ON THE RESTORATION OF LORENTZ INVARIANCE IN SU(2) AND SU(3) LATTICE GAUGE THEORIES

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We calculate the energy-momentum dispersion relation for the $0^{+(+)}$ glueball in SU(2) and SU(3) lattice gauge theories. We find that this relation takes the usual relativistic continuum form $E^2 = p^2 + m^2$ for $\beta \geqslant 2.2$ in the case of SU(2) and $\beta \geqslant 5.5$ in the case of SU(3), thus demonstrating the dynamical restoration of Lorentz invariance. We obtain similar results, albeit with larger statistical errors, for the $2^{+(+)}$ glueball.

The success of lattice [1] Monte Carlo [2] calculations in illuminating the physics of continuum QCD has been made possible by the apparently precocious onset of continuum behaviour: typical non-perturbative quantities seem to have become insensitive to the lattice structure for lattice spacings that are still as large as $a \simeq 1/3$ fm. The onset of continuum behaviour may be deduced, for a particular set of physical quantities, from the β ($\equiv 2Nc/g^2$) independence of dimensionless ratios of these quantities. More directly the onset of continuum physics should be accompanied by a restoration of those symmetries of the continuum gauge theory, which are explicitly broken by the lattice regularization. Some time ago numerical evidence was presented for the dynamical restoration of rotational symmetry in the SU(2) gauge theory [3]. In this letter we present numerical evidence for the dynamical restoration of Lorentz invariance, in both SU(2) and SU(3) lattice regularized gauge theories, in a range of couplings where calculations of physical quantities are currently being performed. To be precise, we shall calculate the energy–momentum dispersion relation for glueballs, and we shall see to what extent we reproduce the relativistic continuum form $E^2 = p^2 + m^2$.

The by now standard technique [4] for calculating glueball masses proceeds roughly as follows [5]. Construct a trial glueball wave-functional, $\phi(\boldsymbol{n}, t)$, "centred" on the site $(\boldsymbol{n}, t)$ with the desired J^{PC} properties. Make the $\boldsymbol{p} = 0$ translation invariant sum $\phi(\boldsymbol{p} = 0, t) = \Sigma_n \phi(\boldsymbol{n}, t)$ and measure the correlation function $G(\boldsymbol{p} = 0, t) \equiv \langle \phi(\boldsymbol{p} = 0, t)\, \phi(\boldsymbol{p} = 0, 0)\rangle$. Vary ϕ over a class of wave-functionals to obtain a large enough projection onto the desired glueball so that $G(\boldsymbol{p} = 0, t)$ is dominated by the lowest mass glueball propagator for $t \geqslant a$. Then obtain the glueball mass from

$$ma = \ln[G(\boldsymbol{p} = 0, t = a)/G(\boldsymbol{p} = 0, t = 2a)] \ . \qquad (1)$$

To obtain the glueball *energy*, E, as a function of the momentum $\boldsymbol{p}$, follow the same steps as above, *except* use a trial wave-functional of momentum $\boldsymbol{p}$,

$$\phi(\boldsymbol{p}, t) = \sum_n \exp(i\boldsymbol{p}\boldsymbol{n})\, \phi(\boldsymbol{n}, t) \ , \qquad (2)$$

and find

$$E(\boldsymbol{p})\, a = \ln[G(\boldsymbol{p}, t = a)/G(\boldsymbol{p}, t = 2a)] \ . \qquad (3)$$

In practice we shall employ as our basic loops in the construction of $\phi(\boldsymbol{n}, t)$ the 1×1 and 2×2 plaquettes.

We know from previous work [5,6] that at least one of these will provide a "good enough" (in the above sense) wave-functional in the range of couplings of interest herein. For the (spacelike) 2 × 2 loop we take $\boldsymbol{n}$ at the centre of the loop. For the 1 × 1 loop we take $\boldsymbol{n}$ to be the vertex from which the loop emanates. Note that in assigning momentum and angular momentum properties the choice of $\boldsymbol{n}$ can lead to some ambiguity. However, as long as one chooses $\boldsymbol{n}$ to be within one lattice spacing of the geometrical centre, this ambiguity is "irrelevant" in the technical sense.

We use the usual Wilson action [1] and employ conventional periodic boundary conditions. For a $L_s^3 \cdot L_t$ lattice the allowed momenta are thus

$$p_i a = (2\pi/L_s)\, n\,, \qquad n = 0, ..., L_s/2\,, \tag{4}$$

for L_s even. The results we present in this paper will be on lattices with $L_s = 8$. However, we do not calculate $E(\boldsymbol{p})$ for all $\boldsymbol{p}$ for several reasons: computer time; for large $|\boldsymbol{p}|$ the signal is lost in noise; when $|\boldsymbol{p}| \gg m$ one must go to $t \gg a$ to isolate the lowest glueball contribution to $G(\boldsymbol{p}, t)$. In fact we limit ourselves to $|\boldsymbol{p}| \lesssim 2m$ (for the $0^{+(+)}$ glueball). Here we expect (3) to be accurate, and we can hope to obtain statistically significant results.

We begin with the SU(2) case. Our most extensive measurements were taken at $\beta = 2.3$ on an $8^3 \cdot 10$ lattice: 24 000 configurations, broken into 24 subsets of 1000 configurations each for the error analysis. We use eq. (3) to extract $[E(\boldsymbol{p})\, a]^2$ versus $(\boldsymbol{p}a)^2$. In fig. 1a we plot the results for the wavefunction based on the 1 × 1 plaquette for the 0^+ glueball. To guide the eye we display the continuum relativistic dispersion relation, $E^2 = \boldsymbol{p}^2 + m^2$. It manifestly interpolates the data very well. The leading order strong coupling $E(\boldsymbol{p})$ is (obviously) independent of $\boldsymbol{p}$. The momentum dependence first comes in at $O(\beta^4)$ [7]. Our data is certainly very distant from that strong coupling limit. For comparison we also show the nonrelativistic continuum dispersion relation. Our data unambiguously prefers the relativistic version.

We have performed a lower statistics calculation at $\beta = 2.2$: 5400 configurations, again on an $8^3 \cdot 10$ lattice. Averaging the results for 1 × 1 and 2 × 2 basic loops we obtain $[E(\boldsymbol{p})\, a]^2$ as shown in fig. 1b. Again the relativistic dispersion relation is well verified, although at an obviously reduced level of statistical significance.

Turning now to SU(3) we show in fig. 1c the 0^{++} glueball energy squared, as a function of momentum, extracted (using the 1 × 1 loop) from 4000 configurations on an 8^4 lattice at $\beta = 5.7$. Again we obtain a very nice verification of the relativistic continuum dispersion relation.

We have also measured $E(\boldsymbol{p})$ for the $2^{+(+)}$ glueball. Since this state is much heavier than the $0^{+(+)}$ the errors are larger and plots such as those in fig. 1, where one *squares* $E(\boldsymbol{p})$, tend to be visually uninformative because of the large errors. As an alternative procedure we extract from $E(\boldsymbol{p})\, a$ the mass ma, using eq. (4) and $E^2 = \boldsymbol{p}^2 + m^2$, and see to what extent the extracted mass is independent of $(\boldsymbol{p}a)^2$. In figs. 2a,b we show the

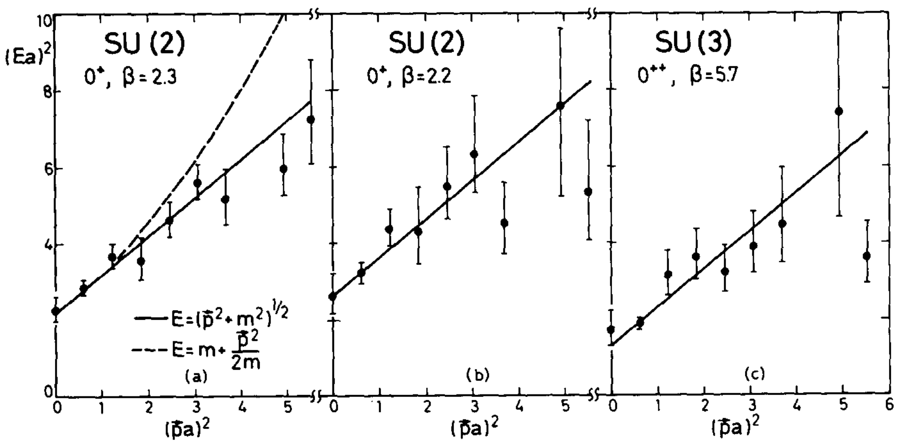

Fig. 1. Energy versus momentum for the 0^{++} glueball: (a) at $\beta = 2.3$ in SU(2); (b) at $\beta = 2.2$ in SU(2); (c) at $\beta = 5.7$ in SU(3).

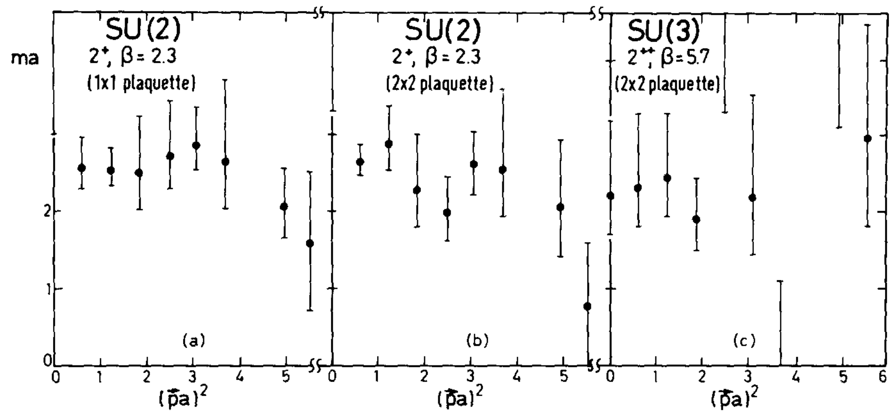

Fig. 2. The 2^{++} glueball mass extracted from $E(p)$ using $E^2 = p^2 + m^2$: (a) 1 × 1 loop wavefunction at $\beta = 2.3$ in SU(2); (b) 2 × 2 loop wavefunction at $\beta = 2.3$ in SU(2); (c) 2 × 2 loop wavefunction at $\beta = 5.7$ in SU(3).

SU(2) results at $\beta = 2.3$ (on $8^3 \cdot 10$) for the wavefunctions based on the 1 × 1 and 2 × 2 loops, respectively. In fig. 2c we show the SU(3) results at $\beta = 5.7$ (on 8^4). We clearly have a significant consistency with the $E^2 = p^2 + m^2$ dispersion relation.

To provide a contrast we have performed a similar calculation at a value of β, where the lattice is not yet expected to have become "irrelevant". In fig. 3 we plot $[E(p)\, a]^2$ for the SU(3) 0^{++} glueball (based on the 1 × 1 loop) as extracted from 1800 configurations

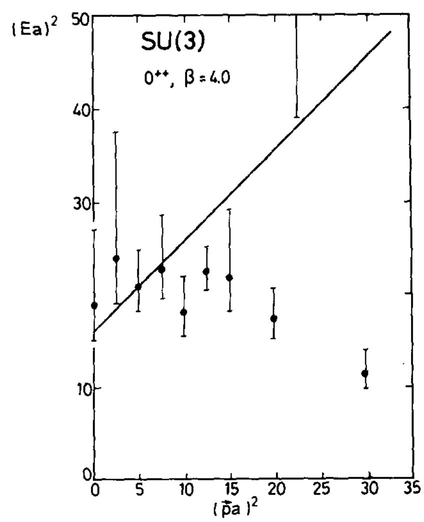

Fig. 3. Energy versus momentum for the 0^{++} glueball at $\beta = 4.0$ in SU(3).

at $\beta = 4.0$ on a $4^3 \cdot 8$ lattice. [We have used $G(a)/G(0)$ which should be accurate enough in this case.] The observed dispersion relation is very different from the continuum one and provides a yardstick against which to measure the significance of our earlier results.

We conclude that we have good numerical evidence for the restoration of Lorentz invariance in the range of couplings $\beta \gtrsim 2.2$ [SU(2)] and $\beta \gtrsim 5.5$ [SU(3)] which are of particular interest for current lattice QCD calculations. In a longer paper we shall give more details from our data taken on large lattices at β = 2.2, 2.3, 2.4, 2.5 in the SU(2) case and at β = 5.5, 5.7, 5.9 in the SU(3) case, together with results on the restoration of rotational invariance. Having verified the continuum dispersion relation one can use it for the lowest momenta to enhance the statistics of glueball calculations on large lattices (as discussed originally in ref. [8]). The utility of this, especially for the hard to get 2^{++}, will be obvious from our fig. 2. The results for the 0^{++} and 2^{++} glueball masses, obtained from the data summarized herein, are presented in an accompanying letter [9].

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