SCALING OF THE QUARK–ANTIQUARK POTENTIAL AND IMPROVED ACTIONS IN SU(2) LATTICE GAUGE THEORY

F. GUTBROD
Deutsches Elektronen-Synchrotron DESY, Hamburg, Fed. Rep. Germany

and

I. MONTVAY [1]
II. Institut für Theoretische Physik, Universität Hamburg, Hamburg, Fed. Rep. Germany

Received 28 October 1983

The scaling behaviour of the quark–antiquark potential is investigated by a high statistics Monte Carlo calculation in SU(2) lattice gauge theory. Besides the standard one-plaquette action we also use Symanzik's tree-level improved action and Wilson's block-spin improved action. No significant differences between Symanzik's action and the standard action have been observed. For small β Wilson's action scales differently. The string tension value κ extracted from the data corresponds to $\Lambda_{\mathrm{latt}} = (0.018 \pm 0.001)\sqrt{\kappa}$ for the one-plaquette action.

The basic assumption in lattice quantum chromodynamics [1] is the existence of a unique continuum limit for zero lattice spacing ($a \to 0$). This implies that physical quantities, like a potential V, calculated on the lattice obey the renormalization group equation (RGE) [2]:

$$\{-a\,\partial/\partial a + \overline{\beta}(g)\,\partial/\partial g\}V = \mathrm{O}(a^2 \ln a)\,. \qquad (1)$$

Here $\overline{\beta}(g) = -a\,\mathrm{d}g/\mathrm{d}a$ is the Callan–Symanzik β-function specific to the lattice action at hand. Asymptotic freedom requires that the continuum limit is realized by an asymptotically small bare coupling $g \to 0$. We then say that V obeys *asymptotic scaling* if the RHS of (1) is negligible and we can take for $\overline{\beta}(g)$ the regularization scheme independent, lowest order expansion, i.e. for SU(2) gauge theory

$$\overline{\beta}_{\mathrm{as}}(g) = -(4\pi)^{-2}\tfrac{22}{3}g^3 - (4\pi)^{-4}\tfrac{136}{3}g^5\,. \qquad (2)$$

The deviations from the continuum limit are represented by the RHS of eq. (1), which may already be very small in a coupling constant region where $\overline{\beta}(g)$ differs still appreciably from $\overline{\beta}_{\mathrm{as}}(g)$. Then we just have *scaling* (and continuum behaviour) in a non-asymptotic form.

It has been pointed out by Symanzik that by a proper choice of the lattice action the RHS of (1) can be reduced to $\mathrm{O}(a^4 \ln a)$ [3]. This will eventually lead to improved scaling, but it does not necessarily imply a more rapid approach for $\overline{\beta}(g) \to_{g\to 0} \overline{\beta}_{\mathrm{as}}(g)$. In fact, an earlier and more accurate scaling has been observed in previous Monte Carlo calculations with improved actions in the two-dimensional O(3) nonlinear σ-model [4]. Recent calculations [5,6] with improved actions in SU(2) LGT [7,8] seem to be promising, too. Another way to improve the approach to scaling has been proposed by Wilson [9]. He chooses the combination of lattice couplings as near as possible to the trajectory of some block-spin renormalization group transformation. In general the improved actions in SU(N_c) gauge theory have the form [7], with $\beta = 2N_c/g^2$:

$$S = -\beta\left\{c_0 \sum_{\square} \mathrm{Re\,tr}\,\square + c_1 \sum_{\square\!\square} \mathrm{Re\,tr}\,\square\!\square + c_2 \sum \mathrm{Re\,tr} + c_3 \sum \mathrm{Re\,tr}\right\}. \qquad (3)$$

[1] Supported by Bundesministerium für Forschung und Technologie, Bonn, Germany.

Here the sums run over the different (unoriented) closed loops of length 4 and 6, as depicted. The standard 1-plaquette action corresponds to $c_0 = 1, c_1 = c_2 = c_3 = 0$. The tree-level improved action of Symanzik, where the $O(a^2 \ln a)$ scale breaking terms in (1) are eliminated only in zeroth order in g^2, corresponds to [8]

$$c_0 = 5/3, \quad c_1 = -1/12, \quad c_2 = c_3 = 0 . \tag{4}$$

Finally, Wilson's block-spin improved action is specified by [9]

$$c_0 = 4.376, \quad c_1 = -0.252, \quad c_2 = 0, \quad c_3 = -0.17. \tag{5}$$

In this paper we shall study the scaling behaviour of the potential $V(r)$ between static quarks in SU(2) pure gauge theory (without fermions). This potential is given on the lattice by

$$aV(r = aR) + c(\beta) = -\lim_{T\to\infty} \frac{\ln W(R,T)}{T} \equiv P_R . \tag{6}$$

Here $c(\beta)$ is introduced to allow for the normalization of V at a given physical distance. $W(R, T)$ are traces of rectangular Wilson loops of lengths $R \times T$. The potential is presently the only quantity which can be calculated numerically with an accuracy on the percent level without being restricted to the smallest distances R. From the RGE (1) it follows that the dimensionless derivative of the potential scales under a change of g according to

$$a^2(g_1)(\mathrm{d}V/\mathrm{d}r)(r = aR) \equiv v'(g_1, R)$$
$$= (1/\xi_{12}^2)\, v'(g_2, R/\xi_{12}) , \tag{7}$$

with

$$\xi_{12} \equiv \frac{a(g_2)}{a(g_1)} = \exp\left(-\int_{g_1}^{g_2} \frac{\mathrm{d}g}{\bar\beta(g)}\right) . \tag{8}$$

This behaviour is tested here as follows: we determine P_R for as many values of R as possible at different $\beta = 4/g^2$. Fitting P_R with the ansatz

$$P_R(\beta) = -c_-(\beta)/R + c_+(\beta)R + c_0(\beta) , \tag{9}$$

we can determine the derivative $v'(g, R)$ by differentiating (9) with respect to R. The scale factor is then found by minimizing with respect to ξ_{12}

$$\Delta = \sum_R \{[v'(g_1, R) - \xi_{12}^{-2} v'(g_2, R/\xi_{12})]^2$$
$$+ [v'(g_1, \xi_{12}R) - \xi_{12}^{-2} v'(g_2, R)]^2\} , \tag{10}$$

where R runs over the measured values. If this minimization leads to $\sqrt{\Delta} \ll v'(g_{1,2}, R)$, then we say, we have observed scaling with a scale factor ξ_{12}. We shall verify this behaviour in the following to a high accuracy. In addition, we shall find significant deviations of ξ_{12} from the asymptotic ratios corresponding to $\bar\beta(g) = \bar\beta_{\mathrm{as}}(g)$ given by eq. (2). Besides determining the scale ratio ξ_{12}, another way to obtain information on the lattice $\bar\beta$-function is to write the RGE (1) for the potential slope in the form

$$\{2 + R\,\partial/\partial R + \bar\beta(g)\,\partial/\partial g\}\, v'(g, R) = 0 . \tag{11}$$

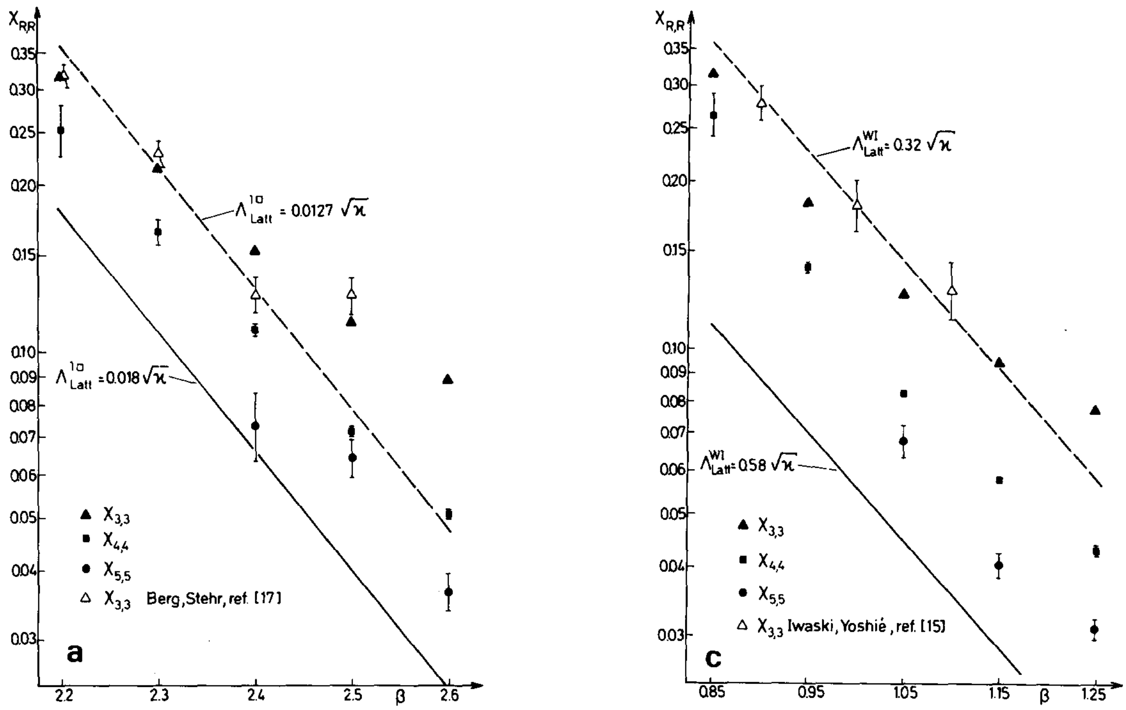
From this equation we have

$$\bar\beta(g) = \frac{-(2 + \partial/\partial \ln R)v'(g,R)}{(\partial/\partial g)\, v'(g,R)} . \tag{12}$$

The partial derivatives of $v'(g, R)$ can be numerically determined e.g. by a quadratic interpolation of the measured neighbouring values.

We performed the Monte Carlo calculations using the icosahedral finite subgroup of SU(2) [10]. For reasons explained below the lattice size was chosen as 15^4 with periodic boundary conditions. The β-ranges investigated were: for the standard one-plaquette action (1□-action) $2.2 \leqslant \beta \leqslant 2.6$, for the tree level improved action of Symanzik (SI-action) $1.6 < \beta \leqslant 2.0$, and for the block-spin improved action of Wilson (WI-action) $0.85 \leqslant \beta \leqslant 1.25$, in all cases with steps $\Delta\beta = 0.1$. In terms of physical scales these intervals are nearly equivalent. For the 1□-action the approximate restoration of the rotation symmetry occurs at $\beta = 2.25$ [11], which sets a lower limit for the range where scaling can be expected. The upper β-limits are close to the critical points where on our 15^4-lattice the Polyakov lines (closed by periodic boundary conditions) get a non-zero expectation value [12]. These critical points may have dangerous effects on the numerical convergence to equilibrium and/or on the extracted potential.

Our data are based on 500–900 measurements for each value of β, separated by 1 sweep with 5 Metropolis hits. In addition, around 200 sweeps were performed to equilibrate the lattice, starting either from



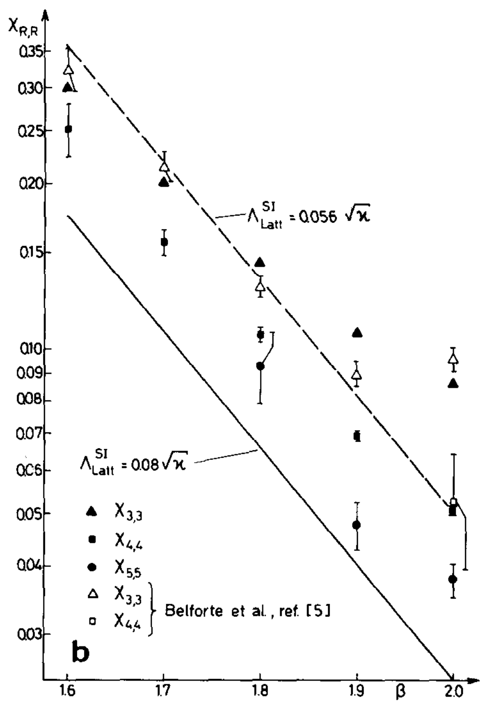


Fig. 1. Creutz ratios $\chi_{R,R}$ as functions of $\beta = 4/g^2$ for various actions. The lines are asymptotic scaling curves corresponding to different values of Λ_{latt} as explained in the text. (a) Standard one-plaquette action. (b) Symanzik's improved action. (c) Wilson's action.

ordered state or (in most cases) from a "neighbouring" lattice. The calculations were done (except for one β-value) on a CYBER 205 Vector Computer in Karlsruhe. Links in distance $3a$ were updated in parallel [13] (which explains our choice of lattice size) allowing vectors of length 5^4. These are processed very efficiently on a 1-pipe CYBER 205. One update took 50 μs per link for the WI-action and 33 μs per link for the SI-action. Updating time and measuring time balanced roughly 1 : 1. Although the icosahedral group algorithm is very fast on a serial computer like the IBM 3081D and about 50% of CPU-time is used for reordering vectors on the CYBER 205, speed ratios of 12 were reached with respect to the IBM. The total data amount to about 37 hours CYBER 205 time.

When we shall quote errors in the following, then they are derived from considering blocks of 25 consecutive iterations as statistically independent from the

Table 1
Creutz ratios $\chi_{R,R}$ and potentials P_R for three lattice actions: 1□: Standard one-plaquette action. SI: Symanzik's improved action. WI: Wilson's action. The numbers in parenthesis are the errors in the last numerals.

	β	χ_{33}	χ_{44}	χ_{55}	P_1	P_2	P_3	P_4	P_5
1□	2.2	0.315 (3)	0.253 (27)	–	0.4975(6)	0.8506(34)	1.129 (19)	–	–
	2.3	0.2173(16)	0.165 (9)	–	0.4272(5)	0.6868(15)	0.871 (5)	1.042(33)	–
	2.4	0.1528(9)	0.1116(30)	0.074(12)	0.3762(3)	0.5701(11)	0.6963(27)	0.798(6)	0.899(14)
	2.5	0.1135(5)	0.0717(15)	0.064(5)	0.3360(2)	0.4887(6)	0.5773(13)	0.646(3)	0.705(5)
	2.6	0.0894(4)	0.0524(9)	0.037(2)	0.3077(2)	0.4345(5)	0.4991(10)	0.542(2)	0.577(2)
SI	1.6	0.299 (2)	0.252 (28)	–	0.4858(5)	0.8426(46)	1.104 (44)	–	–
	1.7	0.2008(19)	0.156 (8)	–	0.4116(4)	0.6654(17)	0.840 (6)	0.991(20)	–
	1.8	0.1428(8)	0.1062(28)	0.093(14)	0.3590(3)	0.5513(9)	0.6675(16)	0.766(6)	–
	1.9	0.1063(8)	0.0690(11)	0.048(5)	0.3205(2)	0.4737(7)	0.5556(16)	0.616(2)	–
	2.0	0.0855(4)	0.509(12)	0.038(3)	0.2922(2)	0.4208(5)	0.4825(9)	0.524(2)	–
WI	0.85	0.315 (2)	0.266 (24)	–	0.4254(3)	0.7787(30)	1.069 (26)	–	–
	0.95	0.1836(11)	0.140 (3)	–	0.3354(3)	0.5613(11)	0.719 (4)	0.849(11)	–
	1.05	0.1251(8)	0.0826(12)	0.068(5)	0.2803(2)	0.4442(8)	0.5334(16)	0.615(3)	
	1.15	0.0940(6)	0.057 (9)	0.041(2)	0.2431(2)	0.3735(5)	0.4437(11)	0.4925(18)	–
	1.25	0.0773(2)	0.0431(6)	0.031(1)	0.2153(2)	0.3244(7)	0.3790(9)	0.4151(14)	–

next one. Only for the smallest errors we found, that using blocks of 50 configurations increases the errors by 20% in the average. The existence of long range fluctuations at a very low level is not excluded.

Before studying the properties of the potential in detail let us comment on the Creutz-ratios [14]

$$\chi_{R,T} = -\ln \frac{W(R,T)\,W(R-1,T-1)}{W(R-1,T)\,W(R,T-1)}. \tag{13}$$

Most scaling tests done so far for improved actions [5,15,16] were analyzing $\chi_{R,R}$ with $R \leqslant 3$ (sometimes $R \leqslant 4$). We show our results for the χ's in table 1 and in figs. 1a–1c, comparing them to previous results. As it can be seen from the figures, the new points for $\chi_{3,3}$ in most cases agree within errors with the older lower statistics ones. Possible exceptions are the points $\beta = 2.4$ (1□ action) and $\beta = 2.0$ (SI-action). We think that this has to do with the expectation value of Polyakov-lines, which is non-zero at $\beta = 2.4$ (1□ action) on a 8^4 lattice [12]. On our larger lattice the Polyakov-lines are still zero at these β-values. This can explain the difference as due either to a critical slowing down of equilibration or to a genuine finite size effect. The values of $\chi_{4,4}$ and $\chi_{5,5}$ in the figures are substantially below the scaling curves corresponding to the previously quoted string tension values.

The conventional procedure [14] to extract the string tension from $\chi_{R,R}$ is to find to lower envelop to all measured points. We see from fig. 1 that this is a rather subjective produce when large Wilson-loops are not available: there is no clear convergence towards a common envelop. As we shall show later, for the largest β-values considered, even $R = 5$ is far from the region, where the potential is linear. The spread of the χ's is a necessary consequence of this, since they measure roughly the derivative of the potential somewhere between $R - 1$ and R.

An inspection of fig. 1 reveals that we cannot expect asymptotic scaling to hold accurately. The spatial scale factor between $\chi_{3,3}$ and $\chi_{5,5}$ lies between 5/3 and 2 which, for asymptotic scaling, corresponds to a change $0.2 \leqslant \Delta\beta \leqslant 0.3$. Pairs of $\chi_{3,3}$ and $\chi_{5,5}$ at such β-values are always connected by a slope steeper than the asymptotic scaling line.

All this will be made more quantitative by the following analysis of the potential. The values $P_R(\beta)$ are extracted by fitting the measured $\ln W(R, T)$ with $3 \leqslant T \leqslant 7$ to a straight line in T [18]. The fits are good on the level of 0.5%. There is, however, a small curvature in T for $R \geqslant 3$, which introduces a systematic error in the order less than 1% in $P_{3,4}(\beta)$. The results for $P_R(\beta)$ are listed in table 1. The local fit parameters $c_{\pm,0}(\beta)$ defined in eq. (9) are given in table 2. The chi-

Table 2
The fitted parameters for the potential. The numbers in parenthesis are the errors in the last numerals. The parameters $c_{\pm,0}(\beta)$ are defined in eq. (9), $c(\beta)$ in eq. (6). $\bar{\beta}(g)$ is the lattice β-function as obtained from eq. (12). ξ_{12} is the fitted scale ratio between consecutive β-values, ξ_{13} is the one between second neighbours ($\Delta\beta = 0.2$). ξ_{16} is the scale ratio to the corresponding β-value for the (cyclically) next action.

	β	$c_-(\beta)$	$c_+(\beta)$	$c_0(\beta)$	$c(\beta)$	ξ_{12}	ξ_{13}	ξ_{16}	$-\bar{\beta}(g)$
1□	2.2	0.22 (7)	0.24 (3)	0.48 (9)	0.42 (3)	1.28(4)	1.67(6)	1.03(12)	
	2.3	0.225(13)	0.147 (6)	0.505(19)	0.453(15)	1.31(2)	1.74(4)	1.03(4)	0.111(15)
	2.4	0.210(7)	0.0893(27)	0.497(9)	0.476(10)	1.32(2)	1.84(4)	1.04(2)	0.100(5)
	2.5	0.198(3)	0.0540(12)	0.480(4)	0.478(6)	1.37(2)		1.04(2)	0.087(3)
	2.6	0.197(2)	0.0287(7)	0.476(3)	0.476*			1.00(2)	
SI	1.6	0.30 (13)	0.20 (6)	0.59 (19)	0.36 (6)	1.31(9)	1.79(18)	0.97(9)	
	1.7	0.238(15)	0.135 (7)	0.515(21)	0.420(19)	1.33(2)	1.82(6)	1.09(2)	0.18 (8)
	1.8	0.228(6)	0.0784(22)	0.508(8)	0.450(14)	1.36(2)	1.88(6)	1.18(2)	0.141(7)
	1.9	0.218(4)	0.0455(16)	0.494(6)	0.463(9)	1.37(2)		1.24(2)	0.125(6)
	2.0	0.204(3)	0.0268(11)	0.470(4)	0.460(8)			1.30(4)	
WI	0.85	0.21 (9)	0.25 (4)	0.38 (12)	0.34 (4)	1.43(4)	2.01(19)	1.01(8)	
	0.95	0.211(9)	0.121 (4)	0.425(13)	0.391(19)	1.41(2)	1.92(6)	0.89(2)	0.29 (4)
	1.05	0.205(5)	0.0619(19)	0.423(7)	0.406(15)	1.37(2)	1.86(6)	0.82(2)	0.277(12)
	1.15	0.187(3)	0.0370(12)	0.393(4)	0.402(11)	1.35(2)		0.80(2)	0.299(14)
	1.25	0.170(3)	0.0245(12)	0.361(5)	0.389(15)			0.83(6)	

squares of all local fits are excellent. It is interesting to notice that for the smallest β-values the dimensionless coupling constant $c_-(\beta)$ is compatible with $\pi/12$, which follows from string roughening [19]. This value could not be determined directly by Stack [18], since for small β he could measure only for $R = 1$ and 2. In addition, the main difference of his analysis compared to ours is, that he assumed asymptotic scaling of the potential from the beginning. This does, of course, influence the values of the extracted potential parameters. For increasing β (decreasing coupling constant) the "Coulomb"-coefficient $c_-(\beta)$ decreases like $1/\beta$ for all three actions. This turns into the logarithmic decrease of the non-abelian Coulomb potential in the asymptotic scaling region.

The potential slopes determined from the coefficients $c_\pm(\beta)$ can be examined for scaling. for this purpose we minimize Δ in eq. (10) by varying ξ_{12}. The resulting ξ_{12} are listed in columns 5–7 of table 2. The statistical errors of the scale ratios as determined by varying the input potential values normally distributed within their errors. In table 2 we have doubled these statistical errors to account for some systematic errors which we guessed by using parametrizations for the interpolating function different from eq. (9). The matching of the potential slopes between the different β-values is good within 5% of the calculated slopes (at most 10% for $R = 4$ and large β). Parenthetically we remark that we finally did not include the χ-ratios when fitting the potential coefficients, since this spoiled the χ^2 dramatically. However, the slopes obtained from the fit usually agree in the region $R = 3$–5 to the values of $\chi_{4,4}$ and $\chi_{5,5}$ within 10–15%.

The scale ratios ξ_{12} can be compared to the asymptotic ones as given by eqs. (2,8):

$$a(g)\Lambda_{\mathrm{latt}} = (\beta_0 g^2)^{-51/121} \exp\{-1/2\beta_0 g^2\} ,\qquad \beta_0 = \tfrac{22}{3}(4\pi)^{-2} . \tag{14}$$

Here Λ_{latt} is the lattice Λ-parameter belonging to the particular choice of lattice action. From this equation we obtain (with $\Delta\beta = 0.1$) $1.285 \leqslant \xi_{12} \leqslant 1.287$ for $2.2 \leqslant \beta \leqslant 2.6$, $1.274 \leqslant \xi_{12} \leqslant 1.281$ for $1.6 \leqslant \beta \leqslant 2.0$ and $1.249 \leqslant \xi_{12} \leqslant 1.264$ for $0.85 \leqslant \beta \leqslant 1.25$. In column 5 of table 2 there are substantial deviations from these numbers for all three actions. The somewhat surprising fact is that for 1□- and SI-actions the deviation seems to increase with increasing β. For the WI-action the deviation is largest, but it is decreasing with increasing β. The very reassuring fact is that, if extrapolated to $\beta = 0.805$ versus $\beta = 1.00$, the scale ratio is within errors equal to 2. This factor 2 was obtained for this action by Wilson [9] from matching the expectation value of block-spin loops on 8^4, 4^4 and 2^4 lattices. (In this respect we disagree with the conclusion of ref. [15], which was based on $\chi_{3,3}$.)

The scaling properties of the force $v'(g, R)$ can be used to determine the lattice $\bar{\beta}$-function directly by eq. (12). The partial derivatives of $v'(g, R)$ with respect to g and R were estimated by quadratic interpolation between three neighbouring values in both variables. The values for $\bar{\beta}(g)$ given in the last column of table 2 are the averages between $R = 3$ and 4, the values for $R = 2$ being systematiclaly lower by 10–20%. At the largest β's, $\bar{\beta}(g)$ is about 15% smaller than the asymptotic $\bar{\beta}_{as}(g)$ in eq. (2), in good agreement with larger scale factors ξ_{12} in table 2.

Finally we construct the physical potential $V(r)$. We shift the scaled potentials $P_R(\beta)\xi_{12}^{-1}$ by constants δ_{12}, such that the expression

$$\sum_{n=1}^{5} \{[P_n(g_1) - \xi_{12}^{-1}\ P_{n/\xi_{12}}(g_2) - \delta_{12}]^2 + [P_{n\xi_{12}}(g_1) - \xi_{12}^{-1} P_n(g_2) - \delta_{12}]^2\}\ , \qquad (15)$$

is minimal. The constant δ_{12} is related to the $c(\beta)$ of eq. (6) by $\delta_{12} = c(\beta_1) - c(\beta_2)/\xi_{12}$. We have listed the $c(\beta)$'s in table 2 with the convention $c(\beta = 2.6) = c_0(\beta = 2.6)$ (1□ action). The errors of δ_{12} are defined in the same way as those of ξ_{12} above.

The resulting potential $V(r)$ is plotted in fig. 2. It can be fitted very well by a Coulomb plus a linear term, the latter one connecting the physical scale to the string tension. (We note that a fit to the potential $V(r)$ with logarithmically changing Coulomb term is also possible with a Λ-parameter around 100 MeV.) We use $\kappa = (420\ \mathrm{MeV})^2$ and obtain (V in GeV, r in Fermi):

$$V(r) = -(0.042 \pm 0.004)/r + 0.046 \pm 0.021 + (0.895 \pm 0.04)r\ . \qquad (16)$$

Determining the lattice spacing at $\beta = 2.6$ from the prescribed value of the string tension, and assuming eq. (14) at this β-value, one has for the lattice Λ-parameter

$$\Lambda_{\mathrm{latt}}^{1\square} = (0.018 \pm 0.001)\sqrt{\kappa}\ . \qquad (17)$$

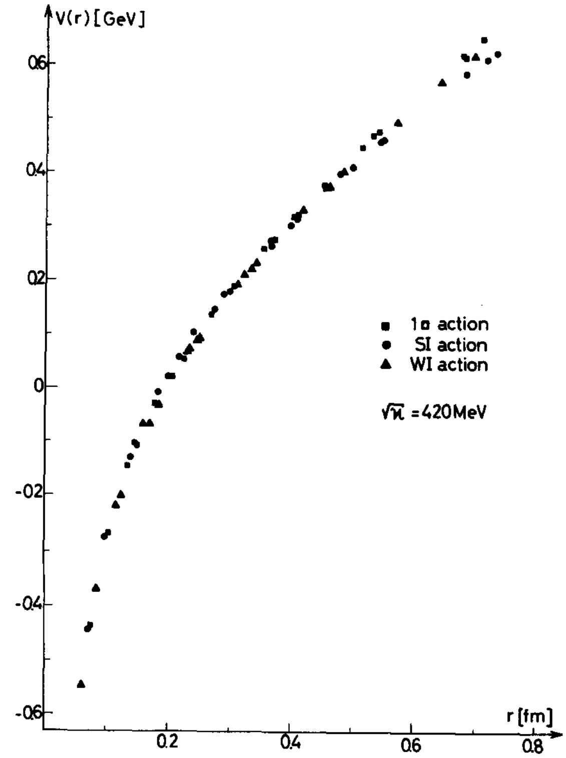


Fig. 2. The potential for static q$\bar{\mathrm{q}}$ pairs. All values for all three actions are included. The involved scale factors ξ_{12} and shift constants $c(\beta)$ are listed in table 2.

According to the scale ratios in table 2 this corresponds to

$$\Lambda_{\mathrm{latt}}^{\mathrm{SI}} = (0.080 \pm 0.005)\sqrt{\kappa}\ , \quad \Lambda_{\mathrm{latt}}^{\mathrm{WI}} = (0.58 \pm 0.03)\sqrt{\kappa}\ . \qquad (18)$$

The ratio $\Lambda_{\mathrm{latt}}^{\mathrm{SI}}/\Lambda_{\mathrm{latt}}^{1\square} \cong 4.5$ is near to the perturbatively calculated value 4.13 [7,20]. The lattice Λ-parameters in eqs. (17, 18) are larger than the previously obtained ones: $\Lambda_{\mathrm{latt}}^{1\square} \cong (0.012–0.013)\sqrt{\kappa}$ [14,17]; $\Lambda_{\mathrm{latt}}^{\mathrm{SI}} = (0.056 \pm 0.003)\sqrt{\kappa}$ [5]; $\Lambda_{\mathrm{latt}}^{\mathrm{WI}} = (0.32 \pm 0.01)\sqrt{\kappa}$ [15, 16]. Eq. (17) agrees very well with $a\ (\beta = 2.3) = (0.80 \pm 0.16)\ \mathrm{GeV}^{-1}$ (or $\Lambda_{\mathrm{latt}}^{1\square} = (0.018 \pm 0.004)\sqrt{\kappa}$) obtained from the ρ-meson mass in the quenched approximation with Wilson-fermions [21]. A lower value of the string tension in SU(3) was reported recently also in refs. [22,23].

In conclusion, our high statistics measurements show that for all three actions considered here scaling for the q$\bar{\mathrm{q}}$-force holds very well. This is a necessary condition for the existence of a unique continuum limit of lattice QCD. It remains to see whether the other physical quantities do scale in the same β-range with the same scale factors. On the other hand, the relevant scale increments $(\xi_{12} - 1)$ differ up to 25% from the asymptotic values, and somewhat uncomfortably, do so with increasing tendency for increasing β. (An exception in this respect is the WI action.) If this holds also on larger lattices, it is unlikely that we are close to the continuum limit at present. Changing the action in order to eliminate correction terms from the RGE does not change the picture within our statistical errors. The differences between the SI- and 1□-action are barely significant. When pressed to decide between maximizing lattice size and improving the action, we would presently favour the first choice.

The good statistical quality of our potential values on a relatively large lattice allow us to determine the lattice Λ-parameter from larger distances than previously possible. We find $\Lambda^{1\square}_{\mathrm{latt}} = (0.018 \pm 0.001)\sqrt{\kappa}$ exceeding older values by 50%.

We are greatly indebted to K. Symanzik, P. Hasenfratz, M. Lüscher and P. Weisz for many illuminating discussions. Thanks are due to the Rechenzentrum der Universität Karlsruhe and to Control Data for their effective help in all questions connected with the CYBER 205. Especially, many programming advices by Dr. G. Schäfer, CD, are gratefully acknowledged. Finally we thank the DESY Direktorium for buying time on the CYBER 205.

References

[1] K.G. Wilson, Phys. Rev. D10 (1974) 2445.
[2] J. Zinn-Justin, in: Cargèse Lecturs in Field theory and critical phenomena, eds. E. Brézin and J.M. Charap (Gordon and Breach, New York, 1975).
[3] K. Symanzik, in: Mathematical problems in theoretical physics, eds. R. Schrader et al., Springer Lecture Notes in Physics (Springer, Berlin, 1982), K. Symanzik, DESY preprints 83-016, 83-026 (1983).
[4] B. Berg, S. Meyer, I. Montvay and K. Symanzik, Phys. Lett. 126B (1983) 467; B. Berg, S. Meyer and I. Montvay, DESY preprint 83-098 (1983).
[5] S. Belforte, G. Curci, P. Menotti and G. Paffuti, Phys. Lett. 131B (1983) 423.
[6] B. Berg, A. Billoire, S. Meyer and C. Panagiotakopoulos, Phys. Lett. 133B (1983) 359.
[7] P. Weisz, Nucl. Phys. B212 (1983) 1; P. Weisz and R. Wohlert, DESY-preprint 83-091 (1983).
[8] G. Curci, P. Menotti and G. Paffuti, Phys. Lett. 130B (1983) 205.
[9] K.G. Wilson, in: Recent developments in gauge theories, Cargèse lectures 1979, eds. G. 't Hooft et al. (Plenum, New York, 1980).
[10] G. Bhanot, C.B. Lang and C. Rebbi, Computer Phys. Commun. 25 (1982) 57; D. Petcher and D.H. Weingarten, Phys. Rev. D22 (1980) 2465.
[11] C.B. Lang and C. Rebbi, Phys. Lett. 115B (1982) 137.
[12] E. Kovács, Phys. Lett. 118B (1982) 125.
[13] D. Barkai and K.J.M. Moriarty, Comp. Phys. Commun. 25 (1982) 57.
[14] M. Creutz, Phys. Rev. D21 (1980) 2308; Phys. Rev. Lett. 45 (1980) 313.
[15] Y. Iwasaki and T. Yoshié, Phys. Lett. 130B (1983) 77.
[16] M. Fukugita, T. Kaneko, T. Niuya and A. Ukawa, Phys. Lett. 130B (1983) 73.
[17] B. Berg and J. Stehr, Z. Physik C9 (1981) 333.
[18] J.D. Stack, Phys. Rev. D27 (1983) 412.
[19] M. Lüscher, K. Symanzik, P. Weisz, Nucl. Phys. B173 (1980) 365; M. Lüscher, Nucl. Phys. B180 [FS2] (1981) 317.
[20] W. Bernreuther and W. Wetzel, Phys. Lett. 132B (1983) 382.
[21] I. Montvay, Phys. Lett. 132B (1983) 393.
[22] F. Gutbrod, P. Hasenfratz, Z. Kunszt and I. Montvay, Phys. Lett. 128B (1983) 415.
[23] G. Parisi, R. Petronzio and F. Rapuano, Phys. Lett. 128B (1983) 418.