A MONTE CARLO SIMULATION OF RANDOM SURFACES

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We use a previously proposed stochastic process and carry out Monte Carlo simulations for two models of random surfaces defined on hypercubic lattices. Tests are carried out in five dimensions. The critical coupling constants are estimated for both models. One of the models allows also results on its entropy.

Random surfaces have attracted a lot of interest [1], particularly because of their relation to non-abelian gauge theories. In ref. [2] a local stochastic process (LSP) was introduced, which allows Monte Carlo (MC) simulations of the grand canonical ensemble of random walks and random surfaces (RS). MC simulations of random walks were done in refs. [2,3]. Here we report a MC simulation of RS.

We work on an 5d cubic lattice and simulate surfaces with a fixed boundary. The surface of minimal area is choosen to be a rectangle of size $I \cdot J$ ($= 1 \cdot 1$, $3 \cdot 3$). For RS details of the LSP [2] are worked out in ref. [4] and we only repeat some main points. We will calculate expectation values with respect to the (modified) partition function

$$Z = \sum_{S} A_S \exp(-\beta A_S). \tag{1}$$

Here A_S is the area of the surface S and the sum goes over all surfaces within a model of RS as specified below. Statistical ensembles of surfaces will be generated by means of local shifts of randomly choosen plaquettes. Only shifts by one lattice unit into a direction perpendicular to the plane spanned by the choosen plaquette are considered. In accordance with detailed balance

$$P(S \to S')/P(S' \to S) = (A_{S'}/A_S)\exp[-\beta(A_{S'} - A_S)], \tag{2}$$

each shift is carried out with a probability $P(S \to S') \equiv P(A_{S'} - A_S) = P(n)$ ($n = 0, \pm 1, ..., \pm 4$). The probabilities $P(n)$ are uniquely fixed [2,4] by optimizing the LSP. The allowed modifications do not change the Euler characteristics of the surface.

We consider two models of RS. In both models each plaquette contains four links and is at each link (except at a boundary link) glued together with precisely one neighbour plaquette. In our first model (BRS) we do allow spikes. These are the planar RS of ref. [5]. In our second model (FRS) we do not allow spikes. The language is choosen in analogy to random walks [2]: Bosonic random surfaces (BRS) and fermionic random surfaces (FRS). (Walks with spikes do not contribute to the hopping expansion of Wilson fermions.)

In the computer implementation of the LSP each plaquette knows the addresses of its neighbours in the computer memory. Using also next and next–next nearest neighbours, the information is sufficient for a complete classification [4] of local "topological" situations. The local topology determines completely the allowed shifts (subtleties of detailed balance are

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Table 1
MC simulations of small surfaces ($A \leqslant 11$). ($N(A)$ = number of surfaces of area A.)

A	BRS, $N(A)$		FRS, $N(A)$	
	MC	exact	MC	exact
1	1.0 ± 0.6	1	1.01 ± 0.02	1
3	30.7 ± 5.5	36	0.0 ± 0.0	0
5	1422 ± 88	1350	6.01 ± 0.10	6
7	79460 ± 1570		167.5 ± 0.5	168
9	(2949 ± 35) × 10^3		151.5 ± 1.5	150
11	(7756 ± 36) × 10^3		≈5300	

discussed in ref. [4] and the optimized probabilities are taken from a table. Our final MC program relies on a set of fast logical decisions. In the present version it is not vectorizable. A major *advantage* is computer memory: In case of the considered two models the needed memory is – in any dimension d – proportional to the allowed maximal area of a surface. Even in large dimensions d we are able to simulate large surfaces. A major *disadvantage* is that programming is very tedious. In fact an enormous number of cross-checks were done to insure that each topological case is implemented correctly. The MC simulations carried out so far have the character of a test. The computer program is written to run in d dimensions. For our simulations we have taken $d = 5$, because this is the smallest dimension where all possible topological cases can be realized. (We always randomly select a plaquette for proposing a shift. If all four neighbour plaquettes point in different directions perpendicular to the selected plaquette, then we can in $d < 5$ not carry out a shift perpendicular to all neighbours.)

Our computer program was first tested for small surfaces, where we can count analytically the entropy factor (strong coupling expansion). Surfaces with an open 1 × 1 boundary (minimal area = 1) are considered. MC and exact results are compared in table 1. The agreement is satisfactory (only surfaces which can be reached within the truncated space $A \leqslant 11$ have to be counted). For FRS also the precision is excellent.

In case of BRS the acceptance rate of our MC procedure is bad and straightforward MC simulations become tedious. A crude estimate of the critical value β, at which the partition function (1) becomes divergent, is

$$2.15 < \beta_c < 2.30 \quad (d = 5). \tag{3}$$

The estimate is based on fig. 1. Surfaces with an open 3 × 3 boundary (minimal surface of area 9) are

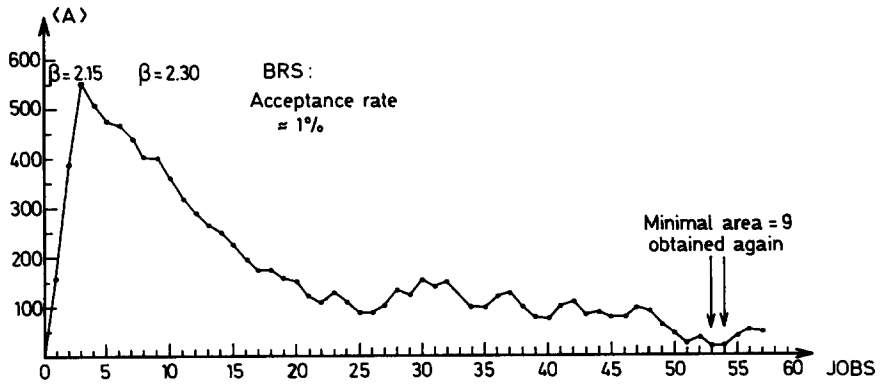

Fig. 1. Determination of β_c for 5d BRS.

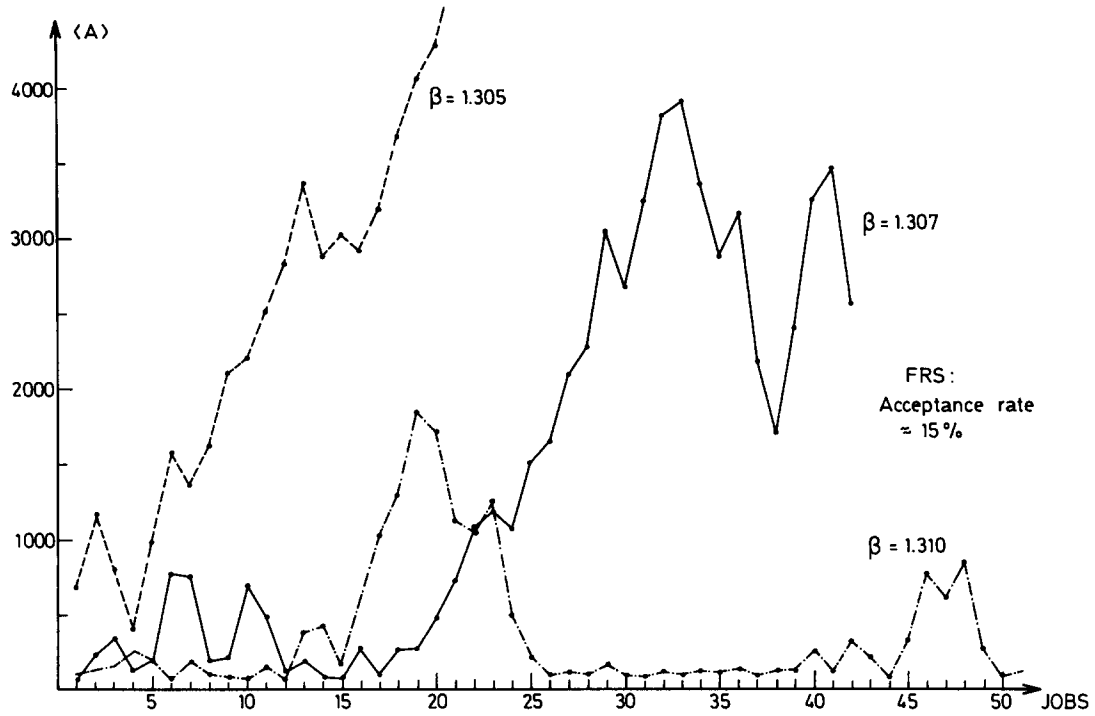

Fig. 2. Determination of β_c for 5d FRS.

simulated. At $\beta = 2.15$ the average area is divergent, whereas the surface contracts again at $\beta = 2.30$. About 50 jobs after the expansion the minimal surface is reached again for the first time. Eq. (3) is (of course) consistent with the theoretical bound [5] $\beta_c < \log [24(d-1)]$. A similar result was obtained taking an open 1×1 boundary (minimal surface of area 1).

Each "job" in fig. 1 attempts $2.4 \cdot 10^6$ iterations, but the acceptance rate for proposed shifts in only about 1% at $\beta = 2.30$. For high β-values the bottleneck of the procedure is adding new plaquettes. In the worst case of adding four plaquettes the decisive probability is $2(d-2)\exp(-4\beta) \approx 6 \cdot \exp(-4 \cdot 2.3) \approx 0.0006$. The MC procedure works much better for FRS, because these surfaces have a smaller entropy factor and therefore a lower β_c. By the same reason we expect an even better performance for self-avoiding RS.

FRS allow the rather accurate estimate

$$1.305 < \beta_c < 1.310 \quad (d = 5). \tag{4}$$

Again surfaces with an open 3×3 boundary are sampled and the evidence for eq. (4) comes from fig. 2. Each job attempts $2 \cdot 10^6$ iterations and at $\beta = 1.31$ the acceptance rate is about 15%. Above the critical point we have carried out a more detailed investigation. In table 2 results for the average area are collected together with the MC statistics. Assuming for the number of surfaces

$$N(A) = \text{const.}\, A^{\gamma} \exp(\beta_c A) \tag{5}$$

we like to determine γ from the data of table 2. As in ref. [6] we replace for large surfaces the summation in (1) by integration and obtain the scaling law

Table 2
Average area of FRS

β	$\langle A \rangle$	(Attempted) iterations
1.305	$\to\infty$	$60 \cdot 10^6$
1.310	336 ± 80	$216 \cdot 10^6$
1.320	58 ± 3	$120 \cdot 10^6$
1.330	40.2 ± 2.0	$96 \cdot 10^6$
1.340	30.9 ± 0.8	$86 \cdot 10^6$
1.350	26.4 ± 0.4	$86 \cdot 10^6$

$$\langle A_S \rangle \approx (\gamma + 2)/(\beta - \beta_c) \quad \text{for } \beta \to \beta_c. \tag{6}$$

Fitting this with our data given the order of magnitude

$$\gamma = -1.1 \pm 0.2 \quad (d = 5). \tag{7}$$

Using exact results [2] for free random walks, the asymptotic behaviour analog to (6) is only found good for very large walks. Therefore the systematic error of eq. (7) is unclear.

In conclusion we have tested a MC method [2,4], which allows direct simulation of the grand canonical ensemble (1) of free random surfaces on hypercubic lattices. In this respect the method differs considerably from other proposals [6,7]. For one of our two models, namely FRS, the simulation has turned out to be efficient. First results are given by means of eqs. (4), (7) and a more detailed analysis of this model may be carried out in the future.

By introducing a "logical" lattice generalization to selfavoiding RS is straightforward and the efficiency is expected to increase.

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