More about the SU(2) 2^+ glueball state

Helmut Kamenzki and Bernd Berg*
II. Institut für Theoretische Physik, Luruper Chaussee 149, D-2000 Hamburg 50, West Germany
and Supercomputer Research Institute, Tallahassee, Florida 32306
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The 2^+ glueball is investigated in four-dimensional SU(2) lattice gauge theory. Using a recently proposed source we carry out a high-statistics Monte Carlo calculation at $\beta=2.25$ and 2.40. We obtain reliable correlation up to distance $t=2$ and a signal/noise ratio >3 for correlations up to distance $t=3$. With increasing distances we find a significantly decreasing $m(2^+)$ mass. There is no indication that the asymptotic limit ($t\to\infty$) has been reached.

Source methods play an important role for investigating the spectrum of pure lattice gauge theories. For a partial reference list see Refs. 1 and 2. Most investigations[1] concentrate on the 0^{++} mass gap. In Ref. 2 sources for the SU(2) 2^+ and the SU(2) 0^- spin state (the definition of spin states on the lattice is given in Ref. 3) were proposed and preliminary Monte Carlo (MC) calculations indicated a reasonable signal in the case of the 2^+ state, whereas the signal for the 0^- state disappeared already at distance $t=1$ into the statistical noise.

The 2^+ source consists in fixing a wave function $|\Psi\rangle$ on a spacelike plane

$$|\Psi\rangle=\prod_l \delta(U(l)-A(l))|0\rangle \ .$$

Here $A(l)=1$ except for those links in the $\hat{e}_2$ direction which have coordinates (n_1,n_2,n_3) with $n_1=0,2,4,\ldots$ (and n_2,n_3 arbitrary). At these links $A(l)=-1$ is taken.

In the present Brief Report we use this 2^+ source and report MC results relying on the high statistics of Table I. The MC procedure creates vacuum field fluctuations and therefore expectation values $\langle\Psi|O(t)|0\rangle$ are calculated. t is the distance of the operator $O(t)$ from the source $|\Psi\rangle$. In the column "measurements" (MEAS) of Table I the first number gives the sweeps performed for each measurement and the second number gives the total number of measurements performed (the product of both is the total number of sweeps). The lattice size is always 8^4. As in Ref. 2 we calculate expectation values of six different operators in the $E+$ representation of the cubic group. This corresponds to spin $2+$ in the continuum limit (see Ref. 3). The six operators considered are depicted in Fig. 1. Error bars are obtained by dividing all data into fixed numbers of bins as given in Table I.

In this Brief Report we only report results from operators which give

$$r=\text{signal/noise}>3 \ , \tag{1}$$

for correlations at distance $t=3$ and $\beta=2.20, 2.40$. The signal to noise ratio is defined as a mean value/error bar. Also for the sake of stability at $t=2,3$ correlations with a sign change at distance $t\geqslant 2$ are not considered. At $\beta=2.15$ a ratio $r\geqslant 3$ is never obtained with our data at distance $t=3$. The correlations with the two highest r ratios are taken into account up to $t=2$. At this β value the 2^+ mass is supposed to be high and correlations at distance $t=2$ should give reliable results. A reasonable wave function may already be constructed from the one-plaquette operator.

The thus selected results are presented in Table II. A detailed discussion of all results will be given in Ref. 4. In Table II the mass-gap estimates

$$m(t_1,t_2)=\frac{-1}{t_2-t_1}\ln\frac{\langle\Psi|O_i(t_2)|0\rangle}{\langle\Psi|O_i(t_1)|0\rangle} \ , \tag{2}$$

are used. The operators O_i ($i=1,\ldots,6$) are taken in the $E+$ representation indicated in Table II. The reliability of our results in the $t\to\infty$ limit (β, volume fixed) has to be discussed critically. An advantage of the $E+$ correlations,[2] as compared with the $A1+$ correlations,[1] is that the vacuum expectation value is known to be exactly zero. Therefore we have one parameter less than, for instance, in the work of de Forcrand, Schierholz, Schneider, and Teper,[1] where the Wilson loop expectation values are treated as free parameters. On the other hand, we only obtain reasonable signals up to the rather short distance $t=3$. More precisely, the mass ratio $m(3,0)$ is clearly out of the statistical noise, whereas $m(3,2)$ already exhibits difficulties concerning the accuracy.

TABLE I. MC statistics: Number of sweeps for reaching equilibrium (EQUI) and for measurements (MEAS). Error bars are calculated with respect to the given number of bins (BINS).

β	EQUI	MEAS	BINS
2.15	1250	3×5000	20
2.25	1250	$2\times 100\,000$	40
2.40	1250	$2\times 50\,000$	20

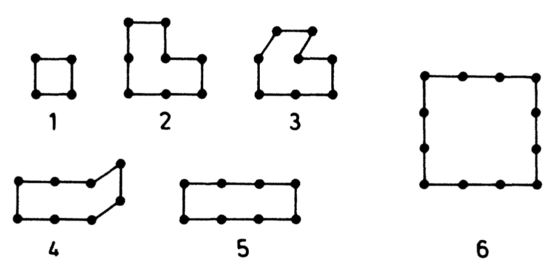

FIG. 1. Operators as considered in the present MC calculation.

TABLE II. Selected MC results. The $E+$ representations (repr.) are defined in Ref. 3 and the selected component (comp.) of these two-dimensional representations is also indicated. Finally, the signal to noise ratio r for correlations at distance $t=3$ is given.

	Operator	$E+$ repr.	$E+$ comp.	$m(0,1)$	$m(0,2)$	$m(1,2)$	$m(0,3)$	$m(1,3)$	$m(2,3)$	r
2.15	1	1	1	3.75(1)	3.64(8)	3.54(16)				
2.15	5	1	1	4.02(1)	3.62(10)	3.22(19)				
2.25	1	1	1	3.72(1)	3.62(2)	3.52(5)	3.12(7)	$2.81(^{+12}_{-9})$	2.11(25)	4.8
2.25	2	1	1	3.92	3.45(2)	2.98(5)	2.92(8)	$2.41(^{+13}_{-10})$	$1.85(^{+29}_{-24})$	4.6
2.25	3	2	1			2.86(5)		$2.37(^{+16}_{-11})$	$1.89(^{+34}_{-28})$	3.8
2.25	4	2	1			2.87(5)		$2.33(^{+31}_{-10})$	$1.79(^{+31}_{-25})$	4.3
2.25	5	1	1	3.92	3.47(3)	3.03(5)	2.97(10)	$2.49(^{+15}_{-11})$	$1.96(^{+33}_{-28})$	4.0
2.40	3	1	2	4.59	4.00(7)	3.42(12)	2.98(11)	$2.18(^{+16}_{-12})$	$0.95(^{+42}_{-37})$	3.8
2.40	6	1	2	4.94(1)	3.42(7)	1.90(16)	2.57(10)	$1.40(^{+15}_{-12})$	$0.90(^{+42}_{-37})$	4.1

The values at $\beta=2.25$ are (at the presented distances) presumably most reliable. Typically we find a series like

$$m(0,1)=3.92, \quad m(1,2)=2.98, \quad m(2,3)=1.85 \ ,$$

and there is no self-consistent evidence that we are already in the asymptotic limit $t\to\infty$. Mass ratios are decreasing systematically for increasing t; at $\beta=2.40$ they even go down rather drastically.

In the strong-coupling limit ($\beta\to 0$) (Ref. 5) as well as in the finite-volume weak-coupling limit ($\beta\to\infty$, volume fixed)[6] a mass ratio

$$m(2^+)/m(0^+)\approx 1 \quad (3)$$

is obtained. In contrast with this high-statistics MC variational (MCV) calculations[7] give the order of magnitude

$$m(2^+)/m(0^+)\approx 1.8 \ . \quad (4)$$

The present work indicates that the mass ratio, given by Eq. (4), may further decrease. Averaging our source method MC results gives (in lattice units) $m(2^+)\approx 1.9$ at $\beta=2.25$ and $m(2^+)\approx 0.93$ at $\beta=2.40$. With the estimate[7] $m(0^+)\approx 190\Lambda_L$ this yields

$$m(2^+)/m(0^+)\approx 1.5 \quad (\beta=2.25) \ . \quad (5)$$

At $\beta=2.40$ the ratio is even down to $m(2^+)/m(0^+)\approx 1.1$ ($\pm 40\%$). However, at this β value the lattice is presumably already too small to reflect true continuum limit behavior.

Finally, we would like to compare standard MCV methods with source methods. Source calculations have the advantage to give a signal for correlations up to larger distances than MCV calculations. In our case the signal at distance $t=3$ is better than the previous (rather bad) signal at distance $t=2$ as obtained in variational calculations.

However, there are several disadvantages of the source method: Firstly, in contrast with the MCV method, the source method does not give upper bounds. Hence, since our results in Table II show no tendency of stabilizing we cannot exclude the possibility that the estimate $m(2,3)$ is, in fact, an underestimate of the asymptotic value. The analysis of all data[4] exhibits further warnings with respect to this point. What one needs would be consistency over several steps in the distance t. This cannot be achieved using the methods proposed so far since the signal dies away too rapidly. Secondly, MCV calculations allow (in principle) to get reliable results already at distance $t=1$ by systematically improving the wave function. On the other hand, a systematic improvement of a source seems to be very difficult. An important last point is that MCV calculations give information about the whole glueball spectrum and the energy-momentum dispersion, whereas a source calculation has to concentrate on a single state.

Despite the points mentioned above our present 2^+ glueball calculation improves previous MCV results.[7] The final aim of a reliable continuum limit extrapolation is, however, not reached.

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*Permanent address: Department of Physics, Florida State University, Tallahassee, Florida 32306.

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