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Higher Order QCD Corrections to the Energy–Energy Correlation Function

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Abstract. We calculate the $O(\alpha_s^2)$ correction to the energy–energy correlation cross section using a Sterman–Weinberg type resolution criterion to account for higher-order/nonperturbative effects. We find that the energy–energy correlation function as well as the asymmetry is sensitive to the choice of resolution parameters.

Basham et al. [1] first introduced energy–energy correlations in e^+e^- annihilation as a possible test of perturbative QCD. Experimentally, one measures the energy weighted correlation defined by

$$\frac{1}{\sigma}\frac{d\Sigma}{d\cos\chi}=\frac{2}{\Delta\chi\sin\chi W^2}\cdot\frac{1}{N}\sum_{A=1}^{N}\sum_{\substack{\text{pairs}\\ \text{in}\Delta\chi}}E_{Aa}E_{Ab}, \tag{1}$$

where χ is the relative angle between two calorimeters and the index specifies the event (1 to N), while a and b specify the individual particles.

In QCD perturbation theory the correlation function can be written to second order in α_s:

$$\frac{1}{\sigma_0}\frac{d\Sigma}{d\cos\chi}=\frac{\alpha_s(W^2)}{\pi}C(\cos\chi)+\left(\frac{\alpha_s(W^2)}{\pi}\right)^2 D(\cos\chi). \tag{2}$$

The first order term $C(\cos\chi)$ receives its contribution from the well-known one-gluon emission diagrams and reads [1]

$$C(\cos\chi)=\frac{3-2\zeta}{6\zeta^5(1-\zeta)}\cdot[2(3-6\zeta+2\zeta^2)\ln(1-\zeta)+3\zeta(2-3\zeta)], \tag{3}$$

where $\zeta=\frac{1}{2}(1-\cos\chi)$. The second order term $D(\cos\chi)$ receives contributions from graphs with $q\bar{q}gg$ and $q\bar{q}q\bar{q}$ final states and also from graphs which consist of the virtual corrections to the $q\bar{q}g$ final state. These 4- and 3-parton cross sections are individually infrared and collinear divergent. To cancel these singularities one introduces a resolution dependent 3-jet cross section

$$d\sigma^{\text{3-jet}}(\varepsilon,\delta)=d\sigma^{\text{3-parton}}+d\sigma^{\text{4-parton}}(\varepsilon,\delta), \tag{4}$$

where $d\sigma^{\sigma\text{-parton}}$ is the $O(\alpha_s^2)$ cross section with 3 partons ($q\bar{q}g$) in the final state, and where $d\sigma^{\text{4-parton}}$ stands for the cross section for $e^+e^-\to q\bar{q}gg$ and $e^+e^-\to q\bar{q}q\bar{q}$, in which one of the partons is not resolved in the Sterman–Weinberg sense, i.e. falls inside the cone of (full) opening angle δ and/or its energy is $\leqq\varepsilon W/2$. This resolution dependent 3-jet cross sections has been calculated analytically [2, 3]. It depends on two scaled energies $x_{1,2}=2E_i/W$ like the lowest order 3-jet cross section. It is straightforward now to calculate the energy–energy correlation from (4). It represents the $O(\alpha_s^2)$ 3-jet contribution to $d\Sigma/d\cos\chi$, to which the genuine (hard) 4-jet contribution has to be added. That is the cross section for the production of 4 partons which fail the ε,δ cuts, and which is obtained by a simple Monte Carlo integration. Altogether $D(\cos\chi)$ is schematically given by

$$D(\cos\chi)=\int dP_3\sum_{i,j=1}^{3}\frac{E_iE_j}{W^2}\cdot\delta(\hat{\mathbf{p}}_i\hat{\mathbf{p}}_j-\cos\chi)\frac{d\sigma^{\text{3-jet}}(\varepsilon,\delta)}{dP_3}+\int dP_4\sum_{i,j=1}^{4}\frac{E_iE_j}{W^2}\cdot\delta(\hat{\mathbf{p}}_i\hat{\mathbf{p}}_j-\cos\chi)\frac{d\sigma^{\text{4-jet}}(\varepsilon,\delta)}{dP_4} \tag{5}$$

where $dP_3(dP_4)$ denotes the 3-body (4-body) phase space integration for massless quanta. Both $d\sigma^{3\text{-jet}}(\varepsilon,\delta)$ and $d\sigma^{4\text{-jet}}(\varepsilon,\delta)$ depend strongly on ε and δ; the 3-jet cross section decreases for $\varepsilon,\delta\to 0$ like $\ln\varepsilon\ln\delta^2$ whereas the 4-jet cross section increases like $\ln\varepsilon\ln\delta^2$. In the sum (5) this strong ε,δ dependence cancels (and also the subdominant single logarithmic terms $\sim\ln\varepsilon$ and $\sim\ln\delta^2$).

The hope is that (5) is only weakly ε,δ dependent. This was not the case for the thrust distribution, which varied by a factor 2 to 7 depending on the thrust value for ε,δ going from 0.2, 40° to 0 [3]. We have calculated $D(\cos\chi)$ for two sets of ε,δ, a large one, $\varepsilon,\delta=0.15, 15°$, and a small one, $\varepsilon,\delta=0.05, 5°$. The results are shown in Fig. 1. We see that $D(\cos\chi)$ changes roughly by a factor of 2 to 3. This is comparable to what has been obtained for the thrust distribution for $T\lesssim 0.8$. We have not tried to calculate the limiting value for $\varepsilon,\delta\to 0$. Comparing with the results of Ali and Barreiro [4], who just computed $D(\cos\chi)$ for $\varepsilon,\delta=0$, $D(\cos\chi)$ is *still 50% smaller for* $\varepsilon,\delta=0.05$, 5°. In Fig. 1 we also show the lowest order contribution $C(\cos\chi)$ for $\varepsilon,\delta=0.15$, 15° and $\varepsilon,\delta=0$. Applying the ε,δ cut to $C(\cos\chi)$ means that one excludes the 2-jet contribution from the $q\bar{q}g$ final state (whereas in $D(\cos\chi)$ it amounts to using ε,δ for separating 3- and 4-jet contributions and excluding the two-jet region). The ratio $D(\cos\chi)/C(\cos\chi)$ for $\varepsilon,\delta=0.15, 15°$ is only 4, so that the effect of the higher order correction in (2) is only 20% for $\alpha_s/\pi=0.05$. This correction is somewhat smaller than in the inclusive thrust distribution for similar ε,δ values [3].

Let us now consider the asymmetry cross section

$$\mathcal{A}(\cos\chi)=\frac{1}{\sigma_0}\left(\frac{d\Sigma}{d\cos\chi}(-\cos\chi)-\frac{d\Sigma}{d\cos\chi}(\cos\chi)\right), \quad (6)$$

which we decompose also into $O(\alpha_s)$ and $O(\alpha_s{}^2)$ contributions:

$$\mathcal{A}(\cos\chi)=\frac{\alpha_s(W^2)}{\pi}A(\cos\chi)+\left(\frac{\alpha_s(W^2)}{\pi}\right)^2 B(\cos\chi). \quad (7)$$

$B(\cos\chi)$ is plotted in Fig. 2 for $\varepsilon,\delta=0.05, 5°$ and $\varepsilon,\delta=0.15, 15°$. $A(\cos\chi)$ is given for $\varepsilon,\delta=0$ only as it is very little dependent on these parameters (which means that the 2-jet region of the $q\bar{q}g$ final state does not contribute, if ε,δ are not too large). The ratio $B(\cos\chi)/A(\cos\chi)$ for $\varepsilon,\delta=0.15, 15°$ is of the same order as D/C, which means that higher order corrections contribute only 20% here too. If ε,δ are varied from $\varepsilon,\delta=0.15, 15°$ to $\varepsilon,\delta=0.05, 5°$, $B(\cos\chi)$ *increases by roughly a factor of* 2 similar to what we obtained for $D(\cos\chi)$. This statement is rigorous only for $|\cos\chi|\gtrsim 0.5$ as our $\varepsilon,\delta=0.05, 5°$ results have too large (Monte Carlo) statistical errors in the range $0\leqq|\cos\chi|\lesssim 0.5$. Our results for $B(\cos\chi)$ and $\varepsilon,\delta=0.05, 5°$ agree approximately with the results of [4] and [5] where $B(\cos\chi)$ was calculated for $\varepsilon,\delta=0$.

We conclude that the $O(\alpha_s{}^2)$ contributions

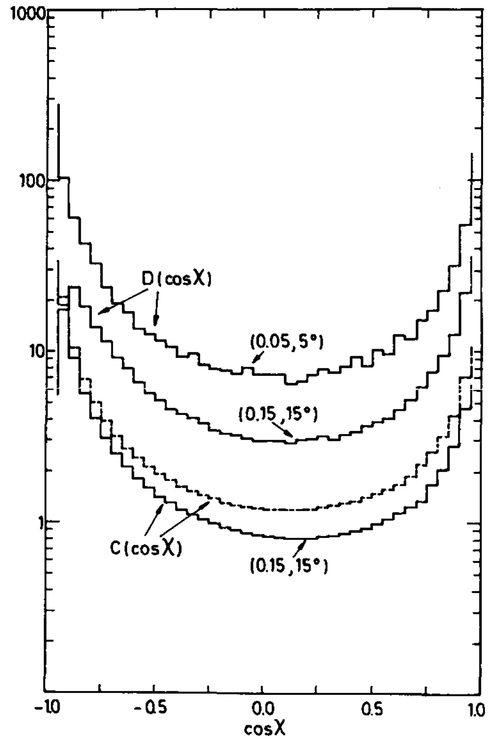

Fig. 1. First and second order contributions $C(\cos\chi)$ and $D(\cos\chi)$ to the energy–energy correlation for $\varepsilon,\delta=0.05, 5°$ and $\varepsilon,\delta=0.15, 15°$

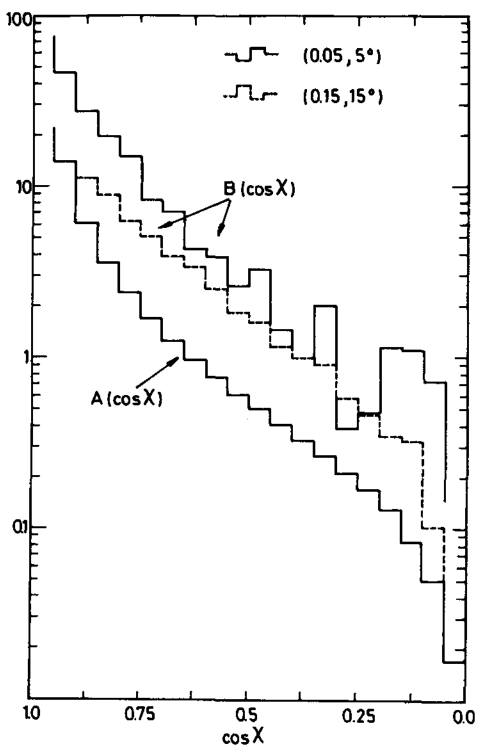

Fig. 2. First and second order contributions $A(\cos\chi)$ and $B(\cos\chi)$ to the asymmetry $\mathcal{A}$ $(\cos\chi)$ of the energy–energy correlation for $\varepsilon,\delta=0.05$, 5° and $\varepsilon,\delta=0.15$, 15°

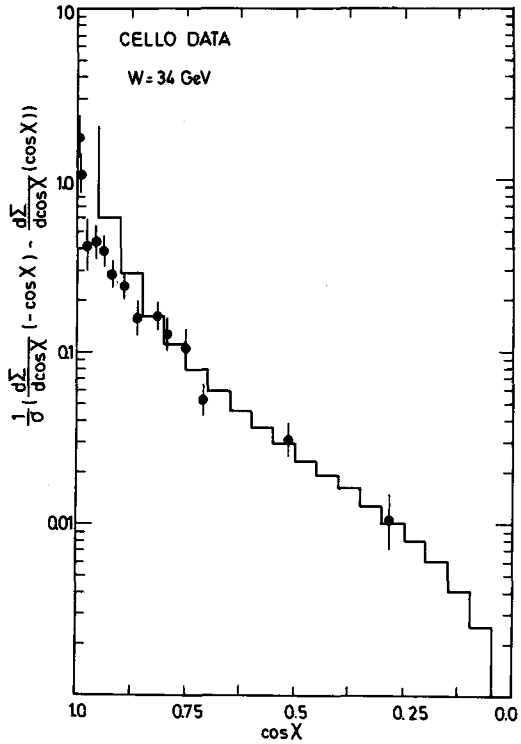

Fig. 3. Comparison of the asymmetry $\mathscr{A}(\cos\chi)$ of the energy–energy correlation for $\varepsilon, \delta = 0.2, 30°$ and $\alpha_s = 0.14$ with the CELLO data

to the energy–energy correlation and to the asymmetry also show an appreciable resolution dependence similar to the inclusive thrust distribution. As the higher order corrections are somewhat smaller than for the thrust distribution, this resolution dependence is, however, of less importance.

To compare our theoretical results to the data we also have calculated $\mathscr{A}(\cos\chi)$ for somewhat larger resolution parameters, which are more appropriate to the nonperturbative jet parameters at PETRA energies [3]. We have chosen $\varepsilon, \delta = 0.2, 30°$ and computed $\mathscr{A}(\cos\chi)$ for $\alpha_s = 0.14$. The result is shown in Fig. 3. The theoretical curve describes well recent CELLO data for $|\cos\chi| < 0.8$ [6]. In this comparison we have to keep in mind that the data are for hadrons and not for jets. It is known that this is responsible for the lower experimental (than theoretical) cross section for $|\cos\chi| > 0.8$. Otherwise the effect of fragmentation is small. We estimate that α_s increases by not more than 10% if independent jet fragmentation is applied [4]. For string fragmentation the effect may be larger.

In the theoretical cross section we excluded the 2-jet region in the lowest order term $A(\cos\chi)$. If we had included this contribution too, the value of α_s would have been $\alpha_s = 0.135$, a very small change indeed. We may also ask what the effect of the jet resolution for 3-jets on α_s is. For $\varepsilon, \delta = 0.05, 15°$ we obtain from the asymmetry $\alpha_s = 0.125$. We see that the resolution dependence is small when determining α_s from the asymmetry [7].

The actual values of α_s, which we obtained by fitting the CELLO data, we repeat, are subject to small changes as fragmentation effects were not properly taken into account (although the CELLO data were corrected for decays and other effects). Since such analysis is being pursued by the CELLO Collaboration [8], we see no point in going further into this matter.

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