ANNALS OF PHYSICS 170, 1–17 (1986)

On Noether's Theorem in Quantum Field Theory

DETLEV BUCHHOLZ

II. Institut für Theoretische Physik, Universität Hamburg, 2000 Hamburg 50, Federal Republic of Germany

AND

SERGIO DOPLICHER AND ROBERTO LONGO

Dipartimento di Matematica, Università di Roma "La Sapienza", 00185 Roma, Italy

Received April 29, 1985

Extending the construction of local generators of symmetries in (S. Doplicher, *Commun. Math. Phys.* **85** (1982), 73; S. Doplicher and R. Longo, *Commun. Math. Phys.* **88** (1983), 399) to space-time and supersymmetries, we establish a weak form of Noether's theorem in quantum field theory. We also comment on the physical significance of the "split property," underlying our analysis, and discuss some local aspects of superselection rules following from our results. © 1986 Academic Press, Inc.

1. INTRODUCTION

One of the general features of field-theoretic models is the appearance of local, conserved currents resulting from internal or space-time symmetries. Within the setting of classical Lagrangian field theory this fact is well understood, and Noether's theorem even provides for each continuous symmetry of a Lagrangian an explicit formula for the corresponding current. In the Lagrangian approach to quantum field theory, the general understanding of the relation between symmetries and currents is, however, less satisfactory. It can happen, for example, that symmetries of a classical Lagrangian disappear at the quantum level due to the effects of renormalization (cf., e.g., 3, Sect. 11.5). Therefore it is unclear how to base a proper quantum version of Noether's theorem on this formalism.

It is the aim of the present article to discuss a different approach to a quantum Noether theorem. In this approach we consider as symmetries of a quantum field theory the set of global space-time or gauge-transformations acting on the physical Hilbert space. So, roughly speaking, we restrict our attention to "visible" symmetries of the solutions of the equations of motion which manifest themselves, e.g., through the presence of superselection rules. The problem of constructing the corresponding currents can then be discussed in the general ("axiomatic") setting of

* Research supported by Ministero della Pubblica Istruzione and CNR-GNAFA.

0003-4916/86 $7.50

quantum field theory. It consists essentially of two parts: first, one must determine to each global symmetry of a quantum field theory a set of local generators. Note that in the presence of Noether currents such generators can be obtained by integrating the (regularized) current densities over finite volumes of space; so the solution of this partial problem may be regarded as a weak form of Noether's theorem. The second step consists then in the reconstruction of the currents from these local generators (integrated densities).

This program has been initiated in [1], where the existence of local charge operators in theories with a global abelian symmetry group was established under very general conditions. An extension of this analysis to theories with a non-abelian global symmetry has been carried out in [2], providing a rigorous variant of local current algebras.

In the present paper we generalize these results to arbitrary symmetries, including space-time and super-symmetries. The proof that local generators of these symmetries exist thus completes the first step towards a quantum Noether theorem. The difficult second step, i.e., the reconstruction of currents, however, requires further investigations and is not touched upon in this article.

The setting used for our analysis is standard[1]: we assume that the physical states are described by vectors in some Hilbert space $\mathscr{H}$ and that the fields underlying the theory generate an irreducible set of local field algebras $\mathfrak{F}(\mathcal{O})$ on $\mathscr{H}$ which are assigned to the bounded regions $\mathcal{O}$ of Minkowski space. It is convenient here to assume that these algebras are von Neumann algebras. Thus each $\mathfrak{F}(\mathcal{O})$ may be regarded as a set of bounded operators built out of fields with localization centers in $\mathcal{O}$. Since it is obvious how to express the covariance properties and spacelike (anti-) commutation relations of fields in terms of the algebras $\mathfrak{F}(\mathcal{O})$, we refrain from listing these properties here.

Let us consider now the cases where the theory has an unbroken internal symmetry. In the present setting this means that there exists some group G (the global gauge group) which is represented on $\mathscr{H}$ by unitary operators $U(g)$, $g \in G$ transforming the vacuum $\Omega \in \mathscr{H}$ into itself,

$$U(g)\Omega = \Omega, \tag{1.1}$$

and leaving the localization of fields unchanged,

$$U(g)\,\mathfrak{F}(\mathcal{O})\,U(g)^{-1} = \mathfrak{F}(\mathcal{O}). \tag{1.2}$$

Space-time symmetries of a theory, such as the translations or the Lorentz-transformations act in a similar manner, the only difference being that on the right-hand side of relation (1.2) the region $\mathcal{O}$ has to be replaced by the transformed region $\mathcal{O}_g$ according to the geometrical action of the group element g. Supersymmetries, however, require a slightly different treatment as will be discussed below.

The structure described so far is familiar from many field-theoretic examples. But it is note-worthy that it can also be derived from first principles. The only input needed is the spacelike commutativity of local observables and the assumption that

[1] For a detailed exposition see the introduction of [4].

the physical states under consideration are well-localized excitations of a vacuum state. Under these circumstances the following results have been established in a series of papers [5–8].

Given the structure of the algebra of all local observables one can reconstruct the Hilbert space $\mathscr{H}$ of physical states, the algebras $\mathfrak{F}(\mathcal{O})$ of charge-carrying fields, the global gauge group G and its representation $U(g)$, $g \in G$. Moreover, the algebra generated by the local observables in a region $\mathcal{O}$ is represented on $\mathscr{H}$ by the algebra $\mathfrak{A}(\mathcal{O})$ of all gauge-invariant elements of $\mathfrak{F}(\mathcal{O})$, i.e.,

$$\mathfrak{A}(\mathcal{O}) = \mathfrak{F}(\mathcal{O}) \cap U'. \tag{1.3}$$

(Here U' denotes the set of bounded operators on $\mathscr{H}$ commuting with all gauge-transformations $U(g)$, $g \in G$.) It should be noticed that the local field algebras $\mathfrak{F}(\mathcal{O})$ are generated by Bose- or Fermi-type operators (with normal commutation relations at spacelike distances), even if there exist super-selection sectors in the theory obeying para-statistics. The latter (intrinsic) property would reveal itself in the non-abelianness of the gauge-group. Hence, summing up, the global symmetries of a theory are fixed by the algebraic structure of the observables and can be determined without any reference to local currents.

These model-independent results show that the present setting covers all theories with localizable charges, such as baryon-number or strangeness in pure quantum chromodynamics. Gauge charges (such as the electric charge in quantum electrodynamics) or quantum-topological charges (as discussed in [9]), however, do not fit into our setting since the corresponding charge carrying fields are necessarily non-local. In view of the latter fact one actually may have doubts that such non-localizable charges are always related to local currents acting on the physical Hilbert space $\mathscr{H}$. We therefore leave aside these cases for the time being, but we will return to them at the end of our paper in a discussion of some local aspects of superselection rules.

Let us now turn to our main objective: given any global (internal or space-time) symmetry transformation $U(g)$ and any bounded region $\mathcal{O}$, we want to exhibit local unitary operators[2] $U_\Lambda(g)$ which induce the same action on $\mathfrak{F}(\mathcal{O})$ as $U(h)$, i.e.,

$$U_\Lambda(g)\, F U_\Lambda(g)^{-1} = U(g)\, F U(g)^{-1} \qquad \text{for} \quad F \in \mathfrak{F}(\mathcal{O}). \tag{1.4}$$

Our local operators $U_\Lambda(g)$ will actually form a representation of the global symmetry group which is covariant in the following sense: if, e.g., h is a global gauge-transformation, then

$$U(h)\, U_\Lambda(g)\, U(h)^{-1} = U_\Lambda(h \cdot g \cdot h^{-1}). \tag{1.5}$$

By virtue of this covariance property and relation (1.3) $U_\Lambda(g)$ is a local observable whenever g commutes with the global gauge group.

[2] The significance of the index Λ will be explained below.

Guided by the example of an internal symmetry transformation $U(g)$, which is the exponential of a local current density integrated over all space, one might expect that one can always find such unitaries $U_\Lambda(g)$ in $\mathfrak{F}(\hat{\mathcal{O}})$, whenever the region $\hat{\mathcal{O}}$ is slightly larger than $\mathcal{O}$.[3] Yet there are models fitting into our general setting (a simple one being the theory of a charged generalized free field) in which no such operators exist for bounded regions $\hat{\mathcal{O}}$ [10]. Fortunately, these physically awkward models can be ruled out by a general and physically significant condition which will be discussed in the following section.

2. Local Preparation of States and the Split Property

We say that a quantum field theory has the split-property if for any bounded region $\mathcal{O}$ there is another bounded region $\hat{\mathcal{O}} \supset \mathcal{O}$ and a type I factor[4] $\mathcal{N}$ such that

$$\mathfrak{F}(\mathcal{O}) \subset \mathcal{N} \subset \mathfrak{F}(\hat{\mathcal{O}}). \tag{2.1}$$

Since the physical significance of this condition is not immediately obvious we give some explanations.

At the level of models the split-property has been established for the free, scalar field [11] (see also [12]) and hence for interacting theories which are locally Fock, such as the $\mathscr{P}(\varphi)_2$ models. These results were extended to arbitrary spins as well as to the Yukawa model in two dimensions in [13]. On the other hand it is known that certain artificial models do not have the split property. Examples are the generalized free field with continuous Källen–Lehmann measure or theories with infinite particle multipletts [10].

The common feature of these counterexamples is the fact that they describe systems with a tremendous number of local degrees of freedom. (As a consequence, there are for example no reasonable temperature states in these models.) This clue of a relation between the split property and the number of local degrees of freedom has been confirmed in [15]. Taking as a measure the energy level-density of localized states in a theory, it has been shown that the split-property holds if the level-density does not grow too fast with the energy. Roughly speaking, the particle spectrum has to be such that the "partition function" $\sum_i e^{-\beta m_i}$ exists for all $\beta > 0$. (The sum is to be taken over all particle types counted according to their multiplicity; m_i are the particle masses.) It seems that this condition is satisfied in most models of physical interest.

[3] The region $\hat{\mathcal{O}}$ has to be larger than $\mathcal{O}$ since integrals over current densities require a regularization which enlarges the localization. Of course, $\hat{\mathcal{O}}$ depends also on g if $U(g)$ is a space-time symmetry transformation.

[4] We recall that a von Neumann algebra $\mathcal{N}$ is called a *factor* if its center is trivial, i.e., if $\mathcal{N} \cap \mathcal{N}' = \mathbb{C} \cdot 1$. As usual, $\mathcal{N}'$ denotes the set of all bounded operators on $\mathscr{H}$ commuting with $\mathcal{N}$. A *type I factor* $\mathcal{N}$ is a factor which contains some minimal projection $E \neq 0$; it is isomorphic to the algebra of all bounded operators on a fixed Hilbert space (cf., e.g., [14]).

If the split inclusion (2.1) holds for the fields it an easy consequence of (1.3) that an analogous relation holds for the observables, i.e., that

$$A(\mathcal{O}) \subset M \subset A(\hat{\mathcal{O}}) \tag{2.2}$$

for some type I factor M [1]. We shall demonstrate now that this property of the algebra $A = \bigcup_{\mathcal{O}} A(\mathcal{O})$ of all local observables can also be grounded on the basic experimental fact that it is possible to fix locally certain specific physical situations (e.g., the vacuum) irrespective of the given initial conditions of the world.

According to the basic principles of quantum theory any physical state corresponds to a positive linear functional φ over the algebra $\mathfrak{A}$, giving the expectation values of observables in this state.[5] Performing a yes–no experiment (corresponding to a selfadjoint projection $E \in \mathfrak{A}$) one can prepare from φ a new state φ_E by rejecting all events where the result of measuring E in φ is zero. This reduced state φ_E is given by

$$\varphi_E(A) = \frac{\varphi(EAE)}{\varphi(E)} \qquad \text{for} \quad A \in \mathfrak{A}, \tag{2.3}$$

provided the probability $\varphi(E)$ of finding the value 1 for the observable E in the state φ is different from 0. A projection E is called a pure (ideal) filter if for every state φ with $\varphi(E) \neq 0$ one obtains $\varphi_E = \omega$, where ω is a fixed state which is independent from φ. It is easy to see that ω must be a pure state of $\mathfrak{A}$, so by measuring a pure filter one can produce ensembles with maximal information.

Pure filters are familiar from systems with a finite number of degrees of freedom. In quantum field theory, however, a pure filter cannot be a (local) observable, because it affects in a sharp way all states at arbitrarily large spacelike distances. On the other hand, one never attempts to measure pure filters. In practice one is content with the possibility of fixing states within limited space-time regions. It is an important empirical fact that this can be achieved with an experimental set-up, where only the parameters of the states in question enter. Phrased differently: by suitable monitoring experiments one can establish a definite state within a given region, irrespective of the unknown and complicated details of the rest of the world. So, locally, such experiments have the same effect as a pure filter.

Translating these facts into the setting of quantum field theory one is led to introduce the concept of a *local filter* for a given state: a projection $E \in \mathfrak{A}$ is called a local filter for ω in the region $\mathcal{O}$ if all reduced states φ_E coincide with ω on the algebra $\mathfrak{A}(\mathcal{O})$, i.e.,

$$\varphi_E(A) = \omega(A), \qquad A \in \mathfrak{A}(\mathcal{O}) \tag{2.4}$$

for any state φ of $\mathfrak{A}$ with $\varphi(E) \neq 0$. The empirical situation just described then suggests that all physically reasonable theories have to admit such local filters. We

[5] Thinking of $\mathfrak{A}$ as an operator algebra on the Hilbert space $\mathscr{H}$ containing all superselection sectors, the states of interest here are of the form $\varphi(A) = \operatorname{Tr} \rho \cdot A$, $A \in \mathfrak{A}$, where ρ is some density matrix on $\mathscr{H}$.

shall demonstrate now that this condition, which expresses a principle of experimental definiteness, implies the split property.

PROPOSITION. *The algebra* $A = \bigcup_{\mathcal{O}} A(\mathcal{O})$ *of local observables in a local quantum field theory has the split property* (2.2) *if and only if there exist local filters for all bounded space-time regions.*

Proof. If a theory has the split-property, then there exists for any bounded region $\mathcal{O}$ a type I factor $\mathcal{M} \subset \mathfrak{A}$ such that $\mathfrak{A}(\mathcal{O}) \subset \mathcal{M}$, cf. relation (2.2). Bearing in mind that the minimal projections $E \in \mathcal{M}$ have (relative to $\mathcal{M}$) the same algebraic properties as a one-dimensional projection on a Hilbert space it is clear that

$$EAE = \omega(A) \cdot E \qquad \text{for} \quad A \in \mathfrak{A}(\mathcal{O}), \tag{2.5}$$

where ω is some state depending on E. One then obtains relation (2.4) by taking the expectation value of this equation in an arbitrary state φ. So the minimal projections in $\mathcal{M}$ act as local filters in the region $\mathcal{O}$. In fact, in theories with the split-property such filters exists for all locally normal states ω of $\mathfrak{A}$.

Conversely, assume that there exists a local filter $E \in \mathfrak{A}$ for some state ω in the region $\mathcal{O}$. Since the physical states φ separate the elements of $\mathfrak{A}$ it follows from (2.4) that relation (2.5) holds for the projection E. Let $\mathcal{R}$ be the von Neumann algebra generated by $\mathfrak{A}(\mathcal{O})$ and E. Since E is a local operator it is clear that $\mathfrak{A}(\mathcal{O}) \subset \mathcal{R} \subset \mathfrak{A}(\hat{\mathcal{O}})$ for some bounded region $\hat{\mathcal{O}}$. Moreover, relation (2.5) implies that

$$E\mathcal{R}E = \mathbb{C} \cdot E, \tag{2.6}$$

which means that E is a minimal projection in $\mathcal{R}$. Thus the proof of the split-property is complete if $\mathcal{R}$ is a factor. Turning therefore to te cases where $\mathcal{R}$ has a center, let C be the entral support of E (i.e., the smallest projection in the center of $\mathcal{R}$ containing E). It then follows from (2.6) that the reduced von Neumann algebra $\mathcal{R} \cdot C$ is a type I factor on $C \cdot \mathcal{H}$ and, by construction,

$$\mathfrak{A}(\mathcal{O}) \cdot C \subset \mathcal{R} \cdot C \subset \mathfrak{A}(\hat{\mathcal{O}}). \tag{2.7}$$

Now given any isometry W mapping $\mathcal{H}$ onto $C \cdot \mathcal{H}$, i.e., $W^*W = 1$ and $WW^* = C$, one can map $\mathcal{R} \cdot C$ onto a type I factor $\mathcal{M} = W^*\mathcal{R} \cdot CW$ on $\mathcal{H}$. It is an important consequence of the Reeh–Schlieder theorem and the fact that $C \in \mathfrak{A}(\mathcal{O})' \cap \mathfrak{A}(\hat{\mathcal{O}})$ that one can find such an isometry W in $\mathfrak{A}(\mathcal{O}_0)' \cap \mathfrak{A}(\hat{\mathcal{O}}_0)$, provided the closure of the region $\mathcal{O}_0$ (resp. $\hat{\mathcal{O}}$) is contained in the interior of $\mathcal{O}$ (resp. $\hat{\mathcal{O}}_0$) [17]. Replacing $\mathcal{O}$ and $\hat{\mathcal{O}}$ in (2.7) by the slightly smaller and larger regions $\mathcal{O}_0$ and $\hat{\mathcal{O}}_0$ and multiplying the resulting relation from the left and right by W^* and W, respectively, one thus arrives at the inclusion $\mathfrak{A}(\mathcal{O}_0) \subset \mathcal{M} \subset \mathfrak{A}(\hat{\mathcal{O}}_0)$, where $\mathcal{M}$ is some type I factor. Q.E.D.

It is an interesting question of whether the split property (2.2) of the observables implies the corresponding relation (2.1) for the fields. In the special case of a theory with a finite abelian gauge group G this is known to be true [1], and there are strong indications that this result can be extended to more general situations [16]. In view of the fact that the split property of the observables admits a direct physical interpretation it would be desirable to clarify this point completely.

It is an easy consequence of the split inclusion (2.1) and relation (2.5) that the reduced states φ_E, where $E \in \mathcal{N}$ is any minimal projection, are product states on $\mathfrak{F}(\mathcal{O}) \cdot \mathfrak{F}(\hat{\mathcal{O}})'$. Actually, there exist product states with certain specific properties which are substantial for the subsequent analysis [2, 10]: given any vector Ω in $\mathcal{H}$ which is cyclic and separating for the algebra $\mathfrak{F}(\mathcal{O})' \cap \mathfrak{F}(\hat{\mathcal{O}})$ (we can take here the vacuum [2]) one can find a vector $\Omega_\Lambda \in \mathcal{H}$ such that

(i) Ω_Λ induces a product state on $\mathfrak{F}(\mathcal{O}) \cdot \mathfrak{F}(\hat{\mathcal{O}})'$ given by

$$(\Omega_\Lambda, FF'\Omega_\Lambda) = (\Omega, F\Omega) \cdot (\Omega, F'\Omega) \tag{2.8}$$

for $F \in \mathfrak{F}(\mathcal{O})$ and $F' \in \mathfrak{F}(\hat{\mathcal{O}})'$.

(ii) Ω_Λ is cyclic for the von Neumann algebra $\mathfrak{F}(\mathcal{O}) \vee \mathfrak{F}(\hat{\mathcal{O}})'$.[6]

(iii) Ω_Λ is an element of the natural cone[7] $P^\natural \subset \mathcal{H}$ associated with Ω and $\mathfrak{F}(\mathcal{O})' \cap \mathfrak{F}(\hat{\mathcal{O}})$.

The vector Ω_Λ is completely fixed by these properties, so it only depends on the triple

$$\Lambda = (\mathfrak{F}(\mathcal{O}), \mathfrak{F}(\hat{\mathcal{O}}), \Omega). \tag{2.9}$$

Moreover, the assignement $\Lambda \to \Omega_\Lambda$ is covariant in the following sense [2, 10]: if a triple

$$\Lambda_0 = (\mathfrak{F}(\mathcal{O}_0), \mathfrak{F}(\hat{\mathcal{O}}_0), \Omega_0) \tag{2.10}$$

is isomorphic to Λ, i.e., if there exists some unitary U_0 on $\mathcal{H}$ such that

$$\Lambda_0 = (U_0 \mathfrak{F}(\mathcal{O})\, U_0^{-1}, U_0 \mathfrak{F}(\hat{\mathcal{O}})\, U_0^{-1}, U_0 \Omega), \tag{2.11}$$

then the corresponding vectors Ω_{Λ_0} and Ω_Λ are related by

$$\Omega_{\Lambda_0} = U_0 \cdot \Omega_\Lambda. \tag{2.12}$$

Note that in the case of a global gauge transformation U_0 one obtains $\Lambda_0 = \Lambda$ (cf. relations (1.1) and (1.2)) and therefore $\Omega_{\Lambda_0} = \Omega_\Lambda$. Thus implies, according to relation (2.12), that Ω_Λ is invariant under the action of U_0.

It may be noticed that the existence of product state vectors Ω_Λ as in equation (2.8) expresses a strong form of statistical independence between the regions $\mathcal{O}$ and $\hat{\mathcal{O}}'$, which is actually equivalent to the split property [11].

[6] The symbol $\mathcal{R}_1 \vee \mathcal{R}_2$ denotes the von Neumann algebra generated by $\mathcal{R}_1$ and $\mathcal{R}_2$.

[7] For a short account of the theory of cones see the Appendix of [10].

3. Local Generators of Symmetries and the Universal Localizing Map

Assuming that the split inclusion (2.1) holds for the fields we now introduce a mapping ψ_Λ of the algebra $\mathscr{B}(\mathscr{H})$ of all bounded operators onto a natural type I factor $\mathscr{N}_\Lambda$ associated with the triple $\Lambda = (\mathfrak{F}(\mathcal{O}), \mathfrak{F}(\hat{\mathcal{O}}), \Omega)$. This *universal localizing map* ψ_Λ will prove to be a convenient tool for the passage from the global symmetries to the corresponding local generators.

Let Ω_Λ be the natural choice of the product state vector of Λ. First, we define an isometry W_Λ of $\mathscr{H}$ onto $\mathscr{H} \otimes \mathscr{H}$ setting

$$W_\Lambda \cdot FF'\Omega_\Lambda = F\Omega \otimes F'\Omega \tag{3.1}$$

for $F \in \mathfrak{F}(\mathcal{O})$ and $F' \in \mathfrak{F}(\hat{\mathcal{O}})'$. (That W_Λ is an isometry as a consequence of (2.8); the assertions on its domain and range follow from the fact that Ω_Λ and Ω are cyclic for $\mathfrak{F}(\mathcal{O}) \vee \mathfrak{F}(\hat{\mathcal{O}})'$ and $\mathfrak{F}(\mathcal{O}), \mathfrak{F}(\hat{\mathcal{O}})'$, respectively.) It is an immediate consequence of this definition that for F, F' as above

$$W_\Lambda \cdot FF' = F \otimes F' \cdot W_\Lambda. \tag{3.3}$$

Now we set

$$\psi_\Lambda(T) = W_\Lambda^{-1}(T \otimes 1) W_\Lambda \qquad \text{for} \quad T \in \mathscr{B}(\mathscr{H}) \tag{3.4}$$

which fixes the universal localizing map[8] ψ_Λ of $\mathscr{B}(\mathscr{H})$ onto the type I factor

$$\mathscr{N}_\Lambda = \psi_\Lambda(\mathscr{B}(\mathscr{H})). \tag{3.5}$$

It follows from (3.3) that

$$\psi_\Lambda(F) = F \qquad \text{for} \quad F \in \mathfrak{F}(\mathcal{O}), \tag{3.6}$$

and taking into account that $\mathscr{B}(\mathscr{H}) \otimes 1 \subset (1 \otimes \mathfrak{F}(\hat{\mathcal{O}})')'$ as well as the fact that $\mathfrak{F}(\hat{\mathcal{O}})'' = \mathfrak{F}(\hat{\mathcal{O}})$ it is also clear that

$$\psi_\Lambda(T) \in \mathfrak{F}(\hat{\mathcal{O}}) \qquad \text{for} \quad T \in \mathscr{B}(\mathscr{H}). \tag{3.7}$$

So, in particular, we obtain the inclusion

$$\mathfrak{F}(\mathcal{O}) \subset \mathscr{N}_\Lambda \subset \mathfrak{F}(\hat{\mathcal{O}}). \tag{3.8}$$

Next, let us determine the transformation properties of ψ_Λ if one proceeds from Λ to any isomorphic triple Λ_0: from the transformation law (2.12) for the product state vectors and the definition (3.1) of the unitaries W_Λ, W_{Λ_0} it follows that

$$W_{\Lambda_0} \cdot U_0 = U_0 \otimes U_0 \cdot W_\Lambda, \tag{3.9}$$

[8] ψ_Λ is actually an isomorphism.

where U_0 is the unitary establishing the isomorphism between Λ and Λ_0. If one makes use of this relation in Eq. (3.4) defining ψ_Λ one obtains

$$\psi_\Lambda(U_0^{-1}TU_0) = U_0^{-1}\psi_{\Lambda_0}(T)\,U_0 \qquad \text{for} \quad T\in\mathscr{B}(\mathscr{H}), \tag{3.10}$$

giving the transformation law for the universal localizing maps. Recalling that $\Lambda_0 = \Lambda$ if U_0 is a global gauge transformation it follows in particular from (3.10) that ψ_Λ commutes with the gauge transformations.

With this information at hand we can turn now to the construction of local generators of the symmetries. We begin by discussing internal symmetries; since in this case we merely reproduce (in the frame of the universal localizing maps) the results obtained in [2, 10], we can be very brief.

Internal Symmetries

Let G be the global gauge group (internal symmetry group) and let $U(g)$, $g\in G$ be the corresponding unitary transformations on $\mathscr{H}$ satisfying the conditions (1.1) and (1.2). Setting

$$U_\Lambda(g) = \psi_\Lambda(U(g)) \tag{3.11}$$

we obtain a new reresentation of G by unitary operators in $\mathfrak{F}(\hat{\mathcal{O}})$, cf. relation (3.4) and (3.7). Since ψ_Λ acts trivially on $\mathfrak{F}(\mathcal{O})$ and since internal symmetry transformations do not change the localization of fields it follows that for $F\in\mathfrak{F}(\mathcal{O})$

$$U_\Lambda(g)\,FU_\Lambda(g)^{-1} = \psi_\Lambda(U(g)\,FU(g)^{-1}) = U(g)\,FU(g)^{-1}. \tag{3.12}$$

So the local operators $U_\Lambda(g)$ induce the same action on $\mathfrak{F}(\mathcal{O})$ as the global transformations $U(g)$.

If G is a Lie group one can proceed from the local symmetry transformations $U_\Lambda(g)$ to the corresponding infinitesimal generators. As has been discussed in [2], these generators are the analogue of locally integrated current densities, and they provide a version of local current algebra. Note that the local symmetry transformations $U_\Lambda(g)$ also exist in the case of discrete symmetries (multiplicative charges). In this respect the information contained in relation (3.11) goes beyond Noether's theorem.

It is also worth mentioning that, under fairly general assumptions, the local symmetry transformation $U_\Lambda(g)$ converge to the global ones if the regions $\mathcal{O}$ and $\hat{\mathcal{O}}$ tend to the whole space [18].

Space-time Symmetries

We now extend this construction of local symmetry transformations to space-time symmetries. Let $\mathscr{P}$ be a group acting on the space-time points x by $x\to Lx$, $L\in\mathscr{P}$ and assume that the theory is symmetric under $\mathscr{P}$, i.e., there exists a continuous, unitary representation $V(L)$, $L\in\mathscr{P}$ on $\mathscr{H}$ such that

$$V(L)\Omega = \Omega \tag{3.13}$$

and

$$V(L)\,\mathfrak{F}(\mathcal{O})\,V(L)^{-1}=\mathfrak{F}(L\mathcal{O}). \tag{3.14}$$

Examples for $\mathscr{P}$ are the translations and the Poincaré-group (resp. its covering group), possibly extended by conformal transformations.

As in the case of internal symmetries one obtains a representation of $\mathscr{P}$ by unitary operators in $\mathfrak{F}(\hat{\mathcal{O}})$, setting

$$V_\Lambda(L)=\psi_\Lambda(V(L)). \tag{3.15}$$

These unitaries induce locally the same action on the fields as the global transformations $V(L)$. Namely, let $\mathcal{O}_0$ be any region contained in the interior of $\mathcal{O}$ and let $\mathscr{P}_0$ be a neighbourhood of the identity in $\mathscr{P}$ such that $L_0\mathcal{O}_0\subset\mathcal{O}$ for all $L_0\in\mathscr{P}_0$. From the fact that ψ_Λ acts trivially on $\mathfrak{F}(\mathcal{O})$ and from relation (3.14) it then follows that for $L_0\in\mathscr{P}_0$ and $F_0\in\mathfrak{F}(\mathcal{O}_0)$,

$$V_\Lambda(L_0)\,F_0\,V_\Lambda(L_0)^{-1}=\psi_\Lambda(V(L_0)\,FV(L_0)^{-1})=V(L_0)\,FV(L_0)^{-1}. \tag{3.16}$$

This establishes the locally correct action of the unitaries $V_\Lambda(L)$.

Assuming that the global transformations $V(L)$ are gauge-invariant, i.e., that

$$U(g)\,V(L)=V(L)\,U(g) \qquad \text{for all} \quad g\in G, \tag{3.17}$$

it also holds true that the operators $V_\Lambda(L)$ are observable. At this point the covariance properties of the universal localizing map ψ_Λ are essential: from relation (3.10) it follows that the set U' of all gauge invariant operators is mapped into itself by ψ_Λ. Hence, using the characterization (1.3) of the observables, one obtains

$$V_\Lambda(L)\in\mathfrak{F}(\hat{\mathcal{O}})\cap U'=\mathfrak{A}(\hat{\mathcal{O}}), \tag{3.18}$$

as claimed.

These results show that the infinitesimal generators of the local spacetime transformations $V_\Lambda(L)$ are the analogue of the $(0, \nu)$-component of the energy-momentum tensor, etc., integrated over a finite volume. Yet in contradistinction to these locally integrated densities the generators of $V_\Lambda(L)$ have the same spectrum as their global counterparts. This follows immediately from the definition (3.15), according to which the representation $V_\Lambda(L)$, $L\in\mathscr{P}$ is unitarily equivalent to the global representation $V(L)$, $L\in\mathscr{P}$ amplified with infinite multiplicity,

$$V_\Lambda(L)\simeq V(L)\otimes 1. \tag{3.19}$$

To give an example: the generators of the translations $V_\Lambda(x)$, $x\in\mathbb{R}^4$ fulfil the relativistic spectrum condition (positivity of the energy); in contrast, the energy-density integrated over a finite volume cannot be a positive operator in a relativistic theory because of the Reeh–Schlieder property of the vacuum. This apparent paradox unravels if one notes that one can add to the integrated densities operators from $\mathfrak{F}(\mathcal{O})' \cap \mathfrak{F}(\hat{\mathcal{O}})$ without affecting their locally correct action (in the sense of relation (3.16)). It follows from our results that there always exist such marginal perturbations which adjust the spectrum of the local generators to that of the global ones.

We conclude this discussion of space-time symmetries with the remark that, similar to the case of internal symmetries, one can define the local transformations (3.15) also in the case of discrete symmetries, e.g., space-inversions. Of course, the regions $\mathcal{O}$, $\hat{\mathcal{O}}$ in the underlying triple Λ should then be symmetric.

Supersymmetries

Our general discussion also applies to theories with supersymmetries (see, e.g., [19, 20]). In a supersymmetric theory there exist Bose as well as Fermi fields which can be identified with the help of the unitary U_s inducing the sign change on Fermi fields. An arbitrary element F of the local field algebras $\mathfrak{F}(\mathcal{O})$ can thus be decomposed into its Bose and Fermi parts F_+ and F_-, respectively, setting

$$F_{\pm} = \tfrac{1}{2}(F \pm U_s F U_s^{-1}). \tag{3.20}$$

A supersymmetry of the theory is given by a $(\pm)$-graded Lie-algebra $\mathcal{Q} = \mathcal{Q}_+ \oplus \mathcal{Q}_-$ represented on $\mathcal{H}$ by selfadjoint operators[9] Q and acting on $F \in \mathfrak{F}(\mathcal{O})$ in a way which is compatible with the $(\pm)$-grading of the fields. Namely,

$$\begin{aligned} \delta_u(F) &= [Q_u, F] && \text{for} \quad u \in \mathcal{Q}_+ \\ \delta_u(F) &= [Q_u, F_+] + \{Q_u, F_-\} && \text{for} \quad u \in \mathcal{Q}_-, \end{aligned} \tag{3.21}$$

where [,] and { , } denote the commutator and anticommutator, respectively. Actually, the expressions (3.21) are not defined for arbitrary elements $F \in \mathfrak{F}(\mathcal{O})$ since the operators Q_u are unbounded. But there should be a common dense domain $\mathfrak{F}_0(\mathcal{O}) \subset \mathfrak{F}(\mathcal{O})$ so that the operators $\delta_u(F)$, $F \in \mathfrak{F}_0(\mathcal{O})$ are affiliated to $\mathfrak{F}(\mathcal{O})$.

The global gauge-group G induces an action on the elements $u \in \mathcal{Q}$ which we denote by $g(u)$, $g \in G$. The corresponding transformation law for the generators Q_u is given by

$$Q_{g(u)} = U(g)\, Q_u\, U(g)^{-1}. \tag{3.22}$$

An analogous statement holds for the space-time symmetries $\mathcal{P}$.

In complete analogy to the cases discussed before one obtains a representation of $\mathcal{Q}$ by selfadjoint operators affiliated to $\mathfrak{F}(\hat{\mathcal{O}})$, setting

$$Q_u^\Lambda = \psi_\Lambda(Q_u). \tag{3.23}$$

These operators induce on $\mathfrak{F}_0(\mathcal{O})$ the infinitesimal supersymmetry transformations δ_u. Moreover,

$$U(g)\, Q_u^\Lambda\, U(g)^{-1} = Q_{g(u)}^\Lambda, \qquad g \in G \tag{3.24}$$

[9] Note that by going to a Majorana representation the supercharges Q become "real."

for any $u \in \mathscr{Q}$. Similar covariance properties of the local generators Q_u^A under Poincaré transformations, etc., follow easily. So the universal localizing map ψ_A supplies adequate local generators also in the case of supersymmetries.

The present results can be viewed as a step towards the construction of local currents which are related to global symmetries. Namely we have established the analogues of finite volume integrals of the zero-components of such currents. It is still a major open problem how to recover these currents from our local generators. In this context the freedom of choosing the region $\hat{\mathcal{O}}$ arbitrarily close to $\mathcal{O}$ and yet having the split inclusion (2.1) may be expected to be crucial [1]. As a matter of fact, this more restrictive form of the split property has been established in models [11–13]; it also follows from a slightly strengthened version of the general assumptions in [15] (cf. [21]).

Our arguments also apply to theories where non-localizable (topological) charges are present [9]. There the construction of the normal field-algebra [8] leads to a set of von Neumann algebras $\mathfrak{F}(\mathscr{S})$ which are associated to spacelike cones $\mathscr{S} \subset \mathbb{R}^4$ ("thickened strings"). In the absence of massless particles it is still reasonable to assume that there exist cones $\hat{\mathscr{S}} \supset \mathscr{S}$ such that the analogue of relation (2.1) holds, i.e.,

$$\mathfrak{F}(\mathscr{S}) \subset \mathscr{N} \subset \mathfrak{F}(\hat{\mathscr{S}}) \tag{3.25}$$

for some type I factor $\mathscr{N}$ [22]. The above analysis then provides a representation of the global symmetries by unitary operators in $\mathfrak{F}(\hat{\mathscr{S}})$ which induce the correct action on $\mathfrak{F}(\mathscr{S})$ (cf. relation (1.4)). But there is no indication (in the case of continuous symmetries) that these operators are related to local currents.

4. Local Aspects of Superselection Rules

We conclude this investigation with a discussion of some local aspects of superselection rules emerging from our analysis. According to its basic definition a superselection rule is just a lable of equivalence classes of irreducible representations of the observable algebra $\mathfrak{A}$. In our present setting these representations can be obtained by restricting $\mathfrak{A}$ to the coherent subspaces (superselection sectors) of $\mathscr{H}$, and the superselection rules (global charges) can be identified with the elements of the center of $\mathfrak{A}''$. All these concepts are of a global nature involving observations at arbitrarily large distances. In practice, however, superselection rules are observed within the limits of a laboratory. So there arises the question of how the superselection structure manifests itself locally within our theoretical setting.

If the global charges are explicitly given as operators acting on $\mathscr{H}$, then an answer can be obtained from [1, 2] as well as from the preceding discussion: with the aid of the universal localizing map ψ_A one can construct from the global charges a family of commuting observables which are localized in $\hat{\mathcal{O}}$ and measure the charges contained in $\mathcal{O}$. Yet this result is not completely satisfactory because it relies on an a priori knowledge of the superselection structure.

If one does not insist on pinpointing specific observables which measure the charges in a given region, a conceptually more satisfactory answer can be given. We shall see that the superselection structure of a theory can be completely uncovered within bounded space-time regions if one knows the "correct" local Hamiltonian. In order to simplify this discussion we assume that the underlying theory has the additivity property, i.e., that

$$\mathfrak{A}(\mathcal{O}) \subset \mathfrak{A}(\mathcal{O}_1) \vee \cdots \vee \mathfrak{A}(\mathcal{O}_n) \tag{4.1}$$

if $\mathcal{O} \subset \mathcal{O}_1 \cup \cdots \cup \mathcal{O}_n$ and $\mathcal{O}, \mathcal{O}_1, \ldots, \mathcal{O}_n$ are regular regions such as double cones. Under this assumption, which is reminiscent of the properties of local Wightman fields, one can determine the suprselection structure even within a fixed, bounded region.

Let us first consider theories with localizable charges, where we can make use of the previous results; we will then extend our analysis to theories with non-localizable charges and long-range forces. Denoting by $V(t)$, $t \in \mathbb{R}$ the global time-translation and by $V_\Lambda(t)$ its local analogue, cf. relation (3.11), we consider the algebra $\mathscr{R}$ generated by $\mathfrak{A}(\mathcal{O})$ and the local Hamiltonian H_Λ (the generator of $V_\Lambda(t)$), i.e.,

$$\mathscr{R} = \mathfrak{A}(\mathcal{O}) \vee H_\Lambda''. \tag{4.2}$$

Taking into account that ψ_Λ acts trivially on $\mathfrak{A}(\mathcal{O})$ and is normal (ultra-weakly continuous) on $\mathscr{B}(\mathscr{H})$ we obtain

$$\mathscr{R} \supset \bigvee_{t \in \mathbb{R}} V_\Lambda(t)\, \mathfrak{A}(\mathcal{O})\, V_\Lambda(t)^{-1} = \psi_\Lambda\Big(\bigvee_{t \in \mathbb{R}} V(t)\, \mathfrak{A}(\mathcal{O})\, V(t)^{-1}\Big). \tag{4.3}$$

Now, as a consequence of the relativistic spectrum condition and the additivity assumption (4.1) the von Neumann algebra generated by the time-translated algebras $V(t)\, \mathfrak{A}(\mathcal{O})\, V(t)^{-1}$, $t \in \mathbb{R}$ is $\mathfrak{A}''$ [23], thus $\mathscr{R} \supset \psi_\Lambda(\mathfrak{A}'')$. On the other hand, assuming that the total energy H is an observable,[10] i.e., $V(t) \in \mathfrak{A}''$, it follows that $\mathscr{R} \subset \psi_\Lambda(\mathfrak{A}'')$, and consequently

$$\mathscr{R} = \psi_\Lambda(\mathfrak{A}''). \tag{4.4}$$

So, in particular, the center of $\mathscr{R}$ is isomorphic to the center of $\mathfrak{A}''$.

This result means, in physical terms, that by combining measurements of the local energy and of observations in $\mathcal{O}$ one comes across a certain specific set of observables (corresponding to the center of $\mathscr{R}$) which are simultaneously measurable with all other observables of this kind. From the spectrum of these specific observables one can then read off the superselection structure.

We now relax the assumption that the theory describes only localizable charges; so we no longer have at our disposal local charge-carrying fields which generate the physical states from the vacuum. But it is still reasonable to assume that there is

[10] By the spectrum condition, such a choice of H is always possible, cf. the remarks below.

some Hilbert space $\mathscr{H}$ of physical states on which the local observables are represented by an algebra $\mathfrak{A}$ of bounded operators. Moreover, on $\mathscr{H}$ there should exist a continuous, unitary representation of the translations $V(x)$, $x \in \mathbb{R}^4$ which acts covariantly on the observables,

$$V(x)\, \mathfrak{A}(\mathcal{O})\, V(x)^{-1} = \mathfrak{A}(\mathcal{O} + x), \tag{4.5}$$

and fulfils the relativistic spectrum condition. Without restriction of generality we may require that $V(x) \in \mathfrak{A}''$ and that the energy-momentum spectrum has a Lorentz-invariant lower boundary in each superselection sector of $\mathscr{H}$; as a matter of fact, these assumptions fix $V(x)$ uniquely (cf. [24 and the references quoted there]). Note that we do not assume the existence of Lorentz-transformations since this would exclude from the outset states carrying an electric charge [25].

In theories with a countable number of superselection rules one may think of $\mathscr{H}$ as a direct sum of all possible superselection sectors. In the presence of massless particles such a construction would, however, lead to a non-separable Hilbert space, since there exist uncountably many superselection sectors due to the numerous possibilities of forming infrared clouds. In view of this abundance of sectors a coarser concept is needed, which groups together sectors differing from one another only by the collective effects of infinitely many massless particles of zero energy, but which still allows to distinguish charges which can be attributed to individual particles as, e.g., the electric charge. Such a concept of *charge classes* has been proposed in [26]. If there exists a countable number of such classes one obtains a separable physical Hilbert space $\mathscr{H}$ by picking from each class a representative and taking the direct sum. This construction is clearly ambiguous, but this ambiguity is physically irrelevant because it merely concerns the infrared behaviour.

So let us assume that $\mathscr{H}$ is separable and that the observables have the split property (2.2). We want to show then that the structure of the center of $\mathfrak{A}''$ can still be uncovered within bounded space-time regions. We begin by noting that, as a consequence of the above assumptions, all superselection sectors of $\mathscr{H}$ are locally equivalent (cf., e.g, [14, Theorem V.5.1]). Taking the subspace $\mathscr{H}_0 \subset \mathscr{H}$ of states carrying the charge-quantum numbers of the vacuum as a reference point and denoting the restriction of the observables $A \in \mathscr{H}$ to $\mathscr{H}_0$ by

$$\pi_0(A) = A \upharpoonright \mathscr{H}_0, \tag{4.6}$$

this equivalence can be expressed as follows: for any bounded region $\hat{\mathcal{O}}$ there exists an isometry W mapping $\mathscr{H}$ onto $\mathscr{H}_0$ such that

$$WAW^{-1} = \pi_0(A) \qquad \text{for} \quad A \in \mathfrak{A}(\hat{\mathcal{O}}). \tag{4.7}$$

Next, using the split property for the observables and taking into account that $\pi_0(\mathfrak{A})$ acts irreducibly on $\mathscr{H}_0$ one can construct (in complete analogy to the discussion in Sect. 3) a universal localizing map $\psi_\Lambda^{(0)}$ corresponding to the triple $\Lambda^{(0)} = (\pi_0(\mathfrak{A}(\mathcal{O})), \pi_0(\mathfrak{A}(\hat{\mathcal{O}})), \Omega)$; $\psi_\Lambda^{(0)}$ maps $\mathscr{B}(\mathscr{H}_0)$ into $\pi_0(\mathfrak{A}(\hat{\mathcal{O}}))$ and acts trivially on $\pi_0(\mathfrak{A}(\mathcal{O}))$.

Now because of relation (4.7) one can lift $\psi_\Lambda^{(0)}$ to a universal localizing map ψ_Λ acting on $\mathscr{B}(\mathscr{H})$, setting[11]

$$\psi_\Lambda(T) = W^{-1} \cdot \psi_\Lambda^{(0)}(WTW^{-1}) \cdot W \qquad \text{for} \quad T \in \mathscr{B}(\mathscr{H}). \tag{4.8}$$

It is obvious that $\psi_\Lambda(\mathscr{B}(\mathscr{H})) \subset \mathfrak{A}(\hat{\mathcal{O}})$ and that $\psi_\Lambda(A) = A$ for $A \in \mathfrak{A}(\mathcal{O})$. So, bearing in mind that $V(x) \in \mathfrak{A}''$, one can still define local translations $V_\Lambda(x) = \psi_\Lambda(V(x))$, $x \in \mathbb{R}^4$. As in the case of localizable charges, it then follows that the center of the algebra $\mathscr{R}$ generated by the local Hamiltonian H_Λ and $\mathfrak{A}(\mathcal{O})$ is isomorphic to the center of $\mathfrak{A}''$. So the superselection structure manifests itself locally in a clearcut way also in the presence of non-localizable charges.

We emphasize that the knowledge of the correct local Hamiltonian H_Λ is crucial, however. Denoting the restriction of the global translations $V(x)$ to the vacuum sector $\mathscr{H}_0$ by $V^{(0)}(x)$ one can, for example, define on $\mathscr{H}$ another local representation of the translations

$$V_\Lambda^{(0)}(x) = W^{-1} \psi_\Lambda^{(0)}(V^{(0)}(x)) \cdot W, \tag{4.9}$$

which also acts correctly on the observables in $\mathcal{O}$. But the spectrum of $V_\Lambda^{(0)}(x)$ coincides with that of the states in the vacuum sector; moreover, the algebra $\mathscr{R}^{(0)}$ generated by $\mathfrak{A}(\mathcal{O})$ and the local Hamiltonian $H_\Lambda^{(0)}$ is isomorphic to $\mathscr{B}(\mathscr{H}_0)$, so its center is trivial. Hence by using this "wrong" local Hamiltonian one would not recognize any superselection structure.

The differences between $H_\Lambda^{(0)}$ and H_Λ can roughly be explained as follows: the observable $H_\Lambda^{(0)}$ measures, in a sense, the energy neeed for the preparation of states in the region $\mathcal{O}$ by perturbations of the vacuum. More precisely, if $\omega_\Lambda^{(0)}$ is a state on $\mathscr{R}^{(0)}$ with spectrum (relative to $H_\Lambda^{(0)}$) about $E^{(0)}$ there exists a unique state $\omega^{(0)}$ of $\mathfrak{A}$ which coincides with $\omega_\Lambda^{(0)}$ on $\mathfrak{A}(\mathcal{O})$ and has finite total energy. This state is given by

$$\omega^{(0)}(A) = \omega_\Lambda^{(0)}(W^{-1} \psi_\Lambda^{(0)}(\pi_0(A)) W) \qquad \text{for} \quad A \in \mathfrak{A}, \tag{4.10}$$

so it belongs to the vacuum sector and, as can easily be seen, has total energy $E^{(0)}$. Hence if the state $\omega_\Lambda^{(0)}$ describes a charged particle which is localized in the region $\mathcal{O}$ one ascribes to it the total energy of a neutral state consisting of this particle and a compensating charge in the causal complement of $\mathcal{O}$. So charged particles are regarded as pieces of extended neutral states, and consequently one does not see any superselection rules. This point of view is, however, artificial because it does not take into account that particles are well-localized concentrations of matter to which one can asign an individual energy. (Note that this holds also true for particles carrying a non-localizable charge, cf. [9].) Accordingly, the energy ascribed to a state consisting of several, sufficiently far separated particles in the region $\mathcal{O}$ should be equal to the sum of the energies of the individual constituents. It is obvious that

[11] The map ψ_Λ depends on the choice of the isometry W establishing the equivalence (4.7). Yet since this dependence is irrelevant here we do not indicate it explicitly.

$H_A^{(0)}$ does not fulfil this requirement, and because of this lack of additivity [27] it has to be discarded as observable defining the local energy.

That H_A leads to a more reasonable definition of local energy can be made plausible as follows: let ω be any state of $\mathfrak{A}$ describing a configuration of particles whose total energy E is concentrated in $\mathcal{O}$. Then there exists a state ω_A on $\mathscr{R}$, given by

$$\omega_A(R) = \omega(\psi_A^{-1}(R)) \qquad \text{for} \quad R \in \mathscr{R}, \tag{4.11}$$

which coincides with ω on $\mathfrak{A}(\mathcal{O})$ and has, with respect to H_A, spectrum about E. Thus H_A assigns to the state ω_A an energy which is compatible with the idea of additivity. A full justification of the interpretation of H_A as local energy requires, however, a proof that the operators H_A converge to the global Hamiltonian H if $\mathcal{O}$ tends to $\mathbb{R}^4$. It is then necessary to remove the remaining ambiguities in the definition of H_A (cf. footnote 11), i.e., to select a coherent set of local Hamiltonians for an increasing net of regions $\mathcal{O}$. That this is possible has recently been shown in [18] for theories of localizable charges. It would be desirable to extend these results to the general case discussed here.

It is another interesting problem to find a characterization of the local space-time and symmetry transformations which does not rely on the existence of the global ones. A better understanding of this point would be important for an extension of our analysis to theories with spontaneously broken symmetries, where the local transformations still ought to exist. We hope to return to these problems elsewhere.

Acknowledgment

We would like to thank Rudolf Haag and Daniel Kastler for stimulating discussions and Jan Lopuzanski for a clarifying remark. We also gratefully acknowledge the hospitality and support extended to us at ZIF, Bielefeld, the University of Hamburg (S.D. and R.L.) and the University of Rome (D.B.).

References

1. S. Doplicher, *Commun. Math. Phys.* **85** (1982), 73.
2. S. Doplicher and R. Longo, *Commun. Math. Phys.* **88** (1983), 399.
3. C. Itzykson and J. B. Zuber, "Quantum Field Theory," Mc Graw–Hill, New York, 1980.
4. S. Doplicher, R. Haag, and J. E. Roberts, *Commun. Math. Phys.* **13** (1969), 1.
5. S. Doplicher, R. Haag, and J. E. Roberts, *Commun. Math. Phys.* **23** (1971), 199.
6. S. Doplicher, R. Haag, and J. E. Roberts, *Commun. Math. Phys.* **35** (1974), 49.
7. S. Doplicher, R. Haag, and J. E. Roberts, *Commun. Math. Phys.* **15** (1969), 173.
8. S. Doplicher and J. E. Roberts, *Bull. Amer. Math. Soc.* **11** (1984), 333; *J. Funct. Anal.*, in press.
9. D. Buchholz and K. Fredenhagen, *Commun. Math. Phys.* **84** (1982), 1.
10. S. Doplicher and R. Longo, *Invent. Math.* **73** (1984), 493.
11. D. Buchholz, *Commun. Math. Phys.* **36** (1974), 287.

12. C. D'ANTONI AND R. LONGO, *J. Funct. Anal.* **51** (1983), 361.
13. S. J. SUMMERS, *Commun. Math. Phys.* **86** (1982), 111.
14. M. TAKESAKI, "Theory of Operator Algebras I," Springer, New York/Heidelberg/Berlin, 1979.
15. D. BUCHHOLZ AND E. WICHMANN, Causal independence and energy level density of localized states in quantum field theory, preprint.
16. Work in progress.
17. H.-J. BORCHERS, *Commun. Math. Phys.* **4**, (1967), 315.
18. C. D'ANTONI, S. DOPLICHER, K. FREDENHAGEN, AND R. LONGO, in preparation.
19. P. FAYET AND S. FERRARA, *Phys. Rep. C* **32** (1977), 249.
20. J. BAGGER AND J. WESS, "Supersymmetry and Supergravity," Princeton Univ. Press, Princeton, N. J., 1983.
21. C. D'ANTONI, D. BUCHHOLZ, AND K. FREDENHAGEN, in preparation.
22. C. D'ANTONI AND K. FREDENHAGEN, *Commun. Math. Phys.* **94** (1984), 537.
23. H. J. BORCHERS, *Nuovo Cimento* **19** (1961), 787.
24. H. J. BORCHERS AND D. BUCHHOLZ, *Commun. Math. Phys.* **85** (1985), 169.
25. H. FRÖHLICH, G. MORCHIO, AND F. STROCCHI, *Phys. Lett. B* **89** (1979), 61.
26. D. BUCHHOLZ, *Commun. Math. Phys.* **85** (1982), 49.
27. A. S. WIGHTMAN, *in* "Trieste Lectures, 1962," Vienna, IAEA, 1963.