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Electroweak Parameters and Leptonic Processes

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Abstract. We calculate for the standard electroweak model the 1-loop radiative corrections to purely leptonic reactions like μ decay, $\overset{(-)}{\nu}_\mu e$ scattering and μ pair production in e^+e^- annihilation. A renormalization scheme with the particle masses M_W, M_Z, M_H, m_f as parameters and the minimal number of wave function renormalization constants leading to finite Green functions is used. We perform a test of the standard model by comparing these low energy data with the results of the $P\bar{P}$ collider experiments for the W and Z boson masses.

1. Introduction

The most remarkable prediction of the Glashow–Salam–Weinberg model was the existence of the W and Z bosons together with numerical values for their masses [1]. Being a renormalizable, spontaneously broken gauge theory the standard model allows a systematic calculation of radiative corrections. These yield a contribution to the W and Z masses of the order of 3–4 GeV [2–9, 21]. The combined results of the $P\bar{P}$ collider experiments give presently [10]

$$M_W = (82.1 \pm 1.7)\,\mathrm{GeV}, \quad M_Z = (93.0 \pm 1.7)\,\mathrm{GeV}. \tag{1.1}$$

These values are in excellent agreement with the radiatively corrected values of M_W, M_Z, whereas the lowest order results are more than one standard deviation lower than (1.1). Thus one may say that the interpretation of the $P\bar{P}$ experiments are now and much more in the future sensitive to radiative corrections.

We want to point out that in spite of this fact there is not yet a commonly accepted, standardized procedure for the calculation of radiative corrections in the standard model [11]. One of the complications in this field comes from different choices of input parameters and from the use of different renormalization schemes. These different schemes have been applied to many physical processes but in view of the complexity of these higher order calculations it is difficult to estimate the consistency of different calculations and to have comprehensive tests of the standard model. Therefore it is desirable to have a well-defined, in detail elaborated renormalization scheme using parameters with a direct physical interpretation. To this end we have proposed and worked out a renormalization of the standard model with the electric charge of the electron e, M_W, M_Z, the Higgs mass M_H and the fermion masses m_f as physical parameters [12]. We have chosen the minimal number of renormalization constants yielding finite Green functions with a simple pole structure and a simple embedding of the usual QED as substructure. The parameters

$$\alpha, M_W, M_Z, M_H, m_f \tag{1.2}$$

constitute a complete set, consequently there is no room for the Weinberg angle Θ_W as an additional independent parameter. Instead $c_W = M_W/M_Z$, $s_W = (1 - M_W^2/M_Z^2)^{1/2}$ are used as abbreviations only, valid in all orders of the perturbation expansion.

In this paper we apply the renormalization scheme

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[12] of the standard electroweak theory to purely leptonic reactions like μ decay, $\nu_\mu e$ scattering and μ pair production in e^+e^- annihilation. We have restricted our analysis to leptonic processes since these are only minimally influenced by the strong interaction thereby allowing the cleanest direct tests of the electroweak interaction provided the experimental data are good enough. The observable quantities for these leptonic reactions are expressed with the parameters (1.2). Having in mind the $P\bar{P}$ results, the experimental data for the μ decay width Γ_μ, the ratio R_ν of the $\nu_\mu e$ to the $\bar{\nu}_\mu e$ cross section and the forward-backward asymmetry A_{FB} in $e^+e^- \to \mu^+\mu^-$ as well as the calculations in other schemes we present the tests of the standard model in the following steps:

—We take the measured values (1.1) of M_W, M_Z as input and deduce from them the results for Γ_μ, R_ν, A_{FB}. This allows to investigate how accurate these masses determine the weak interaction at low and intermediate energies.
—We use Γ_μ, R_ν as input and calculate M_W, M_Z. This serves for a comparison with other calculations of M_W, M_Z. We discuss also the determination of s_W^2 and the effects of radiative corrections on it.
—Finally the relations between M_W and M_Z resulting separately from Γ_μ, R_ν, A_{FB} are compared among each other and with the $P\bar{P}$ collider results. We consider this as the most comprehensive test of the GSW theory which is presently possible using leptonic reactions only.

The dependence of these quantities on the Higgs mass and the quark masses induces uncertainties coming from our almost complete ignorance of M_H and the insufficient understanding of the meaning of the quark masses. We discuss therefore the variation of the results with values of these parameters which lie within a reasonable range.

The paper is organized in the following way: in Sect. 2 the lowest order formulas for Γ_μ, R, A_{FB} are collected. These are completed with radiative corrections in Sect. 3. Section 4 contains the discussions indicated above.

2. Lowest Order Results

In this section we put together the lowest order standard model formulas for μ decay, $\nu_\mu e$ elastic scattering and the forward-backward asymmetry in $e^+e^- \to \mu^+\mu^-$. This shall serve to define our notation and to introduce those observables for which we calculate the radiative corrections. In accordance with the renormalization scheme [12] we express these quantities with help of $\alpha = e^2/4\pi$, M_W, M_Z. For convenience we use also $c_W = M_W/M_Z$ and $s_W^2 = 1 - M_W^2/M_Z^2$.

a) μ Decay. Neglecting terms of order $(m_e/m_\mu)^3$ and $(m_\mu/M_W)^2$ the lowest order expression for the decay width for $\mu^- \to \nu_\mu \bar{\nu}_e e^-$ is given by:

$$\Gamma_\mu^0 = \frac{\alpha^2}{384\pi} m_\mu \left(1 - 8\frac{m_e^2}{m_\mu^2}\right) \frac{m_\mu^4}{M_W^4} \cdot \frac{1}{(1 - M_W^2/M_Z^2)^2} = \frac{\alpha^2}{384\pi} m_\mu \cdot \left(1 - 8\frac{m_e^2}{m_\mu^2}\right)\left(\frac{m_\mu}{M_W s_W}\right)^4. \tag{2.1}$$

b) $\overset{(-)}{\nu}_\mu e$ Scattering. The ratio

$$\Delta_\nu = \frac{\sigma(\nu_\mu e) - \sigma(\bar{\nu}_\mu e)}{\sigma(\nu_\mu e) + \sigma(\bar{\nu}_\mu e)} = \frac{\sigma(\nu_\mu e)/\sigma(\bar{\nu}_\mu e) - 1}{\sigma(\nu_\mu e)/\sigma(\bar{\nu}_\mu e) + 1} = \frac{R_\nu - 1}{R_\nu + 1} \tag{2.2}$$

is well-suited for our purpose since it is sensitive to the ratio M_W/M_Z resp. s_W^2 and less subject to systematic errors than the cross sections themselves; moreover it is free of electromagnetic higher order corrections. With the axial and vector coupling constants of the electron to the Z:

$$a = -1/4 s_W c_W, \quad v = a(1 - 4s_W^2), \quad \xi = v/a = 1 - 4s_W^2 \tag{2.3}$$

this ratio has in lowest order the simple form:

$$\Delta_\nu^0 = \frac{\xi}{1+\xi^2} \quad \text{or} \quad R_\nu^0 = \frac{1+\xi+\xi^2}{1-\xi+\xi^2}. \tag{2.4}$$

c) Forward–Backward Asymmetry in $e^+e^- \to \mu^+\mu^-$. The forward backward asymmetry $A_{FB}(x)$ in e^+e^- annihilation into μ pairs is defined as ($c = \cos\theta$):

$$A_{AB}(x) = \frac{\int_0^x dc \frac{d\sigma}{d\Omega} - \int_{-x}^0 dc \frac{d\sigma}{d\Omega}}{\int_0^x dc \frac{d\sigma}{d\Omega} + \int_{-x}^0 dc \frac{d\sigma}{d\Omega}}. \tag{2.5}$$

$d\sigma/d\Omega$ reads in Born approximation

$$\frac{4s}{\alpha^2} \cdot \frac{d\sigma^0}{d\Omega} = 1 + c^2 + 2\chi(s)[v^2(1+c)^2 + 2a^2 c] + \chi(s)^2[(v^2+a^2)^2(1+c^2) + 4v^2a^2 \cdot 2c] \tag{2.6}$$

where:

$$\chi(s) = \frac{s}{s - M_Z^2}. \tag{2.7}$$

This gives:

$$A_{FB}^0(x,s) = \frac{x}{1 + x^2/3} \cdot 2a^2\chi(s) \cdot \frac{1 + 2v^2\chi(s)}{1 + 2v^2\chi(s) + (v^2+a^2)^2\chi(s)^2}. \tag{2.8}$$

At PETRA/PEP energies one is allowed to neglect the imaginary part $M_Z \Gamma_Z$ in the denominator of χ since $(\Gamma_Z/M_Z)^2 \ll 1$. Also we know that $v^2 \ll a^2$ and consequently may simplify (2.8) and perform a slight redefinition of A:

$$\mathrm{A}^0_{\mathrm{FB}}(s) = \mathrm{A}^0_{\mathrm{FB}}(1, s) \cong \tfrac{3}{2} a^2 \chi/(1 + a^4 \chi^2) \tag{2.9}$$

with

$$a^2 \chi = -\frac{s}{16 M_W^2 (1 - M_W^2/M_Z^2)} \frac{M_Z^2}{M_Z^2 - s}. \tag{2.10}$$

The measurement of Γ_μ, Δ_ν, $\mathrm{A_{FB}}$ and M_W, M_Z yields five experimental numbers. In the lowest order there are two essential parameters in the standard model. Therefore besides the determination of these parameters three independent tests of the model can be performed. The accuracy of the experiments requires the consideration of radiative corrections*. The standard model is renormalizable and consequently allows the calculation of radiative corrections thereby providing the possibility to test also the more subtle parts of the dynamics of the electroweak interaction.

3. Radiative Corrections

In [12] we have presented a renormalization scheme for the standard model using α, M_W, M_Z, the Higgs mass M_H and the fermion masses m_f as parameters. There are contained also explicit expressions for the renormalized self energies and vertex functions. Using these results we obtain for:**

a) μ Decay

$$\begin{aligned}\Gamma_\mu &= \Gamma_\mu^0 \left\{ 1 + \frac{\alpha}{2\pi}\left(\tfrac{25}{4} - \pi^2\right) - 2\Pi^W(0) \right. \\ &\quad \left. + \frac{\alpha}{2\pi s_W^2}\left[6 + \frac{7 - 4 s_W^2}{2 s_W^2} \ln c_W^2 \right] \right\} \\ &= \Gamma_\mu^0 (1 + \delta\Gamma_\mu/\Gamma_\mu^0).\end{aligned} \tag{3.1}$$

The first correction term is the familiar QED correction in the Fermi model [13], the second the contribution of the transverse part of the W self energy Σ_T^W at $s = 0$, $\Pi^W(0) = -\Sigma_T^W(0)/M_W^2$, the last term the sum of the vertex and box diagrams together with the ν_e, ν_μ wave function renormalizations.

b) $\overset{(-)}{\nu}_\mu e$ Scattering

$$\begin{aligned}\Delta_\nu &= \frac{\xi + \Delta^{\gamma Z} - V - \xi A}{1 + \xi^2 - 2 A \xi^2 + 2\xi(\Delta^{\gamma Z} - V)} \\ &= \Delta_\nu^0 + \delta\Delta_\nu\end{aligned} \tag{3.2}$$

with the contribution of the γZ mixing energy

$$\begin{aligned}\Delta^{\gamma Z} &= 4 c_W s_W \Pi^{\gamma Z}(0) + \frac{2\alpha}{3\pi}\left(\ln\frac{M_W^2}{m_\mu^2} + 1 \right) \\ &= -4 c_W^2 \left(\frac{\delta M_Z^2}{M_Z^2} - \frac{\delta M_W^2}{M_W^2} \right)\Bigg|_{\mathrm{fin}} \\ &\quad + \frac{\alpha}{3\pi}\left(3 + 2 c_W^2 + 2 \ln\frac{M_W^2}{m_\mu^2} \right) \\ &\quad - \frac{\alpha}{4\pi}\frac{2}{3}\left(\ln\frac{m_u^2}{m_d^2} + \ln\frac{m_c^2}{m_d^2} + \ln\frac{m_t^2}{m_b^2} \right) \\ &\quad - \frac{\alpha}{4\pi}\frac{c_W^2}{s_W^2}\frac{m_t^2}{M_W^2} \ln\frac{m_t^2}{m_b^2}\end{aligned} \tag{3.3}$$

and of the box diagrams containing two massive gauge bosons:

$$\begin{aligned}V &= -\frac{\alpha}{\pi}\left[\frac{1}{s_W^2} + 3 v a \right], \\ A &= -\frac{\alpha}{\pi}\left[\frac{1}{s_W^2} + \frac{3}{2}(v^2 + a^2) \right].\end{aligned} \tag{3.4}$$

The weak contributions to the renormalized $Z\nu\nu$ and Zee vertex functions vanish in our scheme at zero momentum transfer, yielding the simple expressions above.

c) Forward-Backward Asymmetry in $e^+e^- \to \mu^+\mu^-$. The radiative corrections to $d\sigma/d\Omega$ can be divided into electromagnetic (real and virtual photonic corrections) and purely weak parts:

$$\frac{d\sigma}{d\Omega} = \frac{d\sigma^0}{d\Omega}(1 + C_{\mathrm{em}} + C_w). \tag{3.5}$$

The electromagnetic corrections C_{em} and their influence on $\mathrm{A_{FB}}$ have been treated in [14] and especially in [15]. Therefore we do not reproduce the expressions for C_{em} in this paper but take the formulas of [15] for the numerical evaluation of their contribution to $\mathrm{A_{FB}}$.

The purely weak part C_w is built up from the Z self energy, the gauge part of the γ self energy, the γZ mixing energy, the weak contributions to the e and μ photon and Z form factors and the box graphs with two heavy bosons. For PETRA/PEP energies, neglecting terms of order $\alpha/2\pi \cdot (|t|/M_Z^2)$ these box contributions become independent of $c = \cos\theta$, and therefore the weak corrections can be written in the following way:

$$\begin{aligned}\frac{d\sigma^0}{d\Omega} \cdot C_w &= (1 + c^2)[C_w^{\gamma,+} + 2\chi C_w^{\gamma Z,+} + \chi^2 C_w^{Z,+}] \\ &\quad + 2c[2\chi C_w^{\gamma Z,-} + \chi^2 C_w^{Z,-}].\end{aligned} \tag{3.6}$$

These terms modify the expression (2.9) for the

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* These depend on the additional parameters M_H, m_f whose influence on the results is discussed below

** terms of $O\left(\alpha \frac{m_f^2}{M_W^2}\right)$, $f \neq t$, are neglect forward–backward asymmetry A_{FB} to become:

$$A_{FB} = \frac{3}{4} \cdot \frac{2\chi(a^2 + C_w^{\gamma Z,-}) + \chi^2(4v^2a^2 + C_w^{Z,-})}{1 + C_w^{\gamma,+} + 2\chi(v^2 + C_w^{\gamma Z,+}) + \chi^2((v^2+a^2)^2 + C_w^{Z,+})}. \quad (3.7)$$

Now we write down the explicit form of the corrections C_w:

$$\begin{aligned}
C_w^{\gamma,+} &= -2\Pi_w^\gamma + 4F_{V,w}^{\gamma e},\\
C_w^{\gamma Z,+} &= -v^2(\Pi_w^\gamma + \Pi^{\gamma Z}) - 2v\Pi^{\gamma Z} + 2vF_{V,w}^{Ze}\\
&\quad + 2v(vF_{V,w}^{\gamma e} + aF_{A,w}^{\gamma e})\\
&\quad + 4v^2a^2A_1^{ZZ} + (2s_W)^{-4}V_1^{WW},\\
C_w^{\gamma Z,-} &= -a^2(\Pi_w^\gamma + \Pi^Z) + 2aF_{A,w}^{Ze} + 2a(vF_{A,w}^{\gamma e}\\
&\quad + aF_{V,w}^{\gamma e}) + (v^2+a^2)^2A_1^{ZZ} + (2s_W)^{-4}V_1^{WW},\\
C_w^{Z,+} &= -2(v^2+a^2)^2\Pi^Z - 2v(v^2+a^2)\cdot 2\Pi^{\gamma Z}\\
&\quad + 4(v^2+a^2)(vF_{V,w}^{Ze} + aF_{A,w}^{Ze}) + (v\cdot 2va\\
&\quad + a(v^2+a^2))^2A_1^{ZZ} + (v+a)^2(2s_W)^{-4}V_1^{WW},\\
C_w^{Z,-} &= -8v^2a^2\Pi^Z - 8a^2v\Pi^{\gamma Z} + 8va(vF_{A,w}^{Ze}\\
&\quad + aF_{V,w}^{Ze}) + (v(v^2+a^2) + a\cdot 2va)^2A_1^{ZZ}\\
&\quad + (v+a)^2(2s_W)^{-4}V_1^{WW}
\end{aligned} \quad (3.8)$$

The quantities Π are related to the renormalized transverse self energies $\hat{\Sigma}_T(s)$ (calculated in [12]):

$$\begin{aligned}
\Pi_w^\gamma(s) &= \frac{1}{s}\operatorname{Re}\hat{\Sigma}_{T,w}^\gamma(s) \quad \text{(non-fermionic part)},\\
\Pi^{\gamma Z}(s) &= \frac{1}{s}\operatorname{Re}\hat{\Sigma}_T^{\gamma Z}(s).\\
\Pi^{Z,W}(s) &= \frac{1}{s - M_{Z,W}^2}\operatorname{Re}\hat{\Sigma}_T^{Z,W}(s).
\end{aligned} \quad (3.9)$$

The formfactors $F_{V,w}^{Ze}, \ldots, F_{A,w}^{\gamma e}$ are built from the functions $\Lambda_{2,3}(s, M^2)$ (defined and discussed in [12]) and coupling constants:

$$\begin{aligned}
F_{V,w}^{Ze}(s) &= \frac{\alpha}{4\pi}\left[v(v^2+3a^2)\operatorname{Re}\Lambda_2(s,M_Z^2)\right.\\
&\quad\left. + \frac{1}{8s_W^3c_W}\operatorname{Re}\Lambda_2(s,M_W^2) - \frac{3c_W}{4s_W^3}\Lambda_3(s,M_W^2)\right],\\
F_{A,w}^{Ze}(s) &= \frac{\alpha}{4\pi}\left[a(3v^2+a^2)\operatorname{Re}\Lambda_2(s,M_Z^2)\right.\\
&\quad\left. + \frac{1}{8s_W^3c_W}\operatorname{Re}\Lambda_2(s,M_W^2) - \frac{3c_W}{4s_W^3}\Lambda_2(s,M_W^2)\right],\\
F_{V,w}^{\gamma e}(s) &= \frac{\alpha}{4\pi}\left[(v^2+a^2)\operatorname{Re}\Lambda_2(s,M_Z^2)\right.\\
&\quad\left. + \frac{3}{4s_W^2}\Lambda_3(s,M_W^2)\right],\\
F_{A,w}^{\gamma e}(s) &= \frac{\alpha}{4\pi}\left[2va\cdot\operatorname{Re}\Lambda_2(s,M_Z^2) + \frac{3}{4s_W^2}\Lambda_3(s,M_W^2)\right].
\end{aligned} \quad (3.10)$$

In the renormalization scheme of [12] these form factors vanish for $s=0$ and are for energies $\sqrt{s} \lesssim 45\,\text{GeV}$ smaller than 10^{-3}. Finally the low energy approximations of the ZZ, WW box diagrams have the simple form (coupling constants removed):

$$A_1^{ZZ} = -3\frac{\alpha}{4\pi}, \quad V_1^{WW} = \frac{\alpha}{4\pi} \quad (3.11)$$

yielding terms of the order of magnitude of less than 10^{-3}.

If an accuracy of the relative corrections to A_{FB} at PETRA/PEP energies of 10^{-3} is desired, one is allowed to neglect in the contributions to C_w all terms but the self energies. Then one gets for $A_{FB}^{\text{Born+weak}}$ the following expression:

$$A_{FB}^{\text{Born+weak}}(s) = \frac{3}{2}\chi a^2 \cdot \frac{1 - \Pi_w^\gamma - \Pi^Z + 2\chi v^2}{1 - 2\Pi_w^\gamma + 2\chi v^2(1 - \Pi_w^\gamma - \Pi^Z)} \cdot \frac{\left(1 - 2\Pi^Z - \frac{2}{v}\Pi^{\gamma Z}\right)}{+\chi^2(v^2+a^2)^2\left(1 - \Pi^Z - \frac{4v}{v^2+a^2}\Pi^{\gamma Z}\right)} \quad (3.12)$$

This can be further simplified using the fact that $v^2 \ll a^2$:

$$\begin{aligned}
A_{FB}^{\text{Born+weak}} &\simeq \frac{3}{2}\chi a^2 \cdot \frac{1 - \Pi_w^\gamma - \Pi^z}{1 - 2\Pi_w^\gamma + \chi^2a^4}\\
&\simeq \frac{3}{2}\frac{\chi a^2}{1 + \chi^2a^4}(1 + \Pi_w^\gamma - \Pi^Z).
\end{aligned} \quad (3.13)$$

χ was defined in (2.7) as the ratio of the free Z and γ propagators. Therefore the result (3.13) has the simple interpretation:

$$\begin{aligned}
A_{FB}^{\text{Born+weak}}(s) &= \frac{3}{2}\frac{\chi(s)^{\text{Born+weak}}}{1 + \chi(s)^2a^4}a^2,\\
\chi^{\text{Born+weak}} &= \frac{s + \hat{\Sigma}_{T,w}^\gamma(s)}{s - M_Z^2 + \hat{\Sigma}_T^Z(s)}.
\end{aligned} \quad (3.14)$$

The lowest order expression for $\chi(s)$ has to be replaced by the renormalized one, the radiative corrections to a^2 can be neglected.

The weak i.e. non-Abelian gauge contribution to Π^γ, denoted as Π_w^γ comes from vacuum polarization by W pairs, the corresponding ghosts and unphysical charged Higgses and results in:

$$\begin{aligned}
\Pi_w^\gamma(s) &= \frac{\alpha}{4\pi}\left[\left(3 + 4\frac{M_W^2}{s}\right)F(s; M_W, M_W) - \tfrac{2}{3}\right]\\
&\simeq -\frac{\alpha}{4\pi}\cdot\frac{s}{2M_W^2} + O\left(\frac{\alpha}{4\pi}\left(\frac{s}{M_W^2}\right)^2\right)
\end{aligned} \quad (3.15)$$

($F(s; M_W, M_W)$ again is defined in [12]).

Consequently this can also be neglected at the desired accuracy at PETRA/PEP energies, leaving the

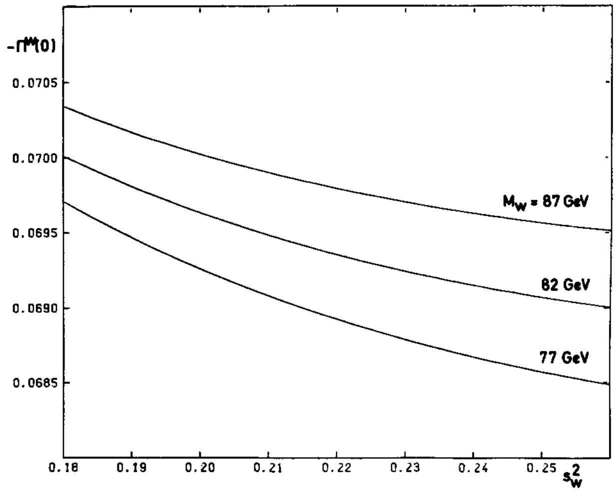

Fig. 1. The W self energy $\Pi^W(0) = -\Sigma_T^W(0)/M_W^2$ as function of s_W^2, M_W for the standard set of parameters (3.17)

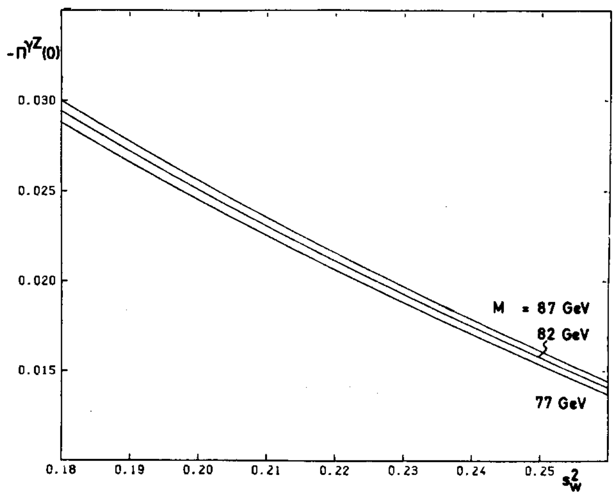

Fig. 2. The γZ mixing $\Pi^{\gamma Z}(0)$ as function of s_W^2, M_W (other parameters like in Fig. 1)

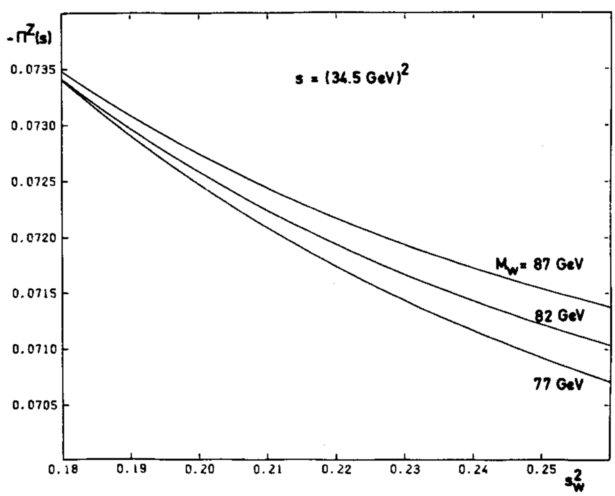

Fig. 3. The Z self energy $\Pi^Z(s)$ as function of s_W^2, M_W for $\sqrt{s} = 34.5\,\text{GeV}$ (other parameter like in Fig. 1)

simple result suited for the practical calculations:

$$A_{FB}^{\text{Born+weak}} = \frac{3}{2}\frac{\chi(s)a^2}{1+\chi^2 a^4}(1 - \Pi^Z(s)). \tag{3.16}$$

We find—in agreement with [16]—that the weak radiative corrections to A_{FB} at low energies and with an accuracy of $\delta A^{\text{weak}}/A^0 < 10^{-3}$ are determined by the transverse Z boson self energy only.

The formulas (3.1)—(3.4) and (3.14) show that the radiative corrections to Γ_μ, Δ_ν and A_{FB} contain besides trivial, only s_W dependent terms the quantities $\Pi^W(0)$, $\Pi^{\gamma Z}(0)$, $\Pi^Z(s)$ which are closely related to the gauge boson self energies. In principle the Πs can depend separately on M_W, M_Z, M_H, m_f and s. In Figs. 1, 2, 3 we present $\Pi^W(0)$, $\Pi^{\gamma Z}(0)$, $\Pi^Z(s)$ as functions of s_W^2 and M_W with our standard choice for the other parameters

$$\begin{aligned} &m_H = 100\,\text{GeV},\\ &m_u = 5\,\text{MeV}, \quad m_d = 7\,\text{MeV}, \quad m_s = 150\,\text{MeV},\\ &m_c = 1.5\,\text{GeV}, \quad m_b = 4.5\,\text{GeV}, \quad m_t = 30\,\text{GeV}. \end{aligned} \tag{3.17}$$

Obviously, in the range of W masses considered (77 − 87 GeV), we find only a weak dependence on M_W. To a good accuracy the Πs and therefore the radiative corrections at low energies depend mainly on s_W^2. For the values (1.1) of M_W, M_Z, i.e. $M_W = 82.1\,\text{GeV}$, $s_W^2 = 0.221$ we have

$$\begin{aligned} &\Pi^W(0) = -0.0694,\\ &\Pi^{\gamma Z}(0) = -0.0210,\\ &\Pi^Z(34.5\,\text{GeV}) = -0.0719. \end{aligned} \tag{3.18}$$

The masses for the u, d, s quarks used above correspond to the values given by Gasser and Leutwyler [17]. Since the quark masses are not known very precisely and since in the literature calculations of radiative corrections using much bigger values for these masses can be found, we have studied the dependence of Π^W, $\Pi^{\gamma Z}$, Π^Z on $m_i = \{m_u, m_d, m_s\}$. Defining $\delta_q \Pi = \Pi(m_{i_1}) - \Pi(m_{i_2})$ we find

$$\begin{aligned} &\delta_q \Pi^W(s) = \delta_q \Pi^Z(s) = -\frac{2\alpha}{\pi}\sum_i Q_i^2 \ln\frac{m_{i_1}}{m_{i_2}},\\ &\delta_q \Pi^{\gamma Z}(s) = \frac{\alpha}{4\pi c_W s_W}\sum_i Q_i(I_i^3 - 2Q_i)\ln\frac{m_{i_1}}{m_{i_2}}. \end{aligned} \tag{3.19}$$

The extreme choice $m_u = m_d = 300\,\text{MeV}$, $m_s = 450\,\text{MeV}$ leads to the curve for $\Pi^W(0)$ shown in Fig. 4a. $\Pi^W(0)$ is lowered by $\simeq 0.011$. In Fig. 4a we present also the variation of $\Pi^W(0)$ with the mass of the top quark. A change from $m_t = 30\,\text{GeV}$ to e.g. $m_t = 60\,\text{GeV}$ increases $\Pi^W(0)$ for $s_W^2 = 0.221$ by 0.002. Finally we do not know the Higgs mass M_H. Therefore we have displayed $\Pi^W(0)$ also for $M_H = 10\,\text{GeV}$ and $M_H = 300\,\text{GeV}$. A light Higgs mass decreases $|\Pi^W(0)|$, a heavy one increases it. The conclusion of this discussion is that our ignorance of the mass parameters gives uncertainties in the calculation of Π^W, Π^Z amounting to ± 0.01 which might be of the same order of magnitude as 2-loop effects.

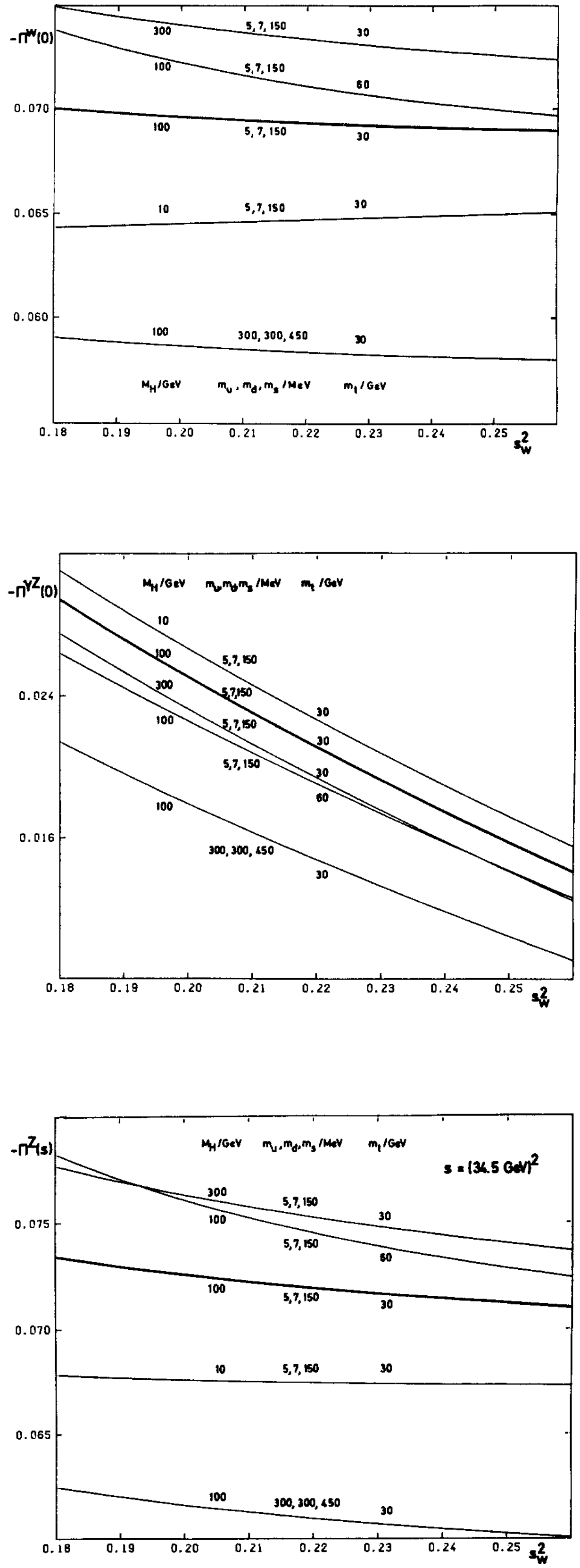

Fig. 4 a–c. $\Pi^W(0)$ **a**, $\Pi^{\gamma Z}(0)$ **b**, $\Pi^Z(s)$ **c** as functions of s_W^2 with $M_W = 82$ GeV fixed. Shown are the variations with the Higgs mass, top quark mass and the masses of the light quarks

4. Discussions

We are now prepared to discuss all the leptonic reactions considered so far at the 1-loop level. The following discussion is based on the results for Γ_μ, R_ν and A_{FB} presented in Sect. 3.

4.1 Input M_W, M_Z from $\bar{P}P$ Collider Experiments

According to the choice of M_W, M_Z as parameters in the renormalization scheme used, the most direct way to compare the predictions of the electroweak standard theory with experimental data is to start with the measured values for M_W, M_Z (or equivalently M_W, s_W^2 resp. M_Z, s_W^2) and to calculate the low energy quantities Γ_μ, R_ν and A_{FB}.

The UA1 and UA2 groups have besides M_W, M_Z also determined a value for $\Delta M = M_Z - M_W$ with an error which is smaller than that resulting from (1.1) because of a partial cancellation of the systematic uncertainties: $\Delta M = (10.9 \pm 1.6)$ GeV [10]. Together with the definition of s_W^2:

$$s_W^2 = 1 - M_W^2/M_Z^2 = \frac{\Delta M}{M_Z}\left(2 - \frac{\Delta M}{M_Z}\right) \qquad (4.1)$$

this gives for the mixing angle:

$$s_W^2 = 0.221 \pm 0.030. \qquad (4.2)$$

We use this value and M_W from (1.1) to calculate Γ_μ, R_ν and A_{FB}.

a) μ Decay. The decay width $\Gamma_\mu = \Gamma_\mu^0 + \delta\Gamma_\mu$ with Γ_μ^0 from (2.1) and $\delta\Gamma_\mu$ from (3.1) depends on both M_W and s_W^2. Using the mean values of (1.1) and (4.2) this gives:

$$\Gamma_\mu = 3.01 \cdot 10^{-16}\,\text{MeV}.$$

whereas the QED corrections to the Fermi model results yields:

$$\Gamma_\mu^0\left[1 + \frac{\alpha}{2\pi}\left(\tfrac{25}{4} - \pi^2\right)\right] = 2.61 \cdot 10^{-16}\,\text{MeV}.$$

This has to be compared with the measured value $\Gamma_\mu^{exp} = 2.9958.10^{-16}$ MeV [18]. With a fixed value of $M_W = 82.1$ GeV we obtain $2.33 \cdot 10^{-16} < \Gamma_\mu < 4.04 \cdot 10^{-16}$ MeV, corresponding to the variation of s_W^2 in (4.2).

The present accuracy of the direct M_W, M_Z measurements does not allow to predict Γ_μ with a precision that can compete with the accuracy of Γ_μ^{exp}. Instead Γ_μ^{exp} can be used as an input quantity from which for a given M_W the corresponding s_W^2 resp. M_Z is obtained as done below in 4.3.

b) $\overset{(-)}{\nu}_\mu e$ Scattering. The quantity R_ν resp. Δ_ν, (3.2), depends on M_W, M_Z mainly via the combination M_W/M_Z, because the variation of the γZ mixing energy with M_W (whence s_W^2 fixed) is small (see Fig. 2). Therefore the value of s_W^2 from (4.2) can directly be

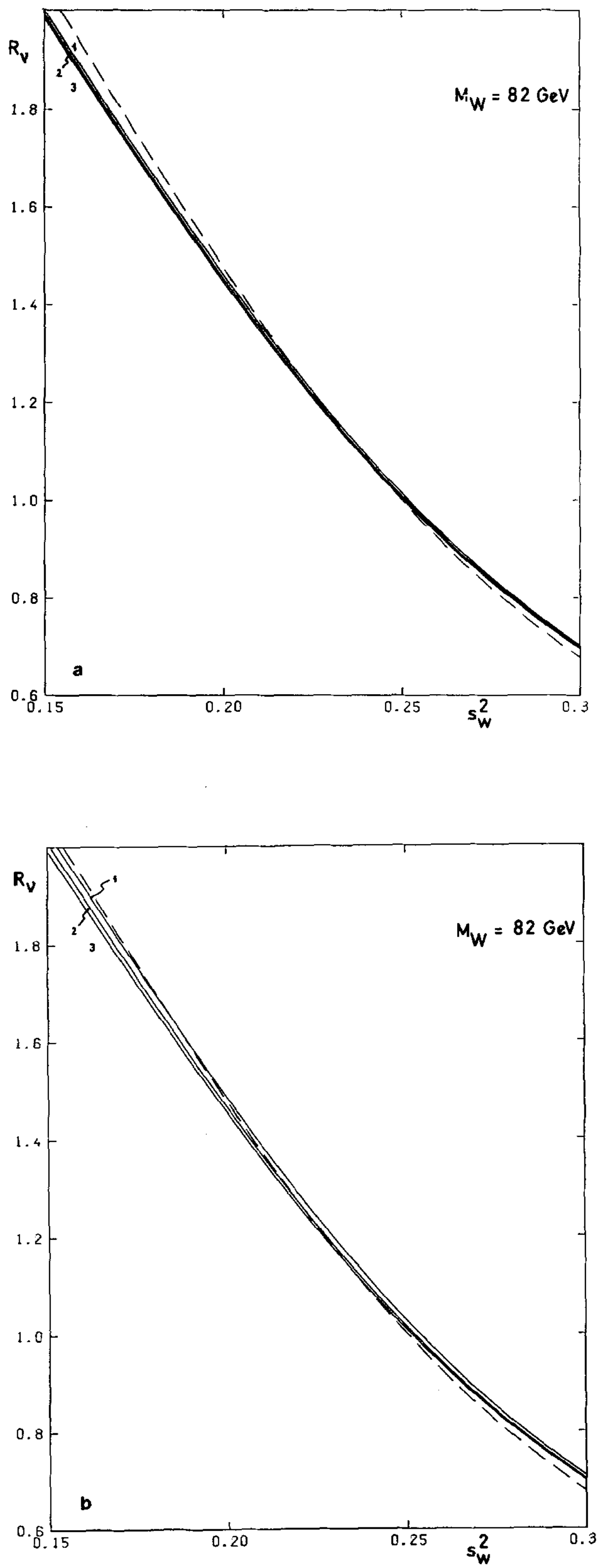

Fig. 5 a and b. R_ν as function of s_W^2 in lowest order (————) and including radiative corrections (————) for several choices of **a** M_H; $1{:}\,M_H = 300\,\mathrm{GeV}$, $2{:}\,M_H = 100\,\mathrm{GeV}$, $3{:}\,M_H = 10\,\mathrm{GeV}$ and **b** quark masses: $1{:}\,(m_u, m_d, m_s) = (300, 300, 450)$ MeV, $m_t = 30\,\mathrm{GeV}$; $2{:}\,(m_u, m_d, m_s) = (5, 7, 150)$ MeV, $m_t = 60\,\mathrm{GeV}$; $3{:}\,(m_u, m_d, m_s) = (5, 7, 150)$ MeV, $m_t = 30\,\mathrm{GeV}$; other parameters from (3.17)

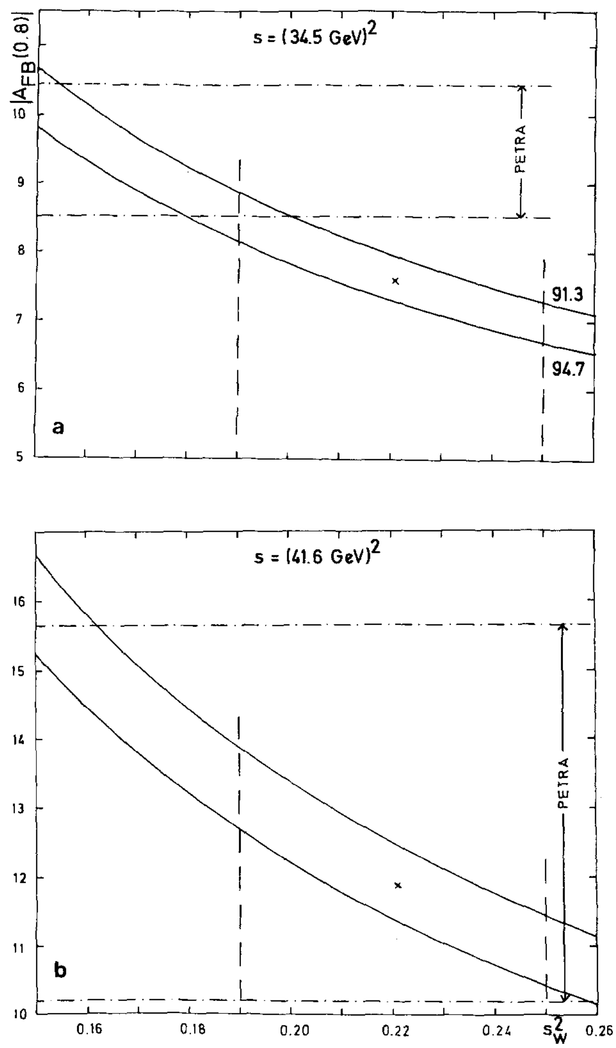

Fig. 6 a and b. $|A_{FB}(0.8)|$ at $\sqrt{s} = 34.5\,\mathrm{GeV}$ **a** and $\sqrt{s} = 41.6$ **b** as function of s_W^2 including complete electroweak radiative corrections. Shown are the curves resulting from the upper (94.7) and lower (91.3) bounds on M_Z from the $P\bar{P}$ collider experiment together with the upper and lower bounds on s_W^2 (dashed lines) from the same experiment. The crosses mark the points corresponding to the mean value $M_Z = 93.0\,\mathrm{GeV}$, $s_W^2 = 0.221$. The dashed-dotted lines mark the PETRA results

converted into the observable R_ν (see also Fig. 5):

$$R_\nu^0 = 1.26\begin{matrix} +0.32 \\ -0.27 \end{matrix}, \quad R_\nu = 1.25\begin{matrix} +0.29 \\ -0.25 \end{matrix}$$

The actual experimental value is [19]:

$$R_\nu = 1.26\begin{matrix} +0.60 \\ -0.40 \end{matrix}$$

c) Forward-Backward Asymmetry in $e^+e^- \to \mu^+\mu^-$. On the basis of the formulas (3.5 – 10) we calculate A_{FB} for the range of the Z mass in (1.1) and s_W^2 in (4.2). The α^3 contribution to A_{FB} which is of pure QED origin is not included because it is model independent and already subtracted in the experimental data. The sum of the QED corrections to Z exchange and γZ interference and the purely weak corrections turn out to be very small over the parameter range considered for realistic cuts (< 0.001 in A_{FB}). Figures 6a, b show the predictions for A_{FB} ($|\cos\theta| < 0.8$) for $\sqrt{s} = 34.5$ and 41.6 GeV with an accolinearity cut of $10°$ and an energy cut of $0.5 E_{beam}$ for the bremsstrahlung part. The experimental PETRA data [20] are shown in the figures, too.

As one can see from Fig. 6a A_{FB} for 34.5 GeV favours values of s_W^2 which are smaller than those following from the M_W/M_Z ratio. The larger experimental uncertainty of A_{FB} at 41.6 GeV (Fig. 6b) does not allow to confirm this tendency.

4.2 Low Energy Data as Input

The left-hand sides of (3.1) and (3.2) correspond to measured physical quantities. We can therefore proceed now in the opposite direction and invert (3.1) and (3.2) to derive values for M_W, M_Z from Γ_μ and R_ν (resp. Δ_ν).

Concerning the observability of radiative corrections the accuracy of the measured ratio R_ν is not good enough at present as was seen in 4.1.b); it is, however, expected to be improved in forthcoming experiments [19]. In order to take care of the uncertainty in R_ν we present out results for M_W, M_Z as functions of R_ν varying over a wide range and keeping Γ_μ, the well known reciprocal μ lifetime, fixed.

a) ν_μ e Scattering and the Weinberg Angle. In the lowest order relation (2.4) R_ν depends on M_W and M_Z only through the combination M_W/M_Z. It is therefore convenient to use $s_W^2 = 1 - M_W^2/M_Z^2$ as variable and consider $s_W^2(R_\nu)$ as a low energy determination of the Weinberg angle. The lowest order relation

$$s_W^{02} = \left(2 + 2\sqrt{\frac{\frac{1}{2} + \Delta_\nu^{\exp}}{\frac{1}{2} - \Delta_\nu^{\exp}}}\right)^{-1},$$

$$\Delta_\nu^{\exp} = \frac{R_\nu^{\exp} - 1}{R_\nu^{\exp} + 1}$$

is displayed in Fig. 5 (dashed line). A measured value of R_ν would yield a "lowest order Weinberg angle" s_W^{02}, which has to be interpreted as a tree level statement about the boson mass ratio. In this order no ambiguity concerning the physical meaning and numerical value of this quantity is involved.

At the 1-loop level $R_\nu(s_W^2, M_W)$ becomes dependent on both M_W^2 and s_W^2 (3.2). Numerically this additional mass dependence turns out to be very weak so that we are allowed to use R_ν as a function of s_W^2 only also in this order. The corrected relation (3.2) is also displayed in Fig. 5a (full curves) for various Higgs masses and in Fig. 5b for several sets of quark masses. We find only a weak dependence on these parameters. A measurement of R_ν now determines a "1-loop Weinberg angle" s_W^2, which in general is different from s_W^{02}.

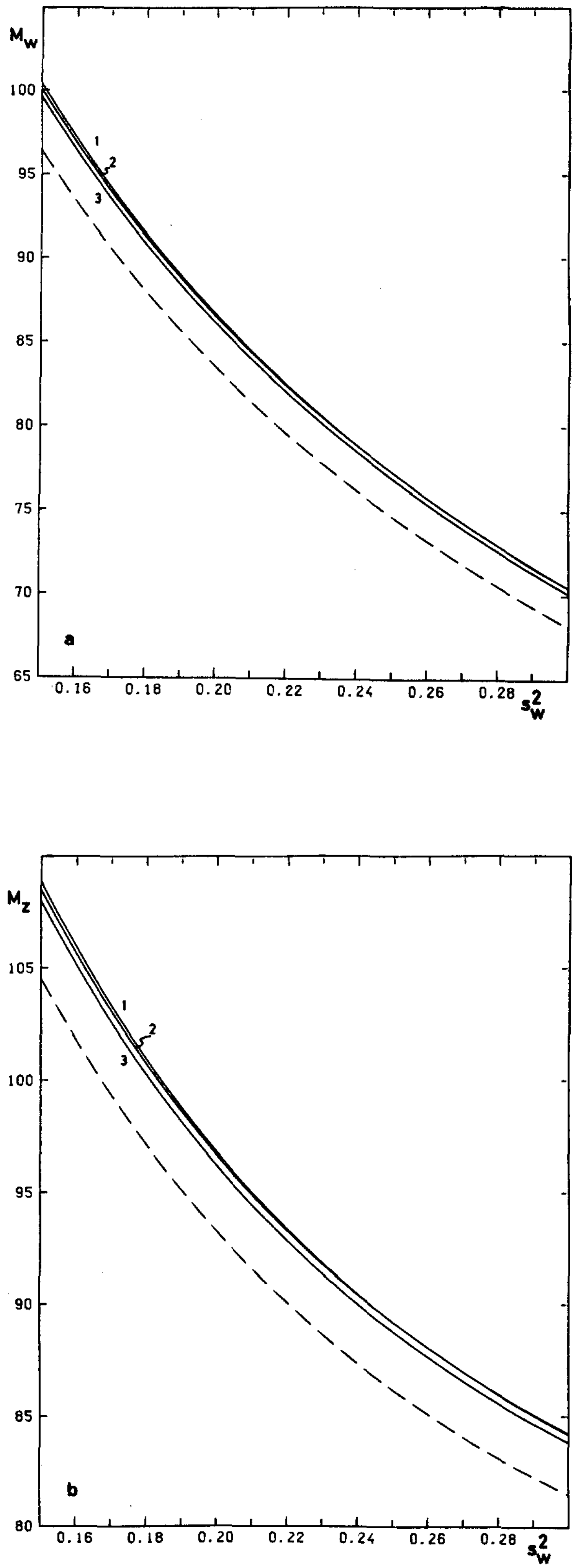

Fig. 7 a and b. M_W **a** and M_Z **b** as function of s_W^2 determined from the μ decay width in lowest order (– – – –) and including 1-loop radiative corrections (———) for: 1: $(m_u, m_d, m_s) = (5, 7, 150)$ MeV, $m_t = 60$ GeV; 2: $(m_u, m_d, m_s) = (5, 7, 150)$ MeV, $m_t = 30$ GeV; 3: $(m_u, m_d, m_s) = (300, 300, 450)$ MeV, $m_t = 30$ GeV; $M_H = 100$ GeV

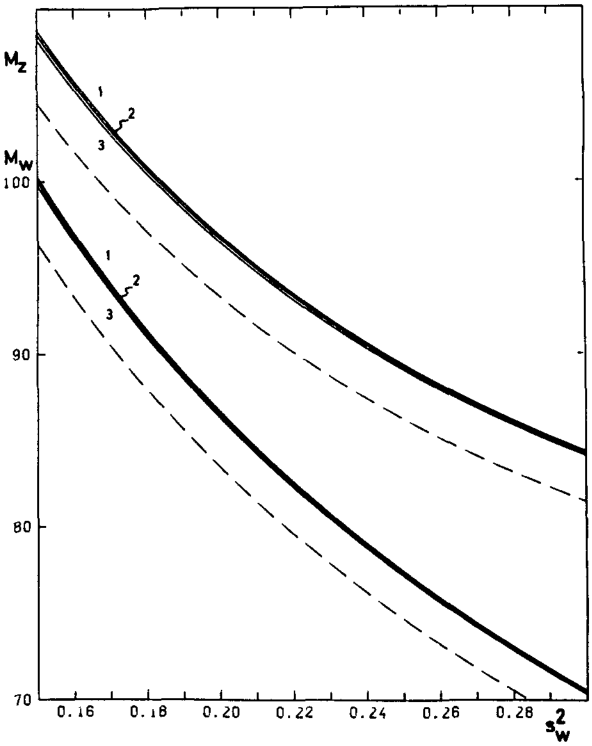

Fig. 8. Same as Figs. 7a, b for several choices of M_H: 1: $M_H = 300\,\mathrm{GeV}$, 2: $M_H = 100\,\mathrm{GeV}$, 3: $M_H = 10\,\mathrm{GeV}$. Quark masses as in (3.17)

Up to order $\alpha/4\pi$ it is possible to get a reasonable value for s_W^2 by first subtracting the radiative corrections $\delta\Delta_\nu$ evaluated for s_W^{02} from the experimental value $\Delta_\nu^{\exp}$ and then using (3.2):

$$s_W^2 \cong \left(2 + 2\sqrt{\frac{\frac{1}{2} + \Delta_\nu^{\exp} - \delta\Delta_\nu(s_W^{02})}{\frac{1}{2} - \Delta_\nu^{\exp} + \delta\Delta_\nu(s_W^{02})}}\right)^{-1}. \tag{4.4}$$

Numerically the value of s_W^2 is lowered if determined by (3.2) compared to the lowest order result s_W^{02}. As an example, an experimental value of $R_\nu = 1.26$ would give:

$s_W^{02} = 0.221$ and $s_W^2 = 0.220$.

We want to repeat that the meaning of $\sqrt{1 - s_W^2}$ is just the ratio M_W/M_Z. This ratio therefore, if it is extracted from R_ν, becomes larger at the 1-loop level. The definition $s_W^2 = 1 - M_W^2/M_Z^2$, thoroughly used in the scheme of [12] and in this analysis, is not the only one encountered in higher order calculations. Other definitions [2, 4, 6, 7, 21] lead to different values for s_W^2 even for the same experimental number of R_ν. In [2] e.g. the relation (2.4) is used to define a renormalized mixing angle $\bar\theta_W$ also in higher order. The differences between such a definition and ours are just the differences between full and dashed curves in Fig. 5: $\sin^2\bar\theta_W = s_W^{02}$.

b) μ Decay and the Boson Masses. The μ decay width $\Gamma_\mu(M_W, M_Z)$ as a precisely measured quantity sets up a relation between the boson masses, in 1-loop order given by (3.1). By inverting this equation we obtain $M_{W,Z}(s_W^2, \Gamma_\mu^{\exp})$ as a function of s_W^2. The dependence of M_W and M_Z on s_W^2 is shown in Figs. 7a, b. A change of the "light" quark masses from the standard set (3.17) to $m_u = m_d = 300\,\mathrm{MeV}$, $m_s = 450\,\mathrm{MeV}$ decreases M_W by 0.39 GeV (comp. Fig. 7a) and M_Z by 0.45 GeV (Fig. 7b). The uncertainty coming from M_H is displayed in Fig. 8. Compared to $M_H = 100\,\mathrm{GeV}$ one gets a shift of $+0.14\,\mathrm{GeV}$ for $M_H = 300\,\mathrm{GeV}$ and $-0.17\,\mathrm{GeV}$ for $M_H = 10\,\mathrm{GeV}$ in the W mass for $s_W^2 \cong 0.22$.

A combined analysis of μ decay and $\nu_\mu e$ scattering yields absolute values of the boson masses expressed by $\Gamma_\mu^{\exp}$ and $R_\nu^{\exp}$. This is done by a numerical solution of the two coupled equations

$$R_\nu(s_W^2, M_W) = R_\nu^{\exp}$$
$$\Gamma_\mu(s_W^2, M_W) = \Gamma_\mu^{\exp}, \tag{4.5}$$

where the l.-h. sides are given by (3.1) and (3.2). The mixing angle as an auxiliary quantity, which is convenient in intermediate steps but is renormalization scheme dependent, has disappeared in the final result. The dependence of M_W, M_Z on $R_\nu^{\exp}$ is shown in Figs. 9a, b. Since only experimental quantities enter these relations the result should be (essentially) independent of the renormalization scheme.

Instead of performing the numerical solution of (4.5) simultaneously it is possible without great loss of accuracy to first evaluate the radiative corrections to Γ_μ and Δ_ν using lowest order results for the parameters M_W, M_Z and then inverting (3.1) and (3.2) retaining only terms of order $\alpha/4\pi$. Because the experimental value of Δ_ν is small it is also possible to drop terms of order Δ_ν^3. Then one gets:

$$M_W = m_\mu \sqrt[4]{\frac{\alpha^2 m_\mu}{24\pi\Gamma_\mu}} \cdot \left(1 + \tfrac{1}{2}\Delta_\nu + \tfrac{3}{8}\Delta_\nu^2 + \frac{1}{4}\frac{\delta\Gamma_\mu}{\Gamma_\mu} - \frac{1}{2}\delta\Delta_\nu\right),$$

$$M_Z = m_\mu \sqrt[4]{\frac{2\alpha^2 m_\mu}{27\pi\Gamma_\mu}} \cdot \left(1 + \tfrac{1}{3}\Delta_\nu + \tfrac{1}{3}\Delta_\nu^2 + \frac{1}{4}\frac{\delta\Gamma_\mu}{\Gamma_\mu} - \frac{1}{3}\delta\Delta_\nu\right). \tag{4.6}$$

The same method to determine M_W, M_Z from R_ν and Γ_μ has also been applied by Aoki et al. [8]. Their scheme deals with the same definition of s_W^2 as ours but gives up manifest gauge invariance. For the value $R_\nu^{\exp} = 1.14$ they find (for $m_u = m_d = m_s = 100\,\mathrm{MeV}$, $M_H = 10\,\mathrm{GeV}$): $M_W = 79.2\,\mathrm{GeV}$, $M_Z = 90.5\,\mathrm{GeV}$, whereas we find (for this set of parameters): $M_W = 79.4\,\mathrm{GeV}$, $M_Z = 90.7\,\mathrm{GeV}$.

We want to compare our results also with other groups having the same definition of s_W^2: For $s_W^2 = 0.217$ Marciano and Sirlin [3] give:

$$M_W^2 = \frac{(37.281\,\mathrm{GeV})^2}{s_W^2}(1 + \Delta r) \tag{4.7}$$

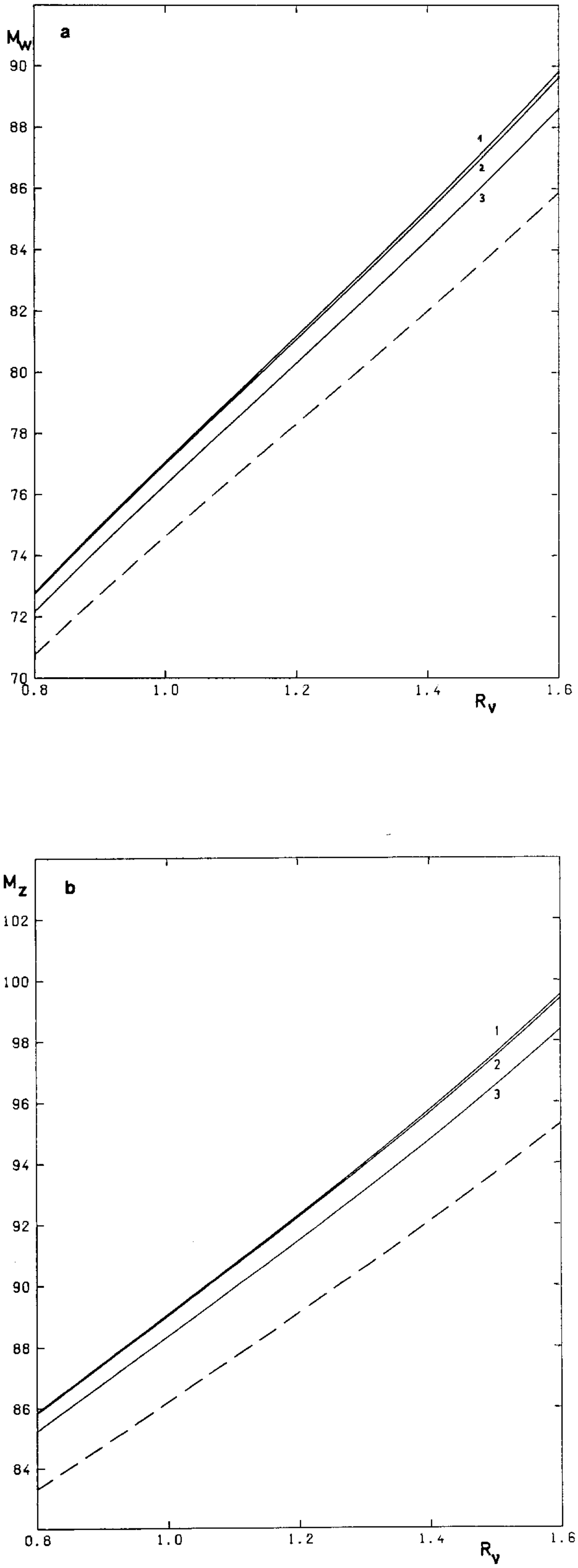

Fig. 9 a and b. M_W **a** and M_Z **b** determined from μ decay and $\nu_\mu e$ scattering as function of R_v in lowest order (— — —) and including radiative corrections (———) for: 1: $(m_u, m_d, m_s) = (5, 7, 150)$ MeV, $m_t = 30$ GeV; 2: $(m_u, m_d, m_s) = (5, 7, 150)$ MeV, $m_t = 60$ GeV; 3: $(m_u, m_d, m_s) = (300, 300, 450)$ MeV, $m_t = 30$ GeV; $M_H = 100$ GeV

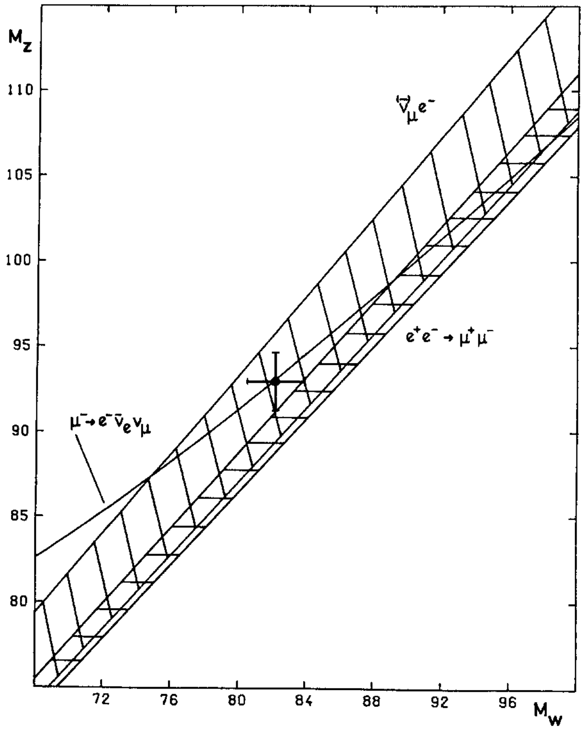

Fig. 10. Comparison of the results for the boson masses in the (M_W, M_Z) plane. Shown are the curve resulting from μ decay (———), the 68% CL band determined from $\overset{(-)}{\nu}_\mu e$ scattering (\\\\) and that from the forward–backward asymmetry in $e^+e^- \to \mu^+\mu^-$ (///). The blob with the error bars represents the combined UA1, UA2 results

with $\Delta r = 0.0696 \pm 0.0002$ $(M_H = M_Z, m_t = 36\,\text{GeV})$. We find for the corresponding quantity (same s_W^2)

$$\Delta r = \begin{cases} 0.0731 & \text{for} \quad (m_u, m_d, m_s) = (5, 7, 150)\,\text{MeV} \\ 0.0628 & \text{for} \quad (m_u, m_d, m_s) = (300, 300, 450)\,\text{MeV}. \end{cases}$$

Consoli et al. [5], who have adopted the same definition of s_w^2 as above give values for M_W, M_Z that are very close to ours (differences less than 0.1 GeV).

c) Comparison of Low and High Energy Experiments. One can now proceed to express the experimental data as results in the (M_W, M_Z) plane. This is done in Fig. 10 at the 1-loop level (for our previously specified standard set of parameters (3.17)) for:

—μ decay, which gives a curve in the (M_W, M_Z) plane;

—$\nu_\mu e$ scattering, yielding relatively weak bounds on M_W, M_Z due to the present experimental errors [19];

—the $e^+e^- + \mu^+\mu^-$ forward–backward asymmetry; the combined PETRA value at $\sqrt{s} = 34.5$ GeV is taken because of its lowest statistical error;

—the direct measurement of M_W and M_Z in the $P\bar{P}$ collider [10].

This picture represents a comprehensive test of the electroweak standard theory at the 1-loop level in the leptonic sector.

Clearly the low energy data (from Γ_μ, R_ν) and the high energy data (M_W, M_Z) are compatible with each other. The agreement would be worse if radiative corrections were not taken into account. But in order to become really sensitive to these corrections improvements in the experimental determination of M_W, M_Z and R_ν are necessary.

The agreement is less evident if A_{FB} is included: the boson masses from $p\bar{p}$ deviate from the common intersection of the other experiments by approximately 2 standard deviations.

4.3 Γ_μ and M_W as Input Parameters

The μ decay width Γ_μ is expressed in (3.1) by the boson masses M_W and M_Z. One can invert this equation to eliminate one of the masses, e.g. M_Z, in favour of Γ_μ. The calculation of other processes in terms of Γ_μ, M_W, M_H, m_f then makes best use of the numbers which are at present known with best accuracy. We apply this mixed parameter set to the following cases:

a) Prediction of M_Z and s_W^2. The determination of s_W^2 from Γ_μ and M_W is the presently best method if one refers only to leptonic processes.

With the value $\Gamma_\mu^{\exp}$ and M_W from (1.1) we get (see Fig. 7a: $M_W(s_W^2) + s_W^2$, Fig. 7b: $M_Z(s_W^2) + M_Z$)

$$s_W^2 = 0.222 \pm 0.009, \quad M_Z = (93.06 \pm 1.38)\,\mathrm{GeV}. \tag{4.8}$$

b) $\overset{(-)}{\nu}_\mu e$ Scattering. The dependence of M_W on R_ν, Fig. 9a, can be used to predict the ratio R_ν to be:

$$R_\nu = 1.24 \pm 0.08. \tag{4.9}$$

c) Forward–Backward Asymmetry in $e^+e^- + \mu^+\mu^-$. We calculate the quantity A_{FB}, (2.5) and (3.5), in two steps:

—for a given value of M_W we take s_W^2 from the relation shown in Fig. 7a ($M_W(s_W^2, \Gamma_\mu)$);

—this value s_W^2 determines the coupling constants v, a in (2.3); together with the propagator (2.7), the electromagnetic and the weak corrections, A_{FB} is obtained using (3.5 – 10).

The electromagnetic corrections to the Z parts are calculated with the same cuts as in Sects. 4.1 and 4.2.

We obtain ($\sqrt{s} = 34.5\,\mathrm{GeV}$) for $M_W = 82.1\,\mathrm{GeV}$:

$$A_{FB}(0.8) = -0.076 \pm 0.001, \tag{4.10}$$

whereas the bounds on M_W in (1.1) yield

$$-0.076 < A_{FB}(0.8) < -0.075. \tag{4.11}$$

The theoretical error comes mainly from the numerical integration of the hard photon part.

The combined PETRA result [20] is for $|\cos\theta| \leq 0.8$:

$$A_{FB}^{\exp} = -0.095 \pm 0.010.$$

This differs from (4.10) by nearly 2 standard deviations and corresponds to the situation encountered in Fig. 10.

The prediction (4.10) is widely independent of the actual values for the Higgs and quark masses. The reason is that the leading term in A_{FB} with weak corrections and with $\Gamma_\mu^{\exp}$ behaves like

$$A_{FB} \sim \frac{M_Z^2}{(\Gamma_\mu^{\exp})^{1/2}} \cdot \frac{1 - \Pi^Z(s)}{I - \Pi^W(0)}, \tag{4.12}$$

where the quark mass dependence cancels (eq. (3.19)) and the residual M_H dependence is negligible (Figs. 4a, c, $\Pi^W(0)$, $\Pi^Z(s)$). The non-leading terms in A_{FB} depend on m_q and m_H only via s_W^2 (Fig. 7b: $M_Z(s_W^2)$), but this dependence is too small to become of practical importance. The electromagnetic corrections to the Z exchange part are free from M_H and depend on m_q also very slightly. These effects are included in the error of the prediction (4.10).

In conclusion we have discussed tests of the electroweak standard model results for purely leptonic processes including 1-loop radiative corrections. The calculations were performed in a renormalization scheme which uses besides the electric charge the particle masses as physical parameters and yields finite Green functions with a minimal number of field renormalization constants. We have—where possible and not too laborious—compared our results for these processes with those of other authors.

We expect that the accuracy of the experiments will improve soon, thereby yielding the possibility of a quantitative comparison with the standard model at the level of 1-loop radiative corrections.

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