Nuclear Physics B239 (1984) 382–394

MULTIPLE BREMSSTRAHLUNG IN GAUGE THEORIES AT HIGH ENERGIES
(III). Finite mass effects in collinear photon bremsstrahlung*

CALKUL Collaboration

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Received 28 December 1983

We present a method for calculating the various spin amplitudes for QED processes in which an arbitrary number of photons is radiated in directions nearly parallel to the fermion directions. This is accomplished by introducing explicit polarization vectors for the photons and by working in the high energy limit, where finite mass effects are treated in leading order.

1. Introduction

In a previous article [1], we developed a general formalism for calculating multiple bremsstrahlung in gauge theories at high energies. This was accomplished by introducing explicit polarization vectors for the radiated photons in a covariant way. By considering the limit of vanishing fermion mass, we were able to obtain simple expressions for the various helicity amplitudes.

The method of ref. [1] needs to be supplemented by a treatment of the finite mass corrections if one wants to describe correctly the kinematical configurations in which the photon is radiated at small angles with respect to the fermion directions. For the simple case of single bremsstrahlung, we showed how this could be done at the level of the cross section [2, 3].

* Work supported in part by NATO research grant no. RG 079.80.
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¶ Work supported in part by the United States Department of Energy under contract no. DE-AS02-76-ER03227.

It is the purpose of this paper to give a general treatment of the finite mass effects for multiple bremsstrahlung when an arbitrary number of photons is radiated nearly parallel to fermion directions. We find that in this case it is essential to consider the mass corrections to the various spin amplitudes of the process, rather than calculating these effects for the cross section as was done in refs. [2, 3]. In the zero mass limit, these spin amplitudes reduce to the helicity amplitudes we considered earlier [1], except for some additional amplitudes which are directly proportional to a fermion mass.

In this article, we shall define collinear photons as being those photons for which the scalar product of their four-momenta k_i with any external fermion four-momentum p is of order m^2, m being the fermion mass. For the case where all $(p \cdot k_i)$ are much larger than m^2, no finite mass corrections have to be introduced, and the techniques of ref. [1] can be applied without modification.

Our method consists again in introducing explicit polarization vectors for the photons in a covariant way, which allows us to consider only a limited number of Feynman diagrams in the collinear situation. In the high energy limit, where fermion masses are small compared to the typical energy of the process, we are then able to calculate the various spin amplitudes up to the necessary powers of m^2. With minor modifications, the same techniques can be applied to the case of collinear fermion-antifermion pairs.

It turns out that the spin amplitudes are related to helicity amplitudes describing lower-order processes in which no collinear photons are radiated. Provided these lower-order processes can be treated with the methods of ref. [1], the technique of this article to generate the finite mass effects for collinear bremsstrahlung is applicable to all QED processes.

This paper is organized as follows. In sect. 2, we present the technique for calculating the spin amplitudes in collinear configurations, where an arbitrary number of photons is radiated. In sect. 3, we show how the general method can be applied to a specific case. We chose radiative Bhabha scattering, which is sufficiently simple for pedagogic purposes, while containing all the complexities of a more general situation. In sects. 4 and 5 we present all the formulae which are necessary for single and double bremsstrahlung in an arbitrary QED reaction. In practice, these are the cases which are most relevant for high-energy physics. Finally, in sect. 6 we discuss our results while in sect. 7 we list our conclusions.

2. General formalism

Let us consider any QED process, in the tree approximation, in which n photons are emitted in directions nearly parallel to a fermion direction described by the four-momentum p (later on we shall consider the more general case where more photons are emitted in directions parallel to other fermion directions). Let k_i $(i = 1, 2 \dots n)$ be the four-momenta of these collinear photons, and let m be the mass of the fermion, i.e. $p^2 = m^2$. There will then be Feynman diagrams describing the process in which fermion propagators will have small denominators:

$$\Delta_i = (p - k_i)^2 - m^2,$$

$$\Delta_{ij} = (p - k_i - k_j)^2 - m^2,$$

$$\dots$$

$$\Delta_{12\dots n} = (p - k_1 - k_2 - \cdots - k_n)^2 - m^2. \tag{2.1}$$

For collinear photons, the quantities $(p \cdot k_i)$ and $(k_i \cdot k_j)$ are of order m^2, which implies that all the quantities Δ are also of order m^2. It is clear, therefore, that even in the high-energy limit the fermion mass must be taken into account.

By introducing a generally positioned lightlike vector q, we can write down the following representation for the polarization vectors of the photons [1]:

$$\not{\varepsilon}_i^{\lambda_i} = N_i\left[\not{k}_i \not{p} \not{q} \omega_{\lambda_i} + \not{q} \not{p} \not{k}_i \omega_{-\lambda_i}\right],$$

$$\omega_\lambda = 1 + \lambda\gamma^5, \qquad \lambda = \pm 1, \qquad i = 1, 2, \dots, n. \tag{2.2}$$

The normalization is $\varepsilon_i^\lambda \cdot \varepsilon_i^{-\lambda} = -1$ which leads to

$$N_i^{-2} = 16(pq)(pk_i)(qk_i) - 8m^2(qk_i)^2. \tag{2.3}$$

Note that all components of ε_i^λ are of order 1, even in the collinear limit. Also, the representation for $\not{\varepsilon}_i^\lambda$ in eq. (2.2) differs from the effective one which was introduced in refs. [1, 3]. This is due to the fact that, for massive fermions, we do not have conservation of axial current, and consequently we cannot omit the $\not{k}_i\gamma^5$ terms in $\not{\varepsilon}_i^\lambda$. However, for $m = 0$ the two representations are gauge equivalent.

With the present choice for $\not{\varepsilon}_i^\lambda$, we now show that, in the collinear situation, the amplitudes are at most of order m^{-n}. To this order in m, the contributions to the amplitude will come only from the Feynman diagrams for which the collinear photons are attached directly next to the external fermion with momentum p. This follows from the fact that only these diagrams have all the denominators Δ (eq. (2.1)).

Consider the case that the photons are collinear with an incoming fermion with momentum p (the remaining cases of an incoming antifermion, or an outgoing (anti)fermion can be treated in the same way). The diagrams with the collinear photons close to the spinor $u(p)$ contain the expression

$$A = \frac{\not{p} - \not{k}_1 - \cdots - \not{k}_n + m}{\Delta_{12\dots n}} \not{\varepsilon}_n^{\lambda_n} \dots \not{\varepsilon}_3^{\lambda_3} \frac{\not{p} - \not{k}_1 - \not{k}_2 + m}{\Delta_{12}} \not{\varepsilon}_2^{\lambda_2} \frac{\not{p} - \not{k}_1 + m}{\Delta_1} \not{\varepsilon}_1^{\lambda_1} u(p)$$

$$+ (n! - 1) \text{ other permutations of } (1, 2, \dots, n). \tag{2.4}$$

But,

$$A_1 = \frac{\not{p} - \not{k}_1 + m}{\Delta_1} \not{\varepsilon}_1^{\lambda_1} u(p) = \frac{1}{\Delta_1} \left[2(p \cdot \varepsilon_1^{\lambda_1}) + \not{\varepsilon}_1^{\lambda_1} \not{k}_1 \right] u(p), \tag{2.5}$$

and, with eq. (2.2), we see that $p \cdot \varepsilon_i^{\lambda_i} = O(m)$. Similarly, as will be shown in sect. 3, $\not{k}_i u(p) = O(m)$. Hence, the whole expression is of order m^{-1}. It also follows that

$$A_2 = \frac{1}{\Delta_{12}\Delta_1} (\not{p} - \not{k}_1 - \not{k}_2 + m) \not{\varepsilon}_2^{\lambda_2} (\not{p} - \not{k}_1 + m) \not{\varepsilon}_1^{\lambda_1} u(p)$$

$$= \frac{1}{\Delta_{12}} \left\{ \left[2(p - k_1 \cdot \varepsilon_2^{\lambda_2}) - \not{k}_2 \not{\varepsilon}_2^{\lambda_2} \right] F_1(k_1, \lambda_1) - \not{\varepsilon}_2^{\lambda_2} \not{\varepsilon}_1^{\lambda_1} \right\} u(p), \tag{2.6}$$

where we introduced the notation

$$F_1(k_1, \lambda_1) = \frac{1}{\Delta_1} \left[2(p \cdot \varepsilon_1^{\lambda_1}) - \not{k}_1 \not{\varepsilon}_1^{\lambda_1} \right]. \tag{2.7}$$

Because of the symmetrization between the photons 1 and 2 contained in eq. (2.4), we can effectively replace $\not{\varepsilon}_2^{\lambda_2} \not{\varepsilon}_1^{\lambda_1}$ by $\varepsilon_2^{\lambda_2} \cdot \varepsilon_1^{\lambda_1}$ in eq. (2.6). Furthermore,

$$\not{k}_2 \not{\varepsilon}_2^{\lambda_2} \not{k}_1 \not{\varepsilon}_1^{\lambda_1} u(p) = 2 N_1 N_2 \delta_{\lambda_1, \lambda_2} \not{k}_2 \not{q} \not{p} \not{k}_2 \not{k}_1 \not{q} \not{p} \not{k}_1 \omega_{-\lambda_1} u(p)$$

$$= N_1 N_2 \delta_{\lambda_1, \lambda_2} \mathrm{Tr} \left[\not{q} \not{p} \not{k}_2 \not{k}_1 \omega_{\lambda_2} \right] \not{k}_2 \not{q} \not{p} \not{k}_1 \omega_{-\lambda_1} u(p)$$

$$= 2 N_1 \delta_{\lambda_1, \lambda_2} (k_1 \cdot \varepsilon_2^{\lambda_2}) \not{k}_2 \not{q} \not{p} \not{k}_1 \omega_{-\lambda_1} u(p). \tag{2.8}$$

Using eq. (2.2), it is easily seen that $(k_i \cdot \varepsilon_j^{\lambda_j}) = O(m)$. Also, $\not{p} \not{k}_i u(p) = O(m^2)$, hence the whole expression (2.8) is of order m^2. It follows that A_2 (eq. (2.6)) is of order m^{-2} and that it can effectively be replaced by

$$F_2(k_1, \lambda_1, k_2, \lambda_2) u(p)$$

$$= \frac{1}{\Delta_{12}} \left\{ \left[2(p - k_1 \cdot \varepsilon_2^{\lambda_2}) - \not{k}_2 \not{\varepsilon}_2^{\lambda_2} \right] F_1(k_1, \lambda_1) - \varepsilon_1^{\lambda_1} \cdot \varepsilon_2^{\lambda_2} \right\} u(p). \tag{2.9}$$

This procedure can now be continued for the remaining photons. We obtain the result that

$$A = F_n(k_1, \lambda_1, \ldots, k_n, \lambda_n) u(p) \tag{2.10}$$

is of order m^{-n} and that F_n is given by the following recursion relation:

$$F_n(k_1,\lambda_1,\dots,k_n,\lambda_n)$$
$$=\frac{1}{\Delta_{1\dots n}}\left\{\left[2(p-k_1-\cdots-k_{n-1}\cdot\varepsilon_n^{\lambda_n})-\not{k}_n\not{\varepsilon}_n^{\lambda_n}\right]F_{n-1}(k_1,\lambda_1,\dots,k_{n-1},\lambda_{n-1})\right.$$
$$\left.-\varepsilon_n^{\lambda_n}\cdot\varepsilon_{n-1}^{\lambda_{n-1}}F_{n-2}(k_1,\lambda_1,\dots,k_{n-2},\lambda_{n-2})\right\}, \tag{2.11}$$

with $F_0=1$. Once F_1 and F_2 (eqs. (2.7) and (2.9)) are known, it is thus possible to evaluate the higher order functions F_n. A very useful property for this purpose is the generalization of eq. (2.8):

$$\not{k}_n\not{\varepsilon}_n^{\lambda_n}\not{k}_{n-1}\not{\varepsilon}_{n-1}^{\lambda_{n-1}}\dots\not{k}_1\not{\varepsilon}_1^{\lambda_1}u(p)=2^{n-1}N_1\delta_{\lambda_n,\lambda_1}\delta_{\lambda_{n-1},\lambda_1}\dots\delta_{\lambda_2,\lambda_1}(k_{n-1}\cdot\varepsilon_n^{\lambda_n})$$
$$\times(k_{n-2}\cdot\varepsilon_{n-1}^{\lambda_{n-1}})\dots(k_1\cdot\varepsilon_2^{\lambda_2})\not{k}_n\not{q}\not{p}\not{k}_1\omega_{-\lambda_1}u(p). \tag{2.12}$$

The remaining part of the diagram, involving the spinor structure A (eq. (2.4)) can now be treated in the massless limit. This means that if we fix the helicities for the remaining fermions, the terms of the type (2.12) will give zero whenever the fermion helicity operators $\frac{1}{2}(1\pm\gamma^5)$ kill the $\omega_{-\lambda_1}$ factor in eq. (2.12). Of course, this term already vanishes when some of the collinear photons have different helicities. Furthermore, for all i and j,

$$(\varepsilon_i^{\lambda}\cdot\varepsilon_j^{\lambda})=0. \tag{2.13}$$

This relation is often useful to eliminate the last term in eq. (2.11).

The other diagrams, which do not have all the collinear photons next to $u(p)$, are necessarily smaller by at least one power of m. This is due to the fact that as soon as an acollinear photon is inserted in the string of collinear photons all the fermion propagators from that point on are of order 1 instead of being of order m^{-1}.

To summarize, we can say that, with our choice of representation for $\not{\varepsilon}_i^{\lambda_i}$ in eq. (2.2), the amplitudes in the collinear situation are at most of order m^{-n} if there are n collinear photons. To this order in m, the relevant Feynman diagrams are those in which the collinear photons are attached immediately next to the external fermion which determines the collinear direction. These Feynman diagrams can easily be evaluated using the function F_n in eq. (2.11), while the rest of the diagram can be treated in the massless fermion limit. The evaluation of F_n is further simplified using the relations (2.12) and (2.13).

This analysis also shows that when all $(p\cdot k_i)$ are small, but much larger than m^2, the same set of diagrams is the only relevant one. In this case, all manipulations in the numerators of the Feynman diagrams can be done in the massless limit, all finite mass effects being of subleading order.

We shall now show on a simple example how our procedure works in practice.

3. A simple example

Consider the process

$$e^+(p_+) + e^-(p_-) \to e^+(q_+) + e^-(q_-) + \gamma(k), \tag{3.1}$$

where the momenta of the particles are given between parentheses. Let us introduce the standard notation:

$$s = (p_+ + p_-)^2, \qquad t = (p_+ - q_+)^2, \qquad u = (p_+ - q_-)^2,$$
$$s' = (q_+ + q_-)^2, \qquad t' = (p_- - q_-)^2, \qquad u' = (p_- - q_+)^2. \tag{3.2}$$

Suppose we want to evaluate this process for $\boldsymbol{k}$ nearly parallel to $\boldsymbol{p}_-$, i.e. for $(p_- \cdot k) = \mathrm{O}(m^2)$, m being the electron mass. We then take for $\not{\varepsilon}$ the following representation:

$$\not{\varepsilon}^{\pm} = N\left[\not{k}\not{p}_-\not{p}_+(1 \pm \gamma^5) + \not{p}_+\not{p}_-\not{k}(1 \mp \gamma^5)\right],$$
$$N^{-2} = 16(p_+p_-)(p_+k)(p_-k) - 8m^2(p_+k)^2. \tag{3.3}$$

From the preceding section, we now know that only the two Feynman diagrams of fig. 1 contribute in leading order in m. They are given by

$$M_1 = \frac{ie^3}{s'}\bar{v}(p_+)\gamma_\mu \frac{\not{p}_- - \not{k} + m}{-2(p_-k)}\not{\varepsilon}u(p_-)\bar{u}(q_-)\gamma^\mu v(q_+),$$

$$M_5 = -\frac{ie^3}{t}\bar{u}(q_-)\gamma_\mu \frac{\not{p}_- - \not{k} + m}{-2(p_-k)}\not{\varepsilon}u(p_-)\bar{v}(p_+)\gamma^\mu v(q_+). \tag{3.4}$$

First, we consider the helicity amplitude $M(+,-,+,-,+)$, where the arguments indicate the helicities of the particles in the order in which they appear in eq. (3.1). We proceed by evaluating $F_1(k,+)u(p_-)$ as given by eq. (2.7). The first term is

$$(p_- \cdot \varepsilon^+) = \tfrac{1}{2}N\,\mathrm{Tr}\left[\not{p}_-\not{k}\not{p}_-\not{p}_+(1+\gamma^5)\right] = \left[4N(p_+k)\right]^{-1}. \tag{3.5}$$

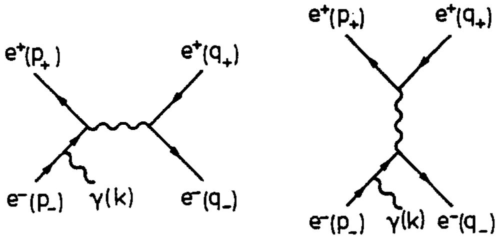

Fig. 1. Feynman diagrams for $\boldsymbol{k}$ nearly parallel to $\boldsymbol{p}_-$.

Next,

$$\not{k}\not{\varepsilon}^+ u(p_-) = N\not{k}\not{p}_+\not{p}_-\not{k}(1-\gamma^5)u(p_-)$$
$$= 2N(p_+k)\not{p}_-\not{k}(1-\gamma^5)u(p_-) + \mathrm{O}(m^2)$$
$$= 2N(p_+k)(1-\gamma^5)[2(p_-k) - m\not{k}]u(p_-) + \mathrm{O}(m^2). \qquad (3.6)$$

For the helicities we are considering, only the component of $\not{k}$ along $\not{p}_-$ is relevant, and the expressions (3.5) and (3.6) become of order m. Consequently, we can perform all the remaining manipulations in the massless limit. Inserting the appropriate fermion helicity operators, we then find

$$M(+,-,+,-,+) \simeq -ie^3\frac{N_0^2}{N}s'\Big[\frac{1}{s'}\bar{v}(p_+)\gamma_\mu(1-\gamma^5)u(p_-)\bar{u}(q_-)\gamma^\mu(1-\gamma^5)v(q_+)$$
$$-\frac{1}{t}\bar{u}(q_-)\gamma_\mu(1-\gamma^5)u(p_-)\bar{v}(p_+)\gamma^\mu(1-\gamma^5)v(q_+)\Big], \qquad (3.7)$$

with

$$N_0^{-2} = 16(p_+p_-)(p_+k)(p_-k), \qquad (3.8)$$

i.e. the normalization factor of $\not{\varepsilon}$ in the massless limit.

Using the techniques of ref. [3] to eliminate the repeated indices, etc., one finds that

$$M(+,-,+,-,+) \simeq \frac{N_0}{N}[M(+,-,+,-,+)]_{m=0}, \qquad (3.9)$$

where $[M(+,-,+,-,+)]_{m=0}$ is the massless helicity amplitude given by eq.(2.56) of ref. [3]. (To obtain this result, one should realize that, in this collinear limit, $st = s't'$.)

The result of eq. (3.9) is quite general: whenever a helicity amplitude in the zero-mass limit exists, its massive counterpart in the single photon collinear situation is proportional to it with the same proportionality factor N_0/N.

In the massive case there exist, however, amplitudes which are vanishing in the massless case. Such a "forbidden" amplitude is, e.g. $M(+,+,+,-,+)$.

For this amplitude, it is convenient to introduce two additional four-vectors t_1 and t_2, obeying the relations

$$(p_-\cdot t_1)=(p_+\cdot t_1)=0, \qquad t_1^2=-1, \tag{3.10}$$

$$t_2^\mu=\varepsilon^\mu_{\alpha\beta\gamma}p_+^\alpha p_-^\beta t_1^\gamma/(p_+\cdot p_-). \tag{3.11}$$

It follows that

$$(p_-\cdot t_2)=(p_+\cdot t_2)=0, \qquad t_2^2=-1, \qquad (t_1\cdot t_2)=0, \tag{3.12}$$

$$\not{t}_2=-i\gamma^5(\not{p}_+\not{p}_-\not{t}_1-\not{t}_1\not{p}_-\not{p}_+)/2(p_+\cdot p_-),$$

$$\not{t}_2\not{p}_+=-i\gamma^5\not{t}_1\not{p}_+. \tag{3.13}$$

As the vectors p_-, p_+, t_1 and t_2 are independent, we can decompose k:

$$k^\mu=\frac{(p_+\cdot k)}{(p_+\cdot p_-)}p_-^\mu+\frac{1}{(p_+\cdot p_-)}\left[(p_-\cdot k)-m^2\frac{(p_+\cdot k)}{(p_-\cdot p_+)}\right]p_+^\mu-(k\cdot t_1)t_1^\mu-(k\cdot t_2)t_2^\mu. \tag{3.14}$$

Introducing the decomposition of k in eq. (3.6), we find that only the terms proportional to $\not{t}_1$ and $\not{t}_2$ contribute to this helicity amplitude:

$$(1+\gamma^5)\not{k}\not{\epsilon}^+u(p_-)=4mN(p_+k)[(k\cdot t_1)\not{t}_1+(k\cdot t_2)\not{t}_2](1+\gamma^5)u(p_-)+\mathrm{O}(m^2). \tag{3.15}$$

However,

$$[(kt_1)\not{t}_1+(kt_2)\not{t}_2](1+\gamma^5)u(p_-)$$
$$=\tfrac{1}{2}[(k\cdot t_1+it_2)(\not{t}_1-i\not{t}_2)+(k\cdot t_1-it_2)(\not{t}_1+i\not{t}_2)](1+\gamma^5)u(p_-). \tag{3.16}$$

But, from eq. (3.13), it follows that

$$i\not{t}_2(1+\gamma^5)u(p_-)=\not{t}_1(1+\gamma^5)u(p_-), \tag{3.17}$$

and only the second term in eq. (3.16) contributes. On the other hand, the form $(\not{t}_1 + i\not{t}_2)(1+\gamma^5)u(p_-)$ is a solution of the Dirac equation and an eigenvector of $1-\gamma^5$. It must therefore be proportional to $(1-\gamma^5)u(p_-)$. Ignoring an irrelevant phase factor, but ensuring a correct normalization, we have

$$(\not{t}_1 + i\not{t}_2)(1+\gamma^5)u(p_-) \mathrel{\hat{=}} 2(1-\gamma^5)u(p_-). \tag{3.18}$$

Throughout this paper, the symbol $\hat{=}$ stands for an equality sign modulo a phase factor. Inserting eq. (3.18) into eq. (3.16), we have

$$[(kt_1)\not{t}_1 + (kt_2)\not{t}_2](1+\gamma^5)u(p_-) \mathrel{\hat{=}} (k\cdot t_1 - it_2)(1-\gamma^5)u(p_-). \tag{3.19}$$

It is now straightforward to write

$$\begin{aligned} M(+,+,+,-,+) \mathrel{\hat{=}} \frac{e^3N}{2(p_-k)} m(p_+k)(k\cdot t_1 - it_2) \\ \times\bigg[\frac{1}{s'}\bar{v}(p_+)\gamma_\mu(1-\gamma^5)u(p_-)\bar{u}(q_-)\gamma^\mu(1-\gamma^5)v(q_+) \\ -\frac{1}{t}\bar{u}(q_-)\gamma_\mu(1-\gamma^5)u(p_-)v(p_+)\gamma^\mu(1-\gamma^5)v(q_+)\bigg]. \end{aligned} \tag{3.20}$$

One recognizes in eq. (3.20) the same spinor structure as in the "allowed" helicity amplitude of eq. (3.7).

One more remark should be made concerning this example. When calculating the helicity amplitude $M(+,-,+,-,+)$ in the massless limit, we had to introduce two different representations for $\not{\epsilon}$ depending on what set of diagrams we were considering (see ref. [3]). This led to certain complications as one had to take into account the appropriate phase factors connecting the different representations. In the collinear configuration, however, one representation (3.3) is sufficient, as the relevant diagrams are those which have the photon next to the spinor $u(p_-)$.

4. Single collinear bremsstrahlung

In this section and the next one, we will present the relevant formulae for the cases where either one or two photons are emitted in collinear configurations. In practice, this corresponds to the most frequent situations.

First, we treat the single bremsstrahlung case. Suppose that a photon with momentum k is nearly parallel to an incoming fermion with momentum p. Let λ and λ_p be their helicities. We already know that the relevant Feynman diagrams are those where $\not\varepsilon^\lambda$ stands next to $u(p)$. On the other end of the fermion line stands another fermion spinor, which can be taken to be massless. If we calculate the helicity amplitudes of the process, this spinor will produce a helicity projection operator $\frac{1}{2}(1+\lambda'\gamma^5)$, with $\lambda' = \pm 1$, multiplying A of eq. (2.4) from the left. We proceed by evaluating $F_1(k, \lambda)$, eq. (2.7), for given values of λ, λ' and λ_p, and denote this quantity by $F_1(\lambda; \lambda', \lambda_p)$.

Suppose we first consider the case $\lambda' = \lambda$. As (see eqs. (2.2) and (2.3))

$$\not k \not\varepsilon^\lambda = N \not k \not q \not p \not k \omega_{-\lambda}, \qquad N^{-2} = 16(pq)(pk)(qk) - 8m^2(qk)^2, \tag{4.1}$$

we find that this term in F_1 does not contribute because of the operator $\omega_{\lambda'} = \omega_\lambda$ which hits it from the left. Hence, for all λ,

$$F_1(\lambda; \lambda, \lambda_p) = -[4N(pk)(qk)]^{-1}\delta_{\lambda,\lambda_p}. \tag{4.2}$$

Next, we take $\lambda' = -\lambda$ and $\lambda_p = -\lambda$. From eqs. (3.6) and (3.14), we see that the terms proportional to $\not t_1$ and $\not t_2$ do not contribute. Hence,

$$F_1(\lambda; -\lambda, -\lambda) = -(q \cdot p - k)/[4N(pq)(pk)(qk)]. \tag{4.3}$$

For $\lambda' = -\lambda$ and $\lambda_p = \lambda$, the $(p \cdot \varepsilon)$ term does not contribute, and, of eq. (3.6), only the two terms proportional to $\not t_1$ and $\not t_2$ give a non-vanishing contribution. They have, however, the effect of flipping the helicity of the spinor $u(p)$. Hence,

$$F_1(\lambda; -\lambda, \lambda) \simeq 2mN|(k \cdot t_1 + it_2)|(qk)/(pk). \tag{4.4}$$

To summarize the single collinear bremsstrahlung case, we find that all helicity amplitudes reduce to a product of a factor F_1 (eqs. (4.2)–(4.4)) times an amplitude for the process in which the collinear photon is removed. For the "allowed" amplitudes, $\lambda_p = \lambda'$, this lower-order amplitude retains the helicities of the fermions, but for the "forbidden" amplitudes, $\lambda_p = -\lambda'$, the helicity of the spinor $u(p)$ is flipped.

5. Double collinear bremsstrahlung

When two collinear photons are emitted, it may happen that they are both nearly parallel to the same fermion, or that each of them is nearly parallel to a different direction.

In the first case, we assume that $\boldsymbol{k}_1$ and $\boldsymbol{k}_2$ are close to the direction of $\boldsymbol{p}$, the momentum of the incoming fermion. We now have to evaluate the function F_2 of eq. (2.9) for the various helicity configurations. The calculation proceeds exactly like in the single collinear bremsstrahlung case, and we merely list the results. Let λ_1, λ_2 and λ_p be the helicities of photons 1 and 2 and of the fermion with momentum p. Let λ' be the signature of the helicity projection operator which is associated with the fermion spinor on the other end of the fermion line which connects with $u(p)$.

First of all, note that the relation

$$F_2(\lambda_1,\lambda_2;\lambda',\lambda_p)=\left[F_2(-\lambda_1,-\lambda_2;-\lambda',-\lambda_p)\right]^* \tag{5.1}$$

holds, which is easily proven by replacing γ^5 by $-\gamma^5$. For the allowed amplitudes, $\lambda'=\lambda_p$, we have

$$\begin{aligned}
F_2(+,+;+,+)&=\frac{1}{\Delta_{12}\Delta_1}\frac{N_2}{2N_1(qk_1)}\mathrm{Tr}\left[(\not p-\not k_1)\not k_2\not p\not q(1+\gamma^5)\right],\\
F_2(+,+;-,-)&=\frac{1}{\Delta_{12}\Delta_1}\frac{N_2(p-k_1-k_2\cdot q)}{2N_1(qk_1)(pq)}\mathrm{Tr}\left[(\not p-\not k_1)\not k_2\not p\not q(1+\gamma^5)\right],\\
F_2(+,-;+,+)&=\frac{1}{\Delta_{12}\Delta_1}\frac{1}{2N_1(qk_1)}\left\{N_2\mathrm{Tr}\left[(\not p-\not k_1)\not k_2\not p\not q(1-\gamma^5)\right]-\frac{1}{2N_2(pq)}\right\}\\
&\quad-\frac{N_2}{\Delta_{12}}\left\{2N_1(pq)\mathrm{Tr}\left[\not k_1\not p\not q\not k_2(1-\gamma^5)\right]-\frac{(qk_2)}{N_1(qk_1)}\right\},\\
F_2(+,-;-,-)&=\frac{1}{\Delta_{12}\Delta_1}\frac{N_2(p-k_1\cdot q)}{2N_1(pq)(qk_1)}\mathrm{Tr}\left[(\not p-\not k_1)\not k_2\not p\not q(1-\gamma^5)\right]\\
&\quad-\frac{N_2}{\Delta_{12}}\left\{2N_1(pq)\mathrm{Tr}\left[\not k_1\not p\not q\not k_2(1-\gamma^5)\right]-\frac{(qk_2)}{N_1(qk_1)}\right\}.
\end{aligned} \tag{5.2}$$

The allowed helicity amplitudes are now given by a sum of expressions $F_2(\lambda_1,\lambda_2;\lambda',\lambda')+F_2(\lambda_2,\lambda_1;\lambda',\lambda')$ times the helicity amplitude for the massless lower-order amplitude in which the two photons are removed, while all other particles retain their helicity assignments.

For the forbidden helicity amplitudes, $\lambda'=-\lambda_p$, we have

$$\begin{aligned}
F_2(+,+;+,-)&=0,\\
F_2(+,+;-,+)&\simeq\frac{2m}{\Delta_{12}\Delta_1}\left\{2N_1N_2(q\cdot k_1+k_2)(k_1\cdot t_1-it_2)\mathrm{Tr}\left[\not k_1\not k_2\not p\not q(1+\gamma^5)\right]\right.\\
&\quad\left.-\frac{N_1(qk_1)}{N_2(qk_2)}(k_1\cdot t_1-it_2)-\frac{N_2(qk_2)}{N_1(qk_1)}(k_2\cdot t_1-it_2)\right\},\\
F_2(+,-;+,-)&\simeq\frac{2m}{\Delta_{12}\Delta_1}\frac{N_2(qk_2)}{N_1(qk_1)}(k_2\cdot t_1+it_2),\\
F_2(+,-;-,+)&\simeq\frac{4m}{\Delta_{12}\Delta_1}N_1N_2(qk_1)(k_1\cdot t_1-it_2)\mathrm{Tr}\left[(\not p-\not k_1)\not k_2\not p\not q(1-\gamma^5)\right].
\end{aligned} \tag{5.3}$$

The forbidden helicity amplitudes are given by $F_2(\lambda_1, \lambda_2; \lambda', -\lambda') + F_2(\lambda_2, \lambda_1; \lambda', -\lambda')$ times the lower-order amplitude without the collinear photons, but with the flipped helicity for $u(p)$.

For the case where the two photons are nearly parallel to two different fermion directions, it becomes cumbersome to give general formulae analogous to eqs. (5.2) and (5.3). The reason is that a given helicity amplitude can become a linear combination of two different lower-order amplitudes, one for which the fermion helicities are unchanged and one for which the two fermions, that specify the parallel directions, have their helicities flipped. Using the techniques of sect. 2, it is, however, straightforward to work out the amplitudes case by case.

6. Discussion

In the case of single bremsstrahlung, a method was presented in ref. [3], which allows one to calculate mass corrections for the cross section in collinear situations. There are, however, kinematical configurations where that method fails while the method of this paper works, namely when the collinear photon and (anti)fermion also make a small angle with another fermion or antifermion. This would be the case, e.g. in radiative Bhabha scattering, when the final state e^+ (e^-) and γ both travel along the e^- (e^+) beam direction. It also occurs in radiative mu-pair production, when the photon and a positive or negative muon are emitted close to any one of the beam directions.

The method of ref. [3] is not applicable in these cases, as it implicitly assumes that only one kinematical invariant is small. When more than two particles have directions nearly parallel to each other, this assumption does not hold. The application of the present method, however, remains straightforward.

7. Conclusions

We have shown that there is a simple way to calculate directly, in the high-energy limit, the various helicity amplitudes for QED processes in the limit where one or more photons are radiated in directions closely parallel to fermion momenta.

This was achieved by introducing explicit polarization vectors, eq. (2.2), for collinear photons. In this way, a gauge choice was made for which only the Feynman diagrams with the collinear photons directly next to the relevant spinors gave the leading contributions.

In sect. 2, we explained in detail the general formalism for the calculation of the collinear limit. As an illustration of the simplicity of our procedure, we calculated in sect. 3 two helicity amplitudes for radiative Bhabha scattering and showed that they were proportional to nonradiative amplitudes.

In sect. 4, we found this property to be a general feature of all single collinear bremsstrahlung processes. We also listed the various proportionality factors F_1 (eqs. (4.2)–(4.4)) for every helicity configuration. Finally, in sect. 5 we treated in an analogous way the case where two photons are collinear to one and the same fermion direction.

This work will be useful in subsequent papers in which we shall write down the cross-section formulae for double bremsstrahlung processes in the high-energy limit which are valid in various regions of phase space. We suspect that it could also be useful for the study of mass singularities in connection with virtual radiative corrections.

We add here a word of caution. Throughout the present considerations, the problem was characterized by one large parameter only, namely the ratio s/m^2. Difficulties may arise if there is a second large parameter. Therefore, in particular, the present approximation may break down if the number of photons, n, is large. We do not know if such a breakdown occurs for $n = \mathrm{O}(\sqrt{s}/m)$ or $n = \mathrm{O}(\ln(s/m^2))$, or perhaps for some other value for n.

One of us (T.T.W.) thanks Professor Hans Joos, Professor Erich Lohrmann, Professor Paul Söding, Professor Volker Soergel, and Professor Thomas Walsh for their kind hospitality at DESY.

References

[1] P. de Causmaecker, R. Gastmans, W. Troost and Tai Tsun Wu, Nucl. Phys. B206 (1982) 53

[2] F.A. Berends, R. Gastmans and T.T. Wu, Univ. of Leuven preprint KUL-TF-79/022, submitted 1979 Int. Symp. Lepton and Photon Interactions at High Energies, Fermilab (Aug. 1979)

[3] F.A. Berends, R. Kleiss, P. de Causmaecker, R. Gastmans, W. Troost and Tai Tsun Wu, Nucl. Phys. B206 (1982) 61