WEAK CORRECTIONS TO POLARIZATION AND CHARGE ASYMMETRIES IN $e^+e^- \to \mu^+\mu^-$ AROUND THE Z^0

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The polarization asymmetry and charge asymmetry (forward–backward asymmetry) in polarized $e^+e^- \to \mu^+\mu^-$ are calculated around the Z^0 including the weak one-loop corrections in the on-shell renormalization scheme of the electroweak standard model. These corrections to the asymmetries are generally small on resonance if the physical boson masses are used as input parameters. Asymmetries from longitudinally polarized electrons turn out to be very sensitive to the value of $\sin^2\theta_W$.

High energy e^+e^- experiments around the Z^0 resonance are expected to become the cleanest investigations of the electroweak interaction in the neutral current sector [1]. Crucial tests of the standard model consist of accurate measurements of the weak boson masses and of the neutral current coupling constants in high energy reactions as well as in low energy processes like neutrino scattering. In particular, experiments on the Z^0 will allow a precise determination of the weak coupling constants and the Z^0 mass. The leptonic vector and axial vector couplings follow from the angular distribution in $e^+e^- \to \mu^+\mu^-$ or equivalently from the integrated cross section and the forward–backward asymmetry.

The use of polarized beams in e^+e^- collisions offers the possibility of additional observables [2]. The inherence of parity violation in the electroweak interaction makes especially a longitudinal beam polarization an important experimental tool at high energies. The parity violating polarization asymmetry and charge asymmetry with polarized beams turn out to be very sensitive to the value of $\sin^2\theta_W$ and allow decisive tests of the standard model when combined with other determinations of the mixing angle.

Precise experimental tests of the electroweak standard model require theoretical predictions that include the electroweak radiative corrections at least at the one-loop level. The one-loop diagrams to $e^+e^- \to \mu^+\mu^-$ contain a subclass of "QED corrections" consisting of real and virtual photon corrections to the γ and Z^0 exchange amplitudes and the QED photon vacuum polarization. They can be made UV finite by the usual QED renormalization. These QED corrections taking into account the finite width of the Z^0 boson have been calculated by Greco et al. [3] and Berends et al. [4] for unpolarized beams and by Böhm and Hollik [5] for general polarizations. The IR divergencies are cancelled in the cross section, for that experimental energy/acollinearity cuts enter the final results.

The full electroweak one-loop corrections require the use of an appropriate renormalization scheme. In ref. [7] an on-shell scheme with finite Green functions for practical high energy calculations has been worked out in detail and has been applied to the unpolarized forward–backward asymmetry at PETRA energies [8] and the one-loop corrections to Bhabha scattering [9]. Other calculations of the radiative corrections to unpolarized $e^+e^- \to \mu^+\mu^-$ have been performed in several different renormalization schemes [6].

In this paper we apply the scheme of ref. [7] to calculate charge and polarization asymmetries around the Z^0 resonance including the weak corrections. This scheme deals with the fine structure constant α, the boson, Higgs and fermion masses M_W, M_Z, M_H, m_f as input parameters, and uses $\sin^2\theta_W = 1 - M_W^2/M_Z^2$ as the definition of the weak mixing angle. It is a natural extension of the common QED renormalization, therefore the QED subdiagrams have after renormalization the same form as in refs. [3–5,10] and can be added independently from the weak contributions considered in this paper.

The basic cross section for $e^+e^- \to \mu^+\mu^-$ (with $P_L^\pm$ as the degrees of longitudinal initial polarization) can be decomposed as follows:

$$\frac{4s}{\alpha^2}\frac{d\sigma}{d\Omega}(P_L^+,P_L^-) = (1-P_L^+P_L^-)(\sigma_u^\gamma + \sigma_u^{\gamma Z} + \sigma_u^Z)$$
$$+ (P_L^+ - P_L^-)(\sigma_L^\gamma + \sigma_L^{\gamma Z} + \sigma_L^Z) . \qquad (1)$$

The essential polarization phenomena can be obtained already with a polarized e^- and an unpolarized e^+ beam. With $P_L^+ = 0, P_L^- \equiv P_L$ we discuss in the following the observables derived from (1) [$d\sigma \equiv (d\sigma/d\Omega) \times d(\cos\theta)$]:

(i) Forward–backward or charge asymmetry A_{FB}^{unpol} (for $P_L = 0$) and A_{FB}^{pol} (for $P_L \neq 0$):

$$\frac{\int_0^1 d\sigma(P_L) - \int_{-1}^0 d\sigma(P_L)}{\int_0^1 d\sigma(P_L) + \int_{-1}^0 d\sigma(P_L)} = A_{FB} . \qquad (2)$$

(ii) Polarization asymmetry A_L:

$$\frac{\int_{-1}^1 d\sigma(P_L) - \int_{-1}^1 d\sigma(-P_L)}{\int_{-1}^1 d\sigma(P_L) + \int_{-1}^1 d\sigma(-P_L)} = -P_L A_L . \qquad (3)$$

(iii) Forward polarization asymmetry A_L^f:

$$\frac{\int_0^1 d\sigma(P_L) - \int_0^1 d\sigma(-P_L)}{\int_0^1 d\sigma(P_L) + \int_0^1 d\sigma(-P_L)} = -P_L A_L^f . \qquad (4)$$

At the tree level the expressions in (1) read (θ = angle between e^- and μ^-):

$$\sigma_u^\gamma = 1 + \cos^2\theta ,$$
$$\sigma_u^{\gamma Z} = 2\,\mathrm{Re}(\chi)\,[v^2(1+\cos^2\theta) + a^2\, 2\cos\theta] ,$$
$$\sigma_u^Z = |\chi|^2[(v^2+a^2)^2(1+\cos^2\theta) + 4v^2a^2\, 2\cos\theta] , \qquad (5)$$
$$\sigma_L^\gamma = 0 , \quad \sigma_L^{\gamma Z} = 2\,\mathrm{Re}(\chi)\, va(1+\cos\theta)^2 ,$$
$$\sigma_L^Z = |\chi|^2\, 2va(v^2+a^2)(1+\cos\theta)^2 , \qquad (6)$$

with

$$a = -1/4s_W c_W , \quad v = a(1-4s_W^2),$$
$$s_W^2 \equiv \sin^2\theta_W = 1 - M_W^2/M_Z^2 . \qquad (7)$$

and

$$\chi = s/(s - M_Z^2 + iM_Z\Gamma_Z) . \qquad (8)$$

The corresponding expressions including the QED corrections are given in ref. [5]. The inclusion of the weak corrections leads to long formulas which will be given in a more detailed publication. Neglecting terms of order $m_e^2/M_W^2, m_\mu^2/M_W^2$ they contain:

the transverse Z^0 self energy ($\hat\Sigma_T^Z$),

the transverse γZ^0 mixing energy ($\hat\Sigma_T^{\gamma Z}$),

the non-photonic vertex corrections to the γee, $\gamma\mu\mu$ and Zee, $Z\mu\mu$ vertices,

the box diagrams with $2Z^0$ and 2W exchange.

The results of our calculation are presented in figs. 1–3. The numerical computations have been done with a Z^0 mass M_Z = 93.0 GeV corresponding to the mean value from measurements at the $p\bar{p}$ collider [11]. In figs. 1, 2 s_W^2 = 0.22 has been used, equivalent to a W mass M_W = 82.1 GeV being the mean value of M_W from $p\bar{p}$ experiments [11]. The tree level curves are obtained by use of eqs. (5)–(8).

Fig. 1 contains the forward–backward asymmetry around the resonance for an unpolarized and a left-handed ($P_L = -1$) e^- beam. Polarization enhances A_{FB} from 4% to 18% at $\sqrt{s} = M_Z$. Near to the peak the weak corrections are very small in both cases (≈0.003). This is a consequence of the on-shell subtraction in the boson self-energies that makes the real part of $\hat\Sigma_T^Z$ vanish at $s = M_Z^2$, which off resonance yields the main part of the weak corrections. The effect of Re $\hat\Sigma_T^Z(s)$ becomes visible in A_{FB} at distances $\gtrsim \Gamma_Z/2$ from M_Z. One has to keep in mind, however, that the QED corrections to A_{FB} give more dramatic effects in the resonance region [3–5] and have to be respected carefully in the final results.

Fig. 2 shows the polarization asymmetry (3) resp. (4) around the Z^0, A_L integrated over the whole θ range and A_L^f integrated only over the forward hemisphere. Both A_L and A_L^f are remarkably high compared to A_{FB} and less energy dependent. Since the forward and backward directions contribute with different signs, A_L^f is bigger than A_L and may be of practical interest.

As in the previous case we find that the on-resonance weak corrections are very small in the on-shell scheme (0.002 in A_L and 0.003 in A_L^f). The corrections outside the peak are somewhat smaller than in A_{FB} because the main parts in the numerators and

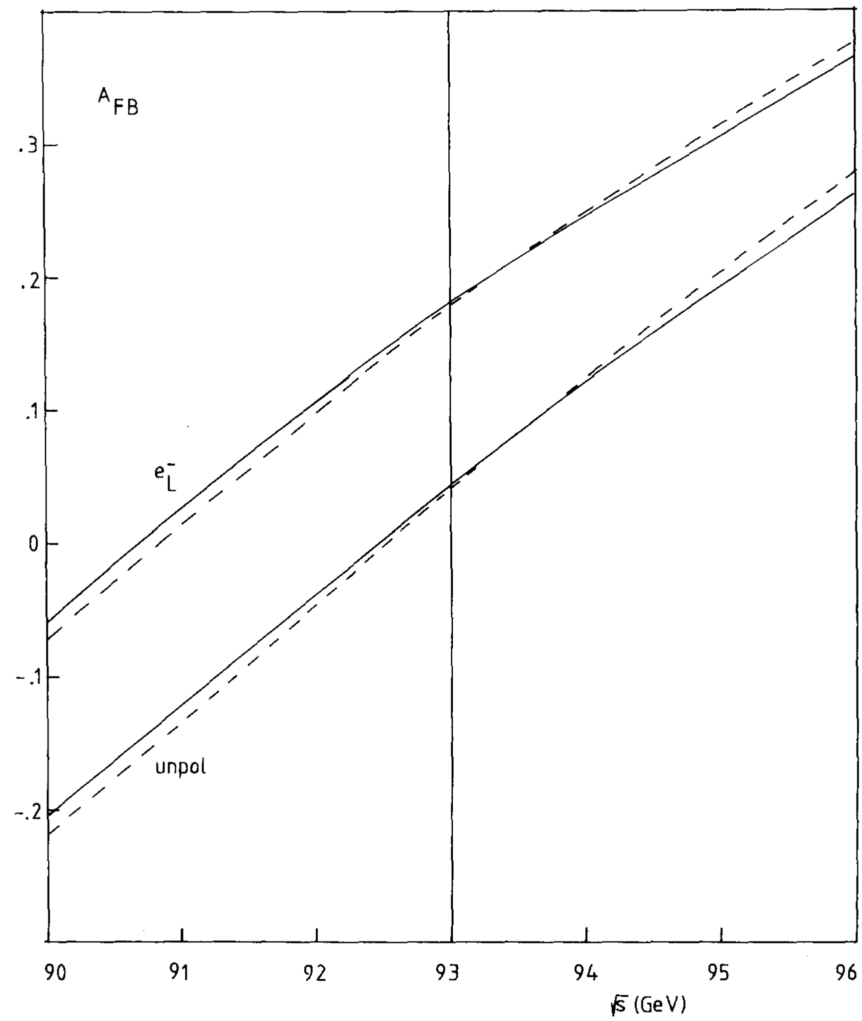

Fig. 1. Forward–backward asymmetry for unpolarized beams (A_{FB}^{unpol}) and for left-handed e^-(A_{FB}^{pol}). – – – tree level; —— with weak corrections (M_H = 100 GeV, m_t = 30 GeV).

denominators of (3) and (4) cancel. Recalling the results of ref. [5] also the QED corrections give only small changes in A_L on resonance.

All the asymmetries discussed so far depend on s_W^2 for a given M_Z. Of most practical importance are the on-resonance asymmetries ($\sqrt{s} = M_Z$) because they can be measured with great precision. In fig. 3 we give the on-resonance values as functions of s_W^2 for M_Z = 93 GeV. The results include the higher order weak effects, which, however, are small as stated above.

The unpolarized forward–backward asymmetry is in leading order proportional to $(v/a)^2 = (1 - 4s_W^2)^2$ and is therefore small for s_W^2 near to 0.25. A_{FB}^{pol} would be a more convenient quantity for a determination of s_W^2; the most sensitive quantities, however, are the polarization asymmetries A_L and A_L^f, which in leading order are proportional to $v/a = 1 - 4s_W^2$.

A value of s_W^2 obtained from the asymmetries can be converted into M_W and compared with direct M_W measurements in $p\bar{p}$. This would give a stringent test

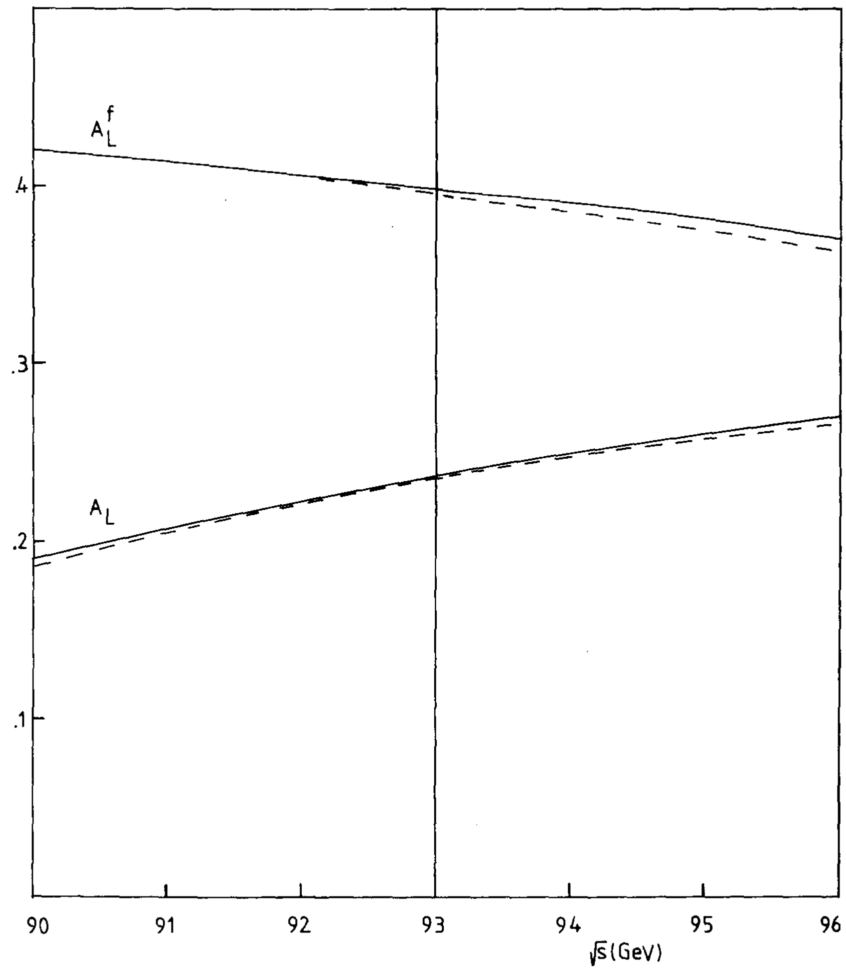

Fig. 2. Polarization asymmetries A_L ($0 \leqslant \theta \leqslant \pi$) and A_L^f ($0 \leqslant \theta \leqslant \pi/2$). – – – tree level; —— with weak corrections (M_H = 100 GeV, m_t = 30 GeV).

of the standard model if the experimental W mass becomes more precise in future.

The combination of the Z^0 asymmetries with low energy experiments like μ decay and neutrino scattering (discussed in the same renormalization scheme) offers further crucial tests:

(a) Determine s_W^2 from neutrino scattering, e.g. from the ratio $R = \sigma(\nu_\mu e)/\sigma(\bar{\nu}_\mu e)$, at the one-loop level as performed in ref. [12] and compare with that obtained from A_{FB} or A_L.

(b) Determine s_W^2 or M_W for a given M_Z from the corrected expression for the μ decay width Γ_μ. For the renormalization scheme considered here this has been done also in ref. [12] [‡1]. The combined results for M_Z = 93 GeV are indicated in fig. 3.

In conclusion the leptonic charge and polarization asymmetries on the Z^0 provide powerful tests of the

‡1 The first calculation of the weak corrections to Γ_μ was done by Sirlin [14] in a simplified scheme without field renormalization. The numerical results are very close in both schemes as shown in ref. [12].

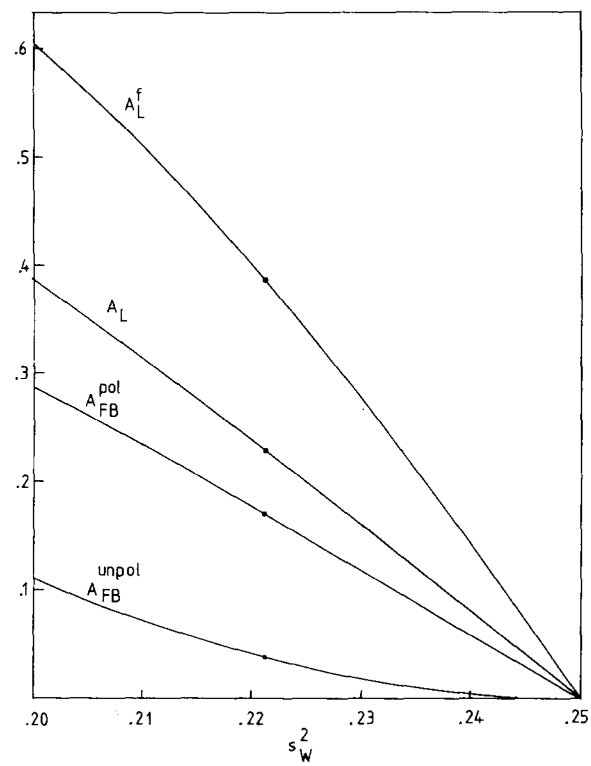

Fig. 3. On-resonance charge and polarization asymmetries for M_Z = 93 GeV as functions of s_W^2. The dots denote the values that follow from a combination with μ decay.

standard electroweak model at the one-loop level if they are combined with precise boson mass measurements and/or with the predictions for the electroweak parameters extracted from low energy experiments.

After finishing this work a paper by Lynn and Stuart [13] on the same topic came to our attention. They use the same set of input parameters except of M_W which is replaced by the μ lifetime τ_μ. This is equivalent to the combined procedure (b) outlined above. The method of using τ_μ instead of M_W leads to differences in the asymmetries if the tree level relations are used. The corrected values, however, as given in ref. [13] for the on-resonance asymmetries, are very close to ours. Small differences (~0.002) may be due to different parametrizations in the hadronic part of the self-energies and the inclusion of the QED photon vacuum polarization in the results of ref. [13].

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