CORRELATIONS IN THE SU(2) FUNDAMENTAL HIGGS MODEL

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Correlations in SU(2) lattice gauge theory with a Higgs doublet are studied by Monte Carlo calculation. Some qualitative consequences for the particle spectrum in isoscalar and isovector channels are discussed.

The SU(2) Higgs model with scalars in the fundamental (doublet) representation is of great theoretical and phenomenological interest, because it is the Higgs sector of the standard SU(2) ⊗ U(1) electroweak model [1], if fermions and electromagnetism are neglected. This model is known to have two phases: the Higgs phase with massive gauge vector bosons (with completely "broken gauge invariance") and a QCD-like confinement phase with composite particle states made out of confined scalar Higgs particles. In both phases there is a mass gap and external colour charges are screened by the scalar doublet (i.e. large Wilson loops obey the perimeter law). In fact, the two phases are not really qualitatively different, there are only quantitative differences [2]. There is a correspondence between the physical states in the two phases: the massive gauge bosons, for instance, can be considered also as a bound state made out of two confined Higgs scalars. A more confinement-like situation may even phenomenologically be a viable alternative [3]. This is due to the fact that low energy electroweak phenomenology is not sensitive to the gauge structure of the theory. It can be explained by global SU(2) invariance and γ–W_0 mixing [4].

After the discovery of the W and Z bosons [5] the problem of the spectrum of the SU(2) ⊗ U(1) model became acute, and it will become even more acute as soon as the 100 GeV energy range is available for a detailed study by the next generation of accelerators. Some, at least qualitative, insight into this question can be obtained by studying different types of correlations in the Higgs model on the lattice.

The phase structure of some simple Higgs models was numerically investigated on the lattice in several previous works (see e.g. ref. [6]). The present status of our knowledge of the phase diagram for the SU(2) Higgs model with a scalar doublet is summarized in a recent paper by Kühnelt, Lang and Vones [7], where the phase diagram was determined, including the effect of the radial degree of freedom for the Higgs field. In accordance with expectation, there is a phase-transition line between the Higgs-like and confinement-like regions. This line, however, ends at some finite values of the couplings, therefore the two regions are continuously connected. The radial degree of freedom does not seem to be crucial for the phase structure, although the position and shape of the phase transition line is somewhat changed.

In this letter some results of a Monte Carlo calculation of correlations in the SU(2) Higgs model with Higgs scalars in the fundamental (doublet) representation will be presented and discussed. The lattice formulation of the model uses the SU(2) gauge link variables $U(x, \mu) \in \mathrm{SU}(2)$ (x is the lattice site; $\mu = \pm 1, 2, 3, 4$ is the direction) and the SU(2) doublet (complex) scalar field ϕ_x on lattice sites. Following the conventions of ref. [7], the euclidean lattice action can be written

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$$S = \sum_x \left(\lambda(\phi_x^+ \phi_x - 1)^2 + \phi_x^+ \phi_x - \kappa \sum_\mu \phi_{x+\hat\mu}^+ U(x,\mu) \phi_x \right) + \beta \sum_P (1 - \tfrac{1}{2}\,\mathrm{Tr}\, U_P). \quad (1)$$

The last term with $\beta \equiv 4/g^2$ is the usual Wilson action for the SU(2) gauge field: P is a positive orientation plaquette and U_P is the plaquette gauge variable (the product of the four link variables around the plaquette). The lattice Higgs field ϕ_x and the lattice Higgs couplings κ, λ are related to the usual continuum variables by

$$a\phi_{\mathrm{cont}}(a,x) \to \phi_x \sqrt{\kappa} \quad (a = \text{lattice spacing}),$$

$$\lambda_{\mathrm{cont}} \to \lambda\kappa^{-2}, \quad (a\mu_{\mathrm{cont}})^2 \to 8 - (1 - 2\lambda)\kappa^{-1}. \quad (2)$$

The tree-level vacuum expectation value in the continuum is $v_{\mathrm{tree}} = \mu_{\mathrm{cont}}/\sqrt{\lambda_{\mathrm{cont}}}$, therefore in the limit $\lambda \to \infty$ we have $v_{\mathrm{tree}}^2 \to 2\kappa a^{-2}$. In this limit the tree-level mass of the physical Higgs particle goes to infinity: $m_{\mathrm{Higgs}} = \sqrt{2}\mu_{\mathrm{cont}}$, and the theory becomes perturbatively non-renormalizable. Nevertheless, the Higgs self-interaction becomes infinitely strong, therefore it is not clear whether perturbation theory has anything to say at all. On the lattice $\lambda \to \infty$ implies that the length of the Higgs field is frozen to $|\phi_x| = 1$. In this paper I shall use this simplification of fixed length Higgs field mainly because of technical reasons. Previous studies showed that in the phase structure there is not very much difference compared to $\lambda < \infty$ [except for very small $\lambda \cong 0$, but this case has presumably not much to do with the situation in standard SU(2) ⊗ U(1)]. In any case, the radial Higgs degree of freedom can be easily restored at some later stage.

In the lattice action (1) it is more convenient to use, instead of ϕ_x, other variables for the Higgs field ($\alpha = 1, 2$):

$$\phi_x^\alpha = \rho_x \sigma_{x,\alpha 1}, \quad \tilde\phi_x^\alpha = \rho_x \sigma_{x,\alpha 2}. \quad (3)$$

Here $\rho_x > 0$ is a real variable and $\sigma_x \in \mathrm{SU(2)}$. (The field $\tilde\phi_x$ is often used in the standard SU(2) ⊗ U(1) model: it has hypercharge $Y = -1/2$, in contrast to the hypercharge $Y = +1/2$ of ϕ_x.) In terms of the new variables the lattice action is

$$S = \sum_x \left(\rho_x^2 + \lambda(1 - \rho_x^2)^2 - \kappa \sum_{\mu>0} \rho_{x+\hat\mu} \rho_x\, \mathrm{Tr}[\sigma_{x+\hat\mu}^+ U(x,\mu) \sigma_x] \right) + \beta \sum_P (1 - \tfrac{1}{2}\,\mathrm{Tr}\, U_P). \quad (4)$$

The integration measure in these variables is $\rho_x^3\, \mathrm{d}\rho_x\, \mathrm{d}^3\sigma_x\, \mathrm{d}^3 U(x,\mu)$ with $\mathrm{d}^3 g$ as the invariant Haar measure in $g \in \mathrm{SU(2)}$. The actions in eqs. (1), (4) have an exact global SU(2) "weak-isospin" symmetry generated by $\sigma_x' = \sigma_x V$, $V \in \mathrm{SU(2)}$. This symmetry is broken in the standard SU(2) ⊗ U(1) model by the fermion mass differences and by electromagnetism.

Using local SU(2) gauge invariance the SU(2) Higgs degree of freedom σ_x can be integrated out. This can be easily seen by making a gauge transformation

$$\sigma_x' = V_x^{-1} \sigma_x = -\mathrm{i}\tau_2 = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}. \quad (5)$$

The choice of $-\mathrm{i}\tau_2$ on the right-hand side is conventional in the standard model. (It corresponds to the electric charge definition $Q = T_3 + Y$.) The action in the remaining variables is

$$S_U = \sum_x \left(\rho_x^2 + \lambda(1 - \rho_x^2)^2 - \kappa \sum_{\mu>0} \rho_{x+\hat\mu} \rho_x\, \mathrm{Tr}\, U(x,\mu) \right) + \beta \sum_P (1 - \tfrac{1}{2}\,\mathrm{Tr}\, U_P). \quad (6)$$

This can be called the "unitary gauge action" because it exhibits the physical degrees of freedom. In the limit $\lambda \to \infty$ the length ρ_x is frozen to $\rho_x = 1$, and we are left with

$$S_U^{\lambda\to\infty} = -\kappa \sum_{x,\mu>0} \mathrm{Tr}\, U(x,\mu) + \beta \sum_P (1 - \tfrac{1}{2}\,\mathrm{Tr}\, U_P). \quad (7)$$

The numerical Monte Carlo simulation I made was based on the action in eq. (7). On an 8^4 lattice the updating of the SU(2) gauge-field link variables was done by the Metropolis method with six hits per link. The correlations were measured in different channels. In all cases the three-momentum was projected out both to $\boldsymbol{p} = 0$ and $a\boldsymbol{p} = \pi/4$ (1 in lattice units) in all possible space orientations.

In the isoscalar spin-zero channel the same operator was taken as in QCD glueball calculations [‡1], namely, the symmetric combination of the three orientations of space-like single plaquettes. Since the C-parity transformation on the lattice is equivalent to the complex conjugation of link variables, the J^{PC} (J = spin, P = parity, C = charge parity) quantum numbers, in this case, are given by $J^{PC} = 0^{++}$. In the isovector spin-one channel a suitable operator can be defined from a single space-like link:

$$O_{Vm}^{(r)} = \mathrm{Tr}\{\tau_r U(x,m)\} \qquad (m, r = 1, 2, 3) . \qquad (8)$$

This has C parity -1. The parity transformation of the zero three-momentum projection is identical to the charge conjugation, because of the summation over space-like points (in the "time-slice") and $U(x, -m) = U(x - \hat{m}, m)^{+}$. Therefore, the operator in eq. (8) has $J^{PC} = 1^{--}$. The quantum number assignment of $O_{Vm}^{(r)}$ is quite clear also in the continuum limit, because for $a \to 0$ the dominant contribution is just the gauge field in point x. Another single link operator, which comes to one's mind, would be simply Tr $U(x, m)$. This is an isoscalar and has $C = +1$. The spin assignment is, however, more complicated: the lowest spin is actually $J^P = 0^+$ [‡2]. Therefore, this operator can be alternatively used for the isoscalar 0^{++} channel. Of course, the coupling to the lowest 0^{++} state can be different and it can also depend on β and κ.

Besides the correlations, also the expectation values of planar and off-axis Wilson loops were measured in order to determine the static energy, E, of an external SU(2) colour charge pair:

$$aE(R) = -\lim_{T\to\infty} T^{-1} \ln W(R, T) . \qquad (9)$$

Here $W(R, T)$ stands for a Wilson loop with length T in the time-direction and euclidean distance R between the endpoints in fixed time-slices. On the 8^4 lattice T is, of course, restricted to $T \leqslant 4$ by the periodic boundary conditions and R has the possible values $R = 1$, $\sqrt{2}, \sqrt{3}, 2, \sqrt{5}$, and 3. (Some other values like $R = \sqrt{6}, \sqrt{8}$, ... were not used in this paper.) In addition to the Wilson loops, the Polyakov lines winding around the periodic lattice in some given direction were also determined for some values of the coupling constants.

The general features of the correlations in the 0^{++} ("glueball") channels are rather similar to the situation in glueball calculations of pure lattice gauge theory, where a large amount of experience was gained recently [8]. The correlations in the 1^{--} (vector boson) channels seem to be even nicer: they are easily determined up to the largest distance on the 8^4 lattice. Representative examples of the 1^- isovector correlations are shown in fig. 1 (for $\boldsymbol{p} = 0$) and in fig. 2 (for $\boldsymbol{p} = 1$). The statistics on the correlations were collected, after 1000–2000 equilibrating sweeps, from 7000–10000 sweeps (a modest number compared to some recent QCD glueball calculations). The inverse correlation lengths (≅ lowest masses) obtained in different points of the (β, κ)-plane are summarized in table 1.

The static energy of an external charge, calculated from eq. (9), and the expectation value of the Polyakov lines $\sqrt{\langle L^2 \rangle}$ are shown in two representative cases in

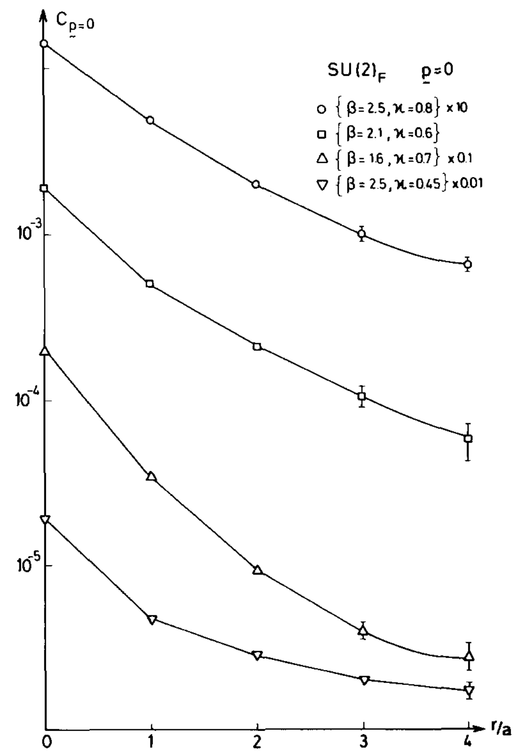

Fig. 1. A sample of the zero momentum correlations in the isovector 1^{--} channel.

[‡1] For a recent review and references see, for instance, ref. [8].

[‡2] In the first version of this paper Tr $U(x, m)$ was erroneously interpreted as being a spin -1 operator. I thank the referee for pointing out to me the correct assignment.

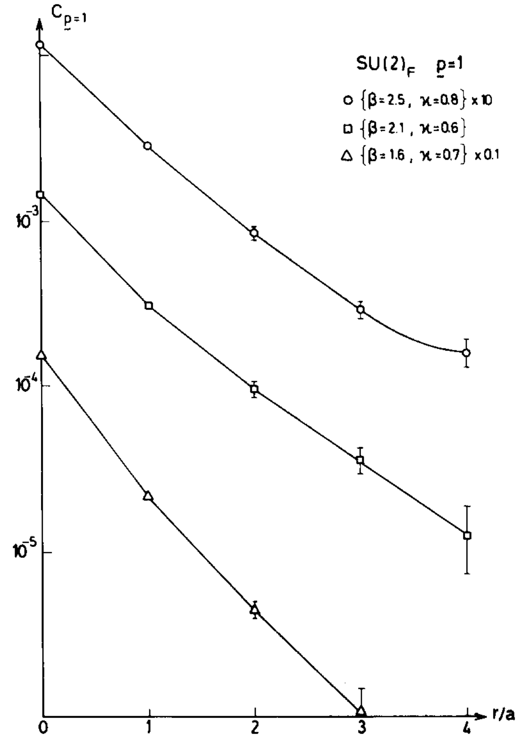

Fig. 2. The same as fig. 1 for the lowest non-vanishing momentum value on the lattice $ap = \pi/4$.

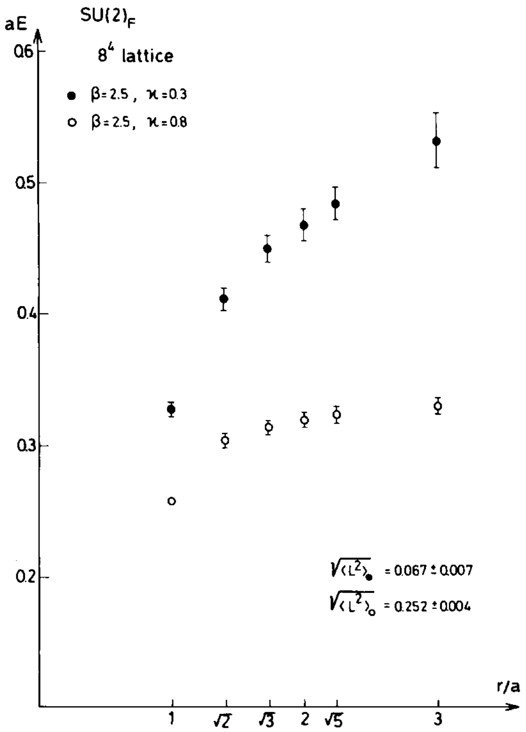

Table 1
The inverse correlations (mass estimates) in lattice units, at different points of the (β, κ)-plane as shown in fig. 4. The mass in the isovector $J^{PC} = 1^{--}$ channel is m_V, in the isoscalar, $J^{PC} = 0^{++}$ channel it is m_S.

β	κ	am_V	am_S
3.0	0.45	0.32 ± 0.08	2.2 ± 0.4
3.0	0.6	0.46 ± 0.05	2.7 ± 0.4
3.0	0.8	0.66 ± 0.05	2.8 ± 0.5
2.6	0.3	2.9 ± 0.6	1.5 ± 0.5
2.5	0.3	3.1 ± 0.6	1.2 ± 0.3
2.5	0.35	2.6 ± 0.6	1.4 ± 0.2
2.5	0.4	0.65 ± 0.14	1.5 ± 0.5
2.5	0.45	0.35 ± 0.07	2.3 ± 0.5
2.5	0.6	0.68 ± 0.07	2.7 ± 0.5
2.5	0.8	0.82 ± 0.08	2.2 ± 0.6
2.3	0.8	0.97 ± 0.08	3.0 ± 0.5
2.1	0.6	0.85 ± 0.07	2.0 ± 0.5
2.0	0.55	0.63 ± 0.05	1.8 ± 0.3
1.9	0.6	0.95 ± 0.08	1.8 ± 0.4
1.8	0.65	0.86 ± 0.08	1.8 ± 0.2
1.8	0.8	1.16 ± 0.05	2.3 ± 0.3
1.6	0.65	1.4 ± 0.2	1.37 ± 0.08
1.6	0.7	1.2 ± 0.2	1.4 ± 0.3
1.5	0.8	1.32 ± 0.08	2.0 ± 0.4
1.4	0.65	1.8 ± 0.4	1.7 ± 0.5
1.4	0.7	1.6 ± 0.3	1.7 ± 0.3

fig. 3: one point in the confinement-like region with ($\beta = 2.5$; $\kappa = 0.3$), and another in the Higgs-like region with ($\beta = 2.5$; $\kappa = 0.8$). The screening of external colour charges is obvious in the Higgs-like region. In the confinement-like region, however, the screening is not evident. Presumably, one can see it only at larger distances. For small distances something like a Coulomb potential dominating, similarly to QCD with dynamical quarks.

An important general feature of the correlations is, according to table 1, that the correlation length in lattice units in the isovector–vector channel is large near the phase-transition line in the (β, κ)-plane. (The measured points and the phase transition line according to ref. [7] are shown in fig. 4.) Above this line, i.e. in the Higgs-like region, the isovector–vector mass is much smaller than the isoscalar–scalar one: $am_V \ll$

Fig. 3. The static energy of an external colour charge pair obtained from eq. (9). The point ($\beta = 2.5$; $\kappa = 0.3$) is in the confinement-like region, the point ($\beta = 2.5$; $\kappa = 0.8$) is in the Higgs-like region. The expectation value $\sqrt{\langle L^2 \rangle}$ of the Polyakov lines is also given.

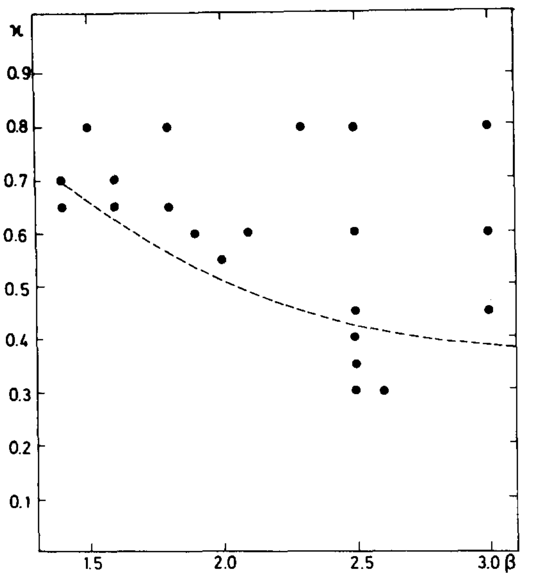

Fig. 4. The position of the points in the (β, κ)-plane, where the correlations were calculated (the results are given in table 1). The dashed line is the phase transition line according to ref. [7].

am_S. Below the phase–transition line, i.e. in the confinement-like region we have, on the contrary, $am_S \ll am_V$. Near the endpoint of the line there is also a region with $am_S \sim am_V$. As a general rule, am_V grows for decreasing β and for increasing κ. By crossing the line somewhere not very near the endpoint, am_V changes rather rapidly. Such a fast change is not observed for am_S. In fact, the measured correlation length in the isoscalar–scalar channel is never large: in all cases smaller than one lattice unit. This may be partly due to the known difficulty of measuring large correlations with single plaquette operators. Strictly speaking, the values in table 1 can only be considered as upper limits for the masses. Therefore, it may be that am_S, in particular, is smaller at some points than given by table 1.

The most probable interpretation of these findings is that the whole phase transition line in the (β, κ)-plane is a critical line corresponding to a second order phase transition with infinite correlation length. At the present level of precision it is, however, also possible that the phase transition line is first order with some small specific heat. In this case the vector meson mass (am_V) could have a jump along the phase transition line and the correlation length could stay finite. A critical line with infinite correlation length would mean that the lattice theory has a continuum limit. This is, of course, a very important matter, which should be investigated in great detail in the future. In particular, the study of the volume dependence of the correlations near the critical line ("finite size scaling") could give important hints in this respect. For the moment, the existence of some continuum limit is suggested by the fact that rotation invariance is restored to a good approximation in the measured points. This can be seen in fig. 3, for instance, by comparing the energies in the points $r/a = \sqrt{3}$, 2 and $\sqrt{5}$. Another sign of the nearby continuum limit is the approximate validity of the Lorentz-invariant energy–momentum relation, shown by the comparison of the $\boldsymbol{p} = 0$ and $\boldsymbol{p} = 1$ correlations. The situation in both aspects is rather similar to what happens in pure gauge theory in the coupling constant region where the correlation length is about 1. It is interesting that the situation along the critical line seems to change qualitatively with β. For instance, $m_S \sim m_V$ occurs only near the endpoint at $\beta \sim 1.5$. If m_S/m_V should stay finite in some continuum limit it would mean something rather remarkable: the physical Higgs particle is, namely, also an isoscalar 0^{++}. Therefore, in spite of the fact that the physical Higgs was formally removed by the limit $\lambda \to \infty$, a state having the same quantum numbers is still there as some glueball-like bound state of gauge vector bosons. Is it possible, that the lattice Higgs theory does not like to become non-renormalizable?

It is quite clear already from the above discussion that the question of the continuum limit and the renormalization group scaling in the SU(2) fundamental Higgs model is of great interest. It should be investigated by different means, for instance, by the direct study of the renormalization group flow, as suggested e.g. in ref. [9]. An important question in this respect is the role of the third coupling parameter λ. A first step in this direction is the extension of the correlation study to the whole (β, κ, λ)-space.

From the point of view of the phenomenology of high energy weak interactions an intriguing possibility is that the mass in the isoscalar–scalar (Higgs) channel is nearly equal to the mass of the W and Z mesons. This occurs in the lattice calculation near the end point of the critical line in the (β, κ)-plane (see table 1). The existence of such a "glueball-like" state in the standard $SU(2) \otimes U(1)$ theory was recently suggested by Veltman

[10]. (His arguments are, however, based on some different theoretical considerations.) Such a state would most probably signal strong interactions, therefore a rich spectrum of glueball-like and other composite states in the mass range of W and Z would follow.

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