ELECTROWEAK RADIATIVE CORRECTIONS TO THE $e^+e^- \to \mu^+\mu^-$ ASYMMETRY

M. BÖHM
Physikalisches Institut der Universität Würzburg, Würzburg, West Germany

and

W. HOLLIK
II. Institut für Theoretische Physik der Universität Hamburg, Hamburg, West Germany

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The electromagnetic and purely weak one-loop corrections to $e^+e^- \to \mu^+\mu^-$ have been calculated in the SU(2) × U(1) standard model using an on-shell renormalisation scheme with finite Green functions. Their influence on the forward backward asymmetry A_{FB} together with soft and hard bremsstrahlung is discussed for PETRA energies. Whereas the electromagnetic corrections to γ and Z^0 exchange diminish A_{FB}, the weak corrections increase A_{FB} almost compensating the QED correction to Z^0 exchange. The main weak contribution comes from the Z^0 self-energy. The other diagrams give only small changes in A_{FB}.

The measurements of the angular distribution in the reaction $e^+e^- \to \mu^+\mu^-$ at PETRA [1] and PEP [2] show a clear negative forward backward asymmetry A_{FB}, which is mainly due to the interference between photon and Z^0 exchange amplitudes. In order to compare the measured value of A_{FB} with the prediction of the electroweak standard model [3], a careful investigation of radiative corrections is necessary. At the one-loop level they can be separated into three classes:

(A) Electromagnetic corrections to γ exchange, consisting of virtual photon and bremsstrahlung contributions together with the fermionic vacuum polarisation of the photon ("reduced QED corrections").

(B) Electromagnetic corrections to Z^0 exchange, i.e. virtual and bremsstrahlung contributions in all possible ways in the Z^0 exchange diagram ("full QED corrections").

(C) Weak (non-photonic) corrections to both γ and Z^0 exchange amplitudes.

The reduced QED corrections (A) are model independent and give a positive contribution to A_{FB} depending on the experimental energy and/or acollinearity cuts [4]. The corrections of type (B) [5,6] give a further positive contribution to A_{FB} depending on the experimental cuts as well as on the model parameters. For $\sin^2\theta_W = 0.23$ this part amounts to +0.6–0.9% at PETRA energies for realistic cuts [6]. The complete QED corrections (A) and (B) are infrared finite since the singularities from virtual and real photons cancel each other. As a consequence of the renormalisability of QED they are also ultraviolet finite if the renormalised QED quantities, photon propagator and electric charge, are used.

In contrast to these QED corrections the weak corrections (C) contain further dynamical aspects of the electroweak model and are sensitive to its renormalizability. A precise knowledge of this part therefore allows a test of the model beyond the tree level. Since the standard model with a non-abelian, non-simple gauge group and a spontaneous symmetry breaking mechanism has besides the fermion masses four basic parameters, the renormalisation becomes already at the one-loop level a non-trivial matter. The choice of the renormalized parameters and their definition via measurable quantities as well as the definition of $\sin^2\theta_W$ is not unique beyond the tree level. This am-

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biguity, the use of different gauges, and the complexity of the formulas make it difficult to compare directly the results obtained in different schemes.

A first calculation of the corrections to $e^+e^- \to \mu^+\mu^-$ was done by Passarino and Veltman [7]. However, they did not include hadronic contributions and effects coming from mass renormalisation of the vector bosons. More recent calculations [8,9] do not give a unique answer for the magnitude of the weak corrections to the forward–backward asymmetry. Also until now no complete work exists containing the electroweak one-loop corrections together with soft and hard bremsstrahlung contributions.

In this paper we present our results for the one-loop and bremsstrahlung corrections to the $e^+e^- \to \mu^+\mu^-$ asymmetry at PETRA/PEP energies. The calculations are performed in a renormalisable 't Hooft gauge involving unphysical degrees of freedom, but leading to UV finite propagators and vertex functions. Wave function renormalisation constants are introduced for the left and right handed fermions, the isovector and isoscalar bosons, Higgs ghosts:

$$\Psi_{L,R} \to Z_{L,R}^{1/2}\Psi_{L,R}\,, \quad W_\mu \to Z_W^{1/2} W_\mu\,, \quad B_\mu \to Z_B^{1/2} B_\mu\,,$$

$$\phi_H \to Z_\phi^{1/2}\phi_H\,, \ldots \qquad (1)$$

for generating the correct counter terms respecting gauge invariance.

Our framework of renormalisation can be characterised as follows:

(1) The renormalised physical parameters are
– the masses of the $W^\pm$ and Z^0 bosons, M_W and M_Z, the Higgs boson mass M_H and the fermion masses m_f;
– the electric charge $e = (4\pi\alpha)^{1/2}$ as measured in the Thomson limit $q^2 \to 0$.

(2) The photon couples as a "usual" real photon to the electron in the limit $q^2 \to 0$ without an intermediate Z^0 boson contribution.

(3) The weak mixing angle is defined by

$$\cos\theta_W = M_W/M_Z\,. \qquad (2)$$

According to (1) the mass counterterms δM_Z, δM_W, δm_f, δM_H are fixed by the on-shell conditions that the poles of the propagators correspond to the physical masses. The charge counterterm δe is determined by the classical Thomson limit. Condition (2) implies that the renormalized propagator matrix for the Z^0 and γ fields corresponding to figs. 1 and 2 be-

Fig. 1. Electroweak one-loop contributions to $e^+e^- \to \mu^+\mu^-$.

comes diagonal in the limit $q^2 \to 0$, if the Z^0 and photon fields are

$$Z_\mu = \cos\theta_W\, W_\mu^0 + \sin\theta_W\, B_\mu\,,$$

$$A_\mu = -\sin\theta_W\, W_\mu^0 + \cos\theta_W\, B_\mu\,, \qquad (3)$$

with the isovector and isoscalar fields W_μ^0, B_μ, and the mixing angle defined by (2).

In order to fix the wave function renormalization constants in (1) we impose the additional conditions:

(i) the charged fermion propagators have residue 1 for $\not{p} = m_f$;

(ii) the photon propagator has residue 1 for $q^2 = 0$;

(iii) the Higgs propagator has residue 1 for $q^2 = M_H^2$.

The choice (i) and (ii) ensures that our scheme is a natural extension of the usual QED renormalisation, such that the photonic corrections in refs. [5,6] can be taken over without modifications.

(iii) is listed for completeness; the one-loop corrections to $e^+e^- \to \mu^+\mu^-$ do not require Higgs wave function renormalisation for practical calculations. All the weak contributions have been calculated analytically for $m_{e,\mu} \ll M_{W,Z}$. Only for the massive box diagrams (fig. 4) an approximation of order $\alpha(s/M_Z^2)$, which is in agreement with the corresponding expression given by Wetzel [8], was used. A list of the explicitly calculated wave function renormalization constants, mass and charge counter terms, and the renormalized finite Green functions will be given in a detailed publication [10]. The forward–backward asymmetry in $e^+e^- \to \mu^+\mu^-$,

$$A_{FB}(|\cos\theta| \leqslant x) = \left(\int_0^x \frac{d\sigma}{d\Omega}\, d\cos\theta - \int_{-x}^0 \frac{d\sigma}{d\Omega}\, d\cos\theta \right)$$

$$\times \left(\int_0^x \frac{d\sigma}{d\Omega}\, d\cos\theta + \int_{-x}^0 \frac{d\sigma}{d\Omega}\, d\cos\theta \right)^{-1}, \qquad (4)$$

is in lowest order given by

Fig. 2. See caption of fig. 1.

$$A_{FB}^{(0)}(|\cos\theta| \leqslant x) = [x/(1+\tfrac{1}{3}x^2)]$$

$$\times \frac{2a^2\,\mathrm{Re}(\chi) + 4v^2a^2|\chi|^2}{1 + 2v^2\,\mathrm{Re}(\chi) + (v^2+a^2)^2|\chi|^2} \qquad (5)$$

with

$$\chi = s/(s - M_Z^2 + iM_Z\Gamma_Z)\,,$$

$$a = -1/4\sin\theta_W\cos\theta_W\,, \quad v = (1 - 4\sin^2\theta_W)\,a\,. \qquad (6)$$

At the one-loop level we have to take for the calculation of (4) the differential cross section

$$\frac{4s}{\alpha^2}\frac{d\sigma}{d\Omega} = \sigma_\gamma[1 + C_{em}^\gamma + C_W^\gamma] + \sigma_{\gamma Z}[1 + C_{em}^{\gamma Z} + C_W^{\gamma Z}]$$

$$+ \sigma_Z[1 + C_{em}^Z + C_W^Z]\,. \qquad (7)$$

$\sigma_{\gamma,\gamma Z,Z}$ are the Born term expressions, $C_{em}^{\gamma,\gamma Z,Z}$ the electromagnetic and $C_W^{\gamma,\gamma Z,Z}$ the weak corrections to γ and Z^0 exchange and their interference.

Including in (4) and (7) step by step the electromagnetic corrections with the photon vacuum polarisation, the Z^0 self-energy (fig. 1), the γZ-transitions (fig. 2), the non-photonic vertex corrections (fig. 3) and the massive box diagrams (fig. 4), we obtain the results given in table 1. Since $m_{e,\mu} \ll M_{W,Z}$, it is sufficient to deal with the transverse parts of the photon and boson propagators only. The various contributions in table 1 are therefore independent of the specific gauge by themselves. The magnitude of the separate parts, however, depends on the renormalisation scheme, whereas their sum should essentially be scheme independent (differences of two-loop order may occur). Table 1 shows that the absolute value of

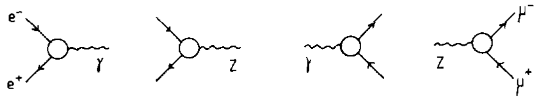

Fig. 3. See caption of fig. 1.

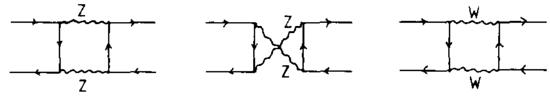

Fig. 4. See caption of fig. 1.

Table 1
The percentage forward–backward asymmetry for $|\cos\theta| \leqslant 0.8$ with QED and weak contributions. M_Z = 93 GeV, M_W = 82.1 GeV ($\hat{=}\ \sin^2\theta_W = 0.22$), M_H = 100 GeV, m_t = 30 GeV. For the bremsstrahlung an acollinearity angle of $\delta = 10°$ is used and the photon energy is restricted to $\Delta E \leqslant E_{beam}/2$.

	$\sqrt{s}$ (GeV)	
	34.5	44
Born	−7.62	−13.63
reduced QED	−5.80	−11.82
full QED	−5.28	−10.85
Z self-energy	−5.89	−11.93
γ−Z transition	−5.89	−11.94
vertex corrections	−5.88	−11.92
box diagrams	−5.89	−11.93

Table 2
Purely weak corrections to A_{FB} ($|\cos\theta| \leqslant 1$) in percent for $\sqrt{s}$ = 34.5 GeV (M_H = 100 GeV, m_t = 30 GeV).

M_Z (GeV)	M_W (GeV)							
	78	79	80	81	82	83	84	85
89	−0.75	−0.80	−0.87	−0.94	−1.03	−1.12	−1.22	−1.28
90	−0.68	−0.73	−0.75	−0.85	−0.93	−1.01	−1.10	−1.20
91	−0.63	−0.67	−0.72	−0.78	−0.84	−0.91	−0.99	−1.09
92	−0.58	−0.62	−0.66	−0.71	−0.76	−0.83	−0.90	−0.98
93	−0.53	−0.57	−0.61	−0.65	−0.70	−0.75	−0.81	−0.88
94	−0.50	−0.53	−0.56	−0.60	−0.64	−0.69	−0.74	−0.80
95	−0.46	−0.49	−0.52	−0.55	−0.59	−0.63	−0.68	−0.73
96	−0.43	−0.46	−0.48	−0.51	−0.55	−0.58	−0.62	−0.67
97	−0.40	−0.43	−0.45	−0.48	−0.51	−0.54	−0.57	−0.61
98	−0.38	−0.40	−0.42	−0.45	−0.47	−0.50	−0.53	−0.57

A_{FB} is reduced by the QED corrections both to γ and Z^0 exchange and is increased again by the weak corrections, essentially by the Z^0 self-energy such that the photonic corrections to Z^0 exchange are compensated. One has to keep in mind, however, that the latter depend on the experimental cuts and the model parameters. A model independent analysis of data including radiative corrections is therefore not possible.

The dependence of the purely weak corrections to A_{FB}, extrapolated to the full θ range, on the renormalized boson masses M_W, M_Z is given in table 2. The corrections are always negative and go down with increasing M_Z and decreasing M_W (increasing $\sin^2\theta_W$). The sensitivity with respect to the Higgs boson mass is very small: a variation of M_H from 10 to 1000 GeV yields a shift in A_B by ca. -0.1%.

In conclusion we have presented the full electroweak one-loop corrections to A_{FB} at PETRA energies in a scheme with physical masses as renormalized model parameters. We found that the expected values for A_{FB} are slightly higher than after applying only QED corrections.

Note added: After finishing this work a paper by J. Cole on the same topic came to our attention which also deals in an on-shell scheme with finite Green functions. His scheme, however, deviates from ours in several parts [11]:

– no condition that makes the renormalized γ–Z^0 mixing vanish in the Thomson limit, instead: definition of $\sin^2\theta_W$ via NC processes at low momentum transfer;

– $\rho = M_W^2/M_Z^2 \cos^2\theta_W = 1 + \delta\rho$ gets radiative corrections, whereas in our scheme $\rho = 1$ to all orders;

– the weak corrections to $e^+e^- \to \mu^+\mu^-$ involve radiative corrections to μ decay (G_μ) and ν scattering ($\sin^2\theta_W$), whereas in our scheme they are directly related to the physical values M_W, M_Z of the boson masses, as is also the case for the electromagnetic corrections to Z^0 exchange [5,6].

In Cole's scheme the weak corrections to A_{FB} are positive and decrease $|A_{FB}|$, whereas we find negative corrections compensating part of the electromagnetic corrections.

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