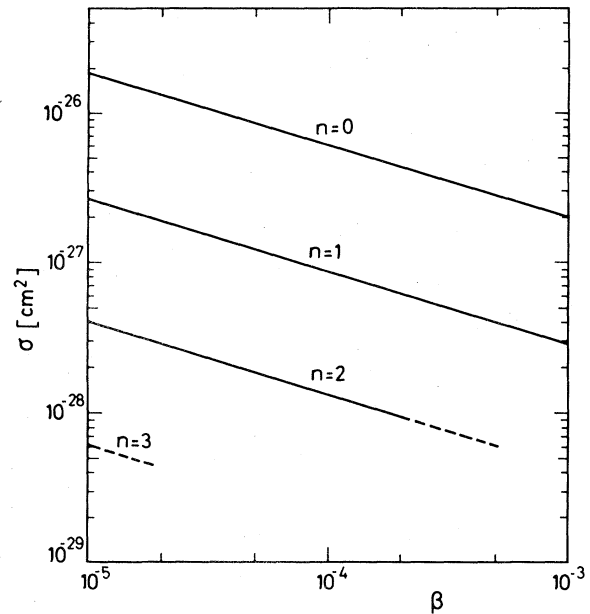


FIG. 1. Cross sections for radiative capture of protons by magnetic monopoles. The labels n refer to the various bound states characterized in Table I. The result for $n = 0$ is a rough estimate. For $n \geq 1$, an evaluation using the Kazama-Yang-Goldhaber relativistic wave functions indicates that the present nonrelativistic approximation is valid to within 5% where the curves are drawn solid, and that it holds within a factor of 2 where they are dashed.

tons would thus indicate the existence of magnetic monopoles capturing protons. If the lowest-energy state exists, the emission of 938-MeV photons would be even more spectacular events.

With $\beta \leq 10^{-3}$, the available energy in subsequent collisions of the bound monopole-proton system with other atoms, $T_{\text{kin}} = \frac{1}{2}M_{\text{atom}}\beta^2$, would not be high enough to cause the $n = 1$ monopole-proton system to break up, in contrast to the more weakly bound states.

We have not carried out any further analysis of how the $n = 1$ monopole-proton system would interact with matter. Such a system might be expected to ionize (or excite) matter more strongly than a bare monopole, because of the proton charge. However, it appears that this system would tend to pick up an electron to regain electric neutrality. The energies by which the electron would bind to the monopole-proton system would be close to those of the hydrogen atom.

It is a pleasure to thank T. F. Walsh and T. T. Wu for many useful discussions. This research has been supported in part by Norges Almenvitenskapelige Forskningsråd.

^(a)Permanent address.

¹Y. Kazama and C. N. Yang, Phys. Rev. D 15, 2300 (1977).

²G. 't Hooft, Nucl. Phys. B79, 276 (1974); A. M. Polyakov, Pis'ma Zh. Eksp. Teor. Fiz. 20, 430 (1974)

[JETP Lett. 20, 194 (1974)]; P. Goddard and D. Olive, Rep. Prog. Phys. 41, 1357 (1978); C. P. Dokos and T. N. Tomaras, Phys. Rev. D 21, 2940 (1980).

³K. Olaussen, H. A. Olsen, P. Osland, and I. Øverbø, Nucl. Phys. B228, 567 (1983).

⁴C. N. Yang, in *Monopoles in Quantum Field Theory*, edited by N. S. Craigie, P. Goddard, and W. Nahm (World Scientific, Singapore, 1982), p. 237.

⁵Y. Kazama, C. N. Yang, and A. S. Goldhaber, Phys. Rev. D 15, 2287 (1977).

⁶Approximate wave functions for the monopole-fermion weakly bound states of nonminimal angular momenta have been found by P. Osland, C. L. Schultz, and T. T. Wu (to be published). The interior-region scattering states are for $\beta \ll 1$ the same as those for the weakly bound states.

⁷A relativistic calculation, taking fully into account the proton anomalous magnetic moment for the scattering states, will be published elsewhere (K. Olaussen, H. A. Olsen, P. Osland, and I. Øverbø, to be published). That calculation will thus extend our results to higher monopole velocities.

⁸For SU(5) monopoles and elementary fermions, the zero-energy state has been shown not to exist by T. F. Walsh, P. Weisz, and T. T. Wu, DESY Report No. DESY 83-022, 1983 (to be published). It is thus unclear whether such a state would exist for SU(5) monopoles and protons.

⁹Cosmological arguments suggest that massive monopoles would typically have velocities in this range; see, e.g., S. Dimopoulos, S. L. Glashow, E. M. Purcell, and F. Wilczek, Nature (London) 298, 824 (1982); M. S. Turner, E. N. Parker, and T. J. Bogdan, Phys. Rev. D 26, 1296 (1982).

¹⁰Th. W. Ruijgrok, J. A. T'jon, and T. T. Wu, Phys. Lett. 129B, 209 (1983).