THE MERGING OF JETS

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In perturbative QCD matrix elements, the invariant mass of parton pairs is normally peaked at low values, due to the singularities of the q → qg and g → gg subprocesses. Whereas pairs at large invariant masses give rise to separately visible jets, one may expect a continuous merging back into one single jet, q or g, when the qg or gg invariant mass becomes small. Within the framework of string fragmentation, we outline an explicit model in which this takes place, and study some of the resulting properties.

In the study of jet phenomena, perturbative QCD can be used to predict the probability for obtaining different parton configurations. For the subsequent fragmentation of these partons into the experimentally observable hadrons, one has to rely on phenomenological models, since one does not yet know how to handle nonperturbative QCD. In this paper we want to discuss the "matching" of perturbative QCD and fragmentation models, and in particular describe a model in which this matching occurs very harmoniously.

Clearly separated jets have been observed in many areas of high energy physics. These jets not only represent the lowest order perturbative QCD processes, but also events associated with the emission of additional gluons. The case example is e^+e^- annihilation where, in addition to ordinary two-jet $q\bar{q}$ events, also three-jet $q\bar{q}g$ are clearly visible. Apart from the fairly small fraction of "textbook" three-jet events, there also exists a large class of events where the gluon jet is either soft or almost collinear with the q or $\bar{q}$ one, since the cross section is divergent for E_g, θ_{qg} or $\theta_{\bar{q}g} \to 0$. The individual event may here be indistinguishable from a two-jet one, whereas the statistical sample as a whole clearly points to the need of gluon emission.

In a jet cascade picture, the q (or $\bar{q}$) is created off mass shell with a virtuality Q^2, and then decays q → qg. The singularities of the matrix element then corresponds to $Q^2 \to 0$. If the new q or g still are not on mass shell, further branchings q → qg, g → gg or g → $q\bar{q}$

may take place. (The latter process is down by more than an order of magnitude compared to g → gg and may normally be neglected.) Such a cascade then leads to events where each "hard" parton, i.e. parton that gives rise to a separately visible jet, normally is composed of several essentially collinear partons.

In a second step, the fragmentation of a fairly complex partonic state would have to be considered. It is then usual to assume that, if partons are closer to each other than some given cut-off value, they are not resolvable, and should be considered to fragment just like one single parton. This "resolution power" thus defines where matrix elements or parton cascades should be cut off. This is a sensible idea but, unfortunately, the actual cut-off value used normally has to be taken on faith because, in most fragmentation models, the fragmentation of two (almost) collinear partons does not mimic that of the single "unresolved" parton. We give two examples. In models where partons are assumed to fragment independently of each other, so that each hadron can only take energy from its own parton, a split of a parton into two will lead to a complete depletion of high-momentum particles, and an increase of low-momentum ones. For models based on phase space decay of clusters (colour singlets formed after nonperturbative splittings g → $q\bar{q}$), lowering the cut off value would result in an increase in the number of clusters but a decrease in their masses, such that the relative production of heavier hadrons, e.g. baryons, would be decreased.

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What we want to discuss next is a fragmentation model which is less dependent on the cutoffs used, in the sense that two collinear partons will actually fragment the same way as a single one with the full energy. In this case the "resolution power" arguments can be applied consistently, but note that the consequences of this model will extend also to the region of separately resolvable jets. Our starting point is the traditional Lund string model, as described in ref. [1]. In previous discussions of this model, soft and collinear partons were explicitly avoided, however, which made several simplifications possible. The present description will be structured according to the three main ingredients of our model: the classical string motion, the tunnelling mechanism for string breakups and the distribution of breakup vertices; for details see ref. [2]. In the following, quarks and gluons will be assumed massless, but massive quarks can be introduced straightforwardly.

In the string model, the expected linear confinement potential between partons is represented by the use of the massless relativistic string, which provides a causal and Lorentz covariant description of the flow of energy and momentum between partons. A two-jet $q\bar{q}$ event is thus represented by a q and $\bar{q}$ at the ends of a simple string, whereas a three-jet $q\bar{q}g$ event initially corresponds to having a string stretched from the q via the g to the $\bar{q}$, i.e. two string pieces, fig. 1a. In the string piece between the g and the q ($\bar{q}$), g four-momentum is flowing towards the q($\bar{q}$) end and q($\bar{q}$) four-momentum towards the g end. When the gluon has lost all its energy, the g four-momentum continues moving away from the middle (i.e. where the gluon was), and instead a third string region is formed there, consisting of in-flowing q and $\bar{q}$ four-momentum, fig. 1a (for details, see refs. [2,3]). If this third region would only appear at a time later than the typical time scale for fragmentation, it could not affect the sharing of momentum between different particles. This is the limit of high momentum, well separated partons covered in refs. [1,4].

For a small E_g, on the other hand, the third region appears early, and the overall drawing of the string is fairly two-jet like, fig. 1b. In the limit $E_g \to 0$, the two initial string regions collapse to naught, and the ordinary two-jet event is recovered. Also for a collinear gluon, i.e. θ_{qg} (or $\theta_{\bar{q}g}$) small, the stretching becomes two-jet like, fig. 1c. In particular, the q string endpoint
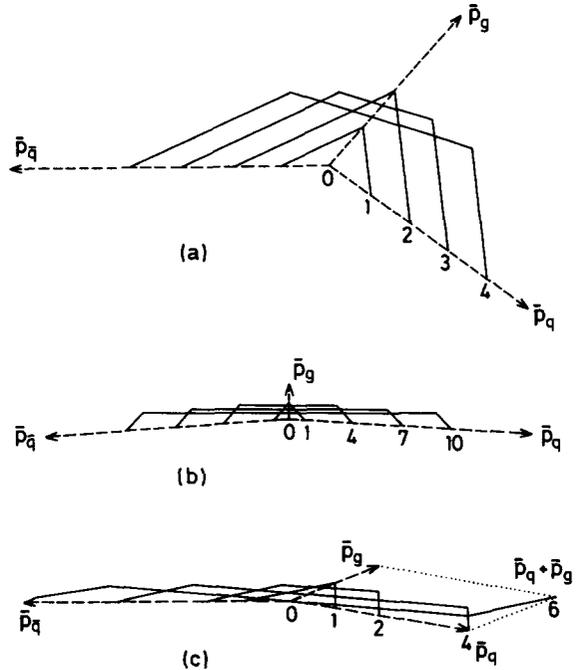

Fig. 1. The string drawing for (a) an ordinary three-jet event, (b) a three-jet event with a soft gluon, (c) a three-jet event with a collinear gluon. Dashed lines give the momenta (and hence trajectories) of the partons; in (c) dots indicate the construction of $\bar{p}_q + \bar{p}_g$. Full lines give the string shape at different times, with numbers representing time in some suitable scale [not necessarily the same in (a), (b) and (c)].

first moves out a distance $\bar{p}_q/\kappa$ (where κ is the string tension) losing energy to the attached string, and then a further distance $\bar{p}_g/\kappa$, a first half accreting energy from the string and a second half reemitting it. (This latter half actually includes yet another string piece; a corresponding piece appears at the $\bar{q}$ end, such that half a period of the system involves five different string regions.) The end result is approximately to draw out a string as if there had only been a single parton with energy $|\bar{p}_q + \bar{p}_g|$, such that the simple two-jet event again is recovered in the limit $\theta_{qg} \to 0$.

The description of the flow of energy and momentum can be generalized to arbitrarily many partons [2], provided that not only the initial momenta of the partons but also their colour arrangement, i.e. the way the string is stretched between them, is specified. This is straightforward if only $q \to qg$, $g \to gg$ or $g \to q\bar{q}$ subprocesses are considered, but not if interference ef-

fects or higher-order corrections are included, as e.g. in the full second-order $e^+e^- \rightarrow q\bar{q}gg$ cross section. Reasonable, practical procedures can still be found, however [5].

For a quark surrounded by a number of more or less collinear gluons, all connected by a string (as is the case in a cascade of an original off-mass-shell quark), the overall string drawing is approximately the same as for a single quark with energy given by the vector sum of momenta, with "wrinkles" on the string mainly carrying information on the relative $p_\perp$ of partons. There is thus normally only one string piece drawn in the central region (cf. fig. 1c), so the multiplicity of low-momentum particles will be considerably less than in independent fragmentation models. It is amusing to note that this behaviour is strongly reminiscent of the one recently shown for soft gluon emission in jet cascades [6] : the large probability for soft gluon emission, obtained when taking an incoherent sum of emission from a number of collinear partons, is largely cancelled by interference terms. The physical reason is that a soft (i.e. long-wavelength) gluon cannot resolve the individual colour charges of a quark surrounded by collinear gluons, and therefore the production probability should only be related to the total colour charge around. Obviously the string model knows nothing of quantum mechanical interference effects, but it does have a built-in "collective behaviour" for partons with small invariant mass, which reflects the total colour charge rather than the individual charges.

After this brief description of the classical flow of energy and momentum in a multi-parton event, we now turn to the fragmentation proper. In the Lund model [1] the production of $q\bar{q}$ pairs, which break the string, is described by the tunnelling mechanism: whereas a massless $q\bar{q}$ pair classically could be produced in one point and then be pulled apart by the field, the q (and $\bar{q}$) of a massive pair would have to tunnel out a length, the string energy of which corresponds to the transverse mass $m_\perp$ of the q (which is the same as that of $\bar{q}$, by flavour and momentum conservation). This gives a quantum mechanical suppression factor

$$\exp[-(\pi/\kappa)m_\perp^2] = \exp[-(\pi/\kappa)p_\perp^2] \, \exp[-(\pi/\kappa)m^2] \,, \tag{1}$$

where the string tension $\kappa \approx 1$ GeV/fm ≈ 0.2 GeV^2. This simultaneously gives a gaussian $p_\perp$ spectrum and

a suppression of heavy flavour production. The tunnelling mechanism can also be used to describe the production of baryons, if the creation of diquark–antidiquark pairs is assumed given by effective diquark masses, with corrections for the number of spin states etc. (SU(6) factors).

The production vertex of a massless $q\bar{q}$ pair can be trivially defined, but also for massive pairs an effective vertex may be defined, by the crossing point of the asymptotes to the classical motion of the q and $\bar{q}$. The position of this vertex also gives the sharing of energy and (longitudinal) momentum between the two string pieces formed by the $q\bar{q}$ breakup [1–3]. In general, many breakups of the string will occur, with the q (or $\bar{q}q$) of one vertex joining the $\bar{q}$ (or qq) of adjacent one to form a hadron. The positions of the two breakup vertices must then be related in such a way that the produced hadron ends up on mass shell. In particular, this implies that all vertices have a spacelike separation i.e. are causally disconnected.

It is therefore convenient to first consider the production of the hadron closest to the q end of the system, then the next closest, etc., such that for each new hadron the position of one vertex is already known and the position of the other is constrained by the mass requirement. Obviously one could also start at the $\bar{q}$ end, and should then obtain the same event (in an average sense). In two-jet events this requirement o "left–right symmetry" leads to an almost unique shape for the fragmentation function $f(z)$, where z is the fraction of remaining $E + p_L$ that is taken by a given hadron with transverse mass $m_\perp$ [1,7] :

$$f(z) = z^{-1}(1 - z)^a \exp(-bm_\perp^2/z) \tag{2}$$

(a slight generalization to different a's for different flavours is possible, but is not required experimentally and will be neglected here). The values of the phenomenological parameters a and b are not well known, and also slightly depend on what matrix element cuts are used, but typical values would be $a = 1$, $b = 0.5$ GeV^{-2}.

The fragmentation function of eq. (2) is tied to an effective distribution of breakup vertices in invariant time τ which, if expressed in terms of the variable $\Gamma = (\kappa\tau)^2$, has the form

$$P(\Gamma) = \Gamma^a \exp(-b\Gamma) \,, \tag{3}$$

with the same a and b values as in eq. (2). This only gives the universal distribution of vertices, however,

and the correlation between adjacent Γ values is found by applying eq. (2) and the recursive relation

$$\Gamma' = (1 - z)(\Gamma + m_\perp^2/z) , \qquad (4)$$

where Γ' is the new value obtained from Γ when taking an $E + p_L$ fraction z for the hadron with transverse mass $m_\perp$.

The generalization of these two-jet results to the multijet case is not unique. We have made the physical assumption that the fragmentation should still be characterized by eq. (3), i.e. that the distribution in invariant time of the breakup vertices does not depend on the exact shape of the string. The z variable no longer has any simple physical interpretation, but eq. (2) and eq. (4) taken together still provide a valid recipe for the relationship between adjacent Γ values. These Γ values themselves are always well defined and, if taken together with the constraint of hadrons on mass shell, uniquely defines the position of each new vertex.

Some difficulties do arise, both in the basic longitudinal fragmentation scheme and when introducing tunnelling $p_\perp$. These are generally related to the fact that, in an entirely classical description, a single hadron can be described by a fairly complicated string configuration. Quantum mechanically, however, the hadron is only characterized by its total momentum (and spin). In particular, the introduction of small "negative" string pieces does allow for considerable simplifications in the fragmentation scheme. Details are given in ref. [2], in particular the stability of the resulting physical picture to the various basic assumptions and approximations is checked and found satisfactory. The amount of necessary bookkeeping is considerable, also after simplifications, but is conveniently implemented in the form of a Monte Carlo program, an outgrowth of the traditional Lund Monte Carlo [4].

This fragmentation model can, if combined with the matrix elements of the initial hard interaction, be used to study the final state properties in a large number of processes. In particular, studies by PETRA groups find a generally good agreement with data using second order QCD [8]. Here we only point to some properties of the model.

As promised in the introduction, two jets do indeed merge to one when the invariant mass between them becomes small. This is illustrated in fig. 2, which shows the longitudinal fragmentation function of the qg side of a $q\bar{q}g$ event for different m_{qg}, assuming E_q
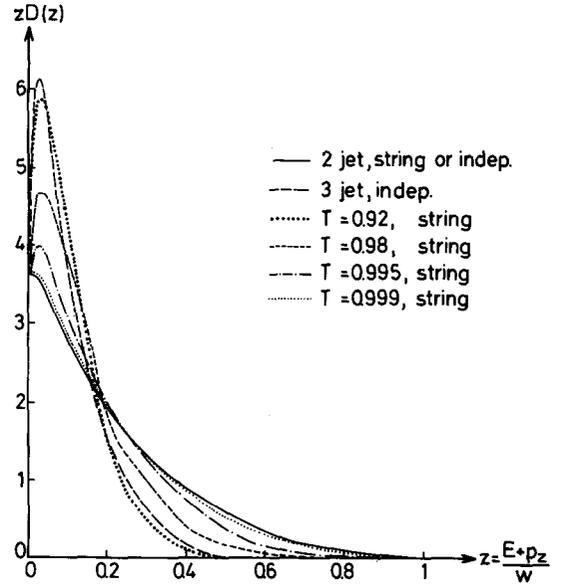

Fig. 2. Longitudinal fragmentation functions for $u\bar{u}$ and $u\bar{u}g$ events at 30 GeV. Full line gives $u\bar{u}$ events. The remaining give $u\bar{u}g$ three-jets, with $E_u = E_g$, and looking at the fragmentation of the ug side: $T = 0.92$ ($m_{ug} = 8.5$ GeV) large dots, $T = 0.98$ ($m_{ug} = 4.2$ GeV) short dashes, $T = 0.995$ ($m_{ug} = 2.1$ GeV) dash–dotted, $T = 0.999$ ($m_{ug} = 0.9$ GeV) dotted. For comparison we also show the independent fragmentation case (with a gluon fragmenting like a quark). Two-jets are the same as in the string case whereas three-jets, which essentially give the same longitudinal behaviour for the four T values considered here, are indicated by the dashed line.

$= E_g$ for simplicity. In contrast, the shape stays essentially constant in independent fragmentation models, such that there is no continuous transition between two- and three-jet events.

The string fragmentation procedure even helps reduce matrix element cut-off dependence, in the sense that a splitting of a single parton into two nearby ones gives a larger difference on the parton level than in the hadronic final state, when looking at event measures like e.g. thrust. This is an extension of the well-known observation that three-jet events "look more two-jet like" in string fragmentation [1,9]. Two partons with invariant mass smaller than 1 GeV (and directly connected by a string) leave just about no trace of their separate existence. Even up to 2–4 GeV the effect of soft and collinear gluon emission are fairly small, and can be approximated by using a slightly larger fragmentation $p_\perp$ and softer fragmentation function. Final-

ly we note that, even if the multiplicity of an event grows somewhat when more gluons are added (keeping the total energy fixed), i.e. when the string contains more "wrinkles" and hence is "longer", the relative flavour composition of particles produced in stable (neglecting the few hadrons containing the initial quark flavours).

In summary, we have developed a model for the fragmentation of an arbitrary parton configuration, with a number of very nice and nontrivial features. At present energies, this can be combined with QCD matrix elements for comparison with experimental data. At higher energies, for studies of multijet configurations, the combination with jet cascade algorithms offer interesting possibilities well worth further study.

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