CORRELATIONS BETWEEN ADJOINT POLYAKOV LOOPS

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Received 9 September 1985

We consider 4D SU(2) lattice gauge theory and report a high statistics MC investigation of correlations between Polyakov loops in the adjoint SU(2) representation. For large β-values and on lattices with small sized spatial volumes these correlations allow glueball estimates improving results of the literature by several orders of magnitude. Our data for the mass gap and the string tension exhibit a very sharp crossover between the small-volume limit and the infinite-volume limit. This is prohibitive to extracting physics from Lüscher's weak coupling expansion by matching it with numerical data.

Let us address the problem of the spectrum of 4D SU(2) lattice gauge theory with the Wilson action

$$S = \tfrac{1}{2}\beta \sum_p (1 - \mathrm{Tr}\, U_p). \quad (1)$$

Analytic methods allow calculating the spectrum in the strong coupling (SC) $\beta \to 0$ limit [1,2]. The mass gap of the theory is the mass of the 0^+ glueball. In the "crossover" region at $\beta \sim 2.0$ the $m(0^+)$ SC series of ref. [2] breaks, however, down and Padé extrapolations [3] to the physical limit (first volume $V \to \infty$ then $\beta \to \infty$) are unreliable due to the complicated singularity structure.

Monte Carlo variational (MCV) calculations [4] [‡1] on an $L^3 N_t$ lattice allow reliable mass gap calculations beyond the region where the SC expansion breaks down. More precisely: Upper bounds on the mass gap are obtained from correlations at rather small distances $t = 0, 1, 2$ and to some extend also $t = 3$. These bounds are supposed to be reliable final estimates up to $\beta \lesssim 2.4$. Beyond $\beta \gtrsim 2.4$ the projection of the considered operators on the mass gap wave function becomes negligibly small. Consequently only a bad upper bound is obtained from short distance correlations, whereas at larger distances the correlations disappear into the statistical noise. For the SU(3) gauge group some improvement has been achieved by means of a high statistics MC cold wall calculation [6]. But at large β-values the method becomes impractical again, because the cold wall projects no longer significantly onto the mass gap wave function. This method does not even give bounds on the true mass gap, as positivity is lost.

The outlined shortcomings of MC calculations prevented so far studying the crossover to another notable limit in which analytic mass gap calculations are feasible [7], namely the limit $\beta \to \infty$ of an $L^3 \times \infty$ continuous box. The natural control parameter for the finite-volume theory is

$$z = m(0^+)L. \quad (2)$$

It may be thought of as the box length in physical units of the correlation length. For large L z rises linearly with L, but as $L \to 0$ the weak coupling expansion ($\beta \to \infty$) applies and z goes to zero only logarithmically. Lüscher's [7] weak coupling calculation of $m(0^+)/\Lambda_{\overline{\mathrm{MS}}}$ breaks down around $z \sim 1.5$, and for decreasing $z \lesssim 1.5$ one finds $m(0^+)/\Lambda_{\overline{\mathrm{MS}}}$ rising extremely rapidly. Therefore the crossover to the asymptotic behaviour $m(0^+)/\Lambda_{\overline{\mathrm{MS}}} \to$ const. for $z \to \infty$ is probably very sharp. This is not unexpected because of the finite temperature phase transition, which exists on an $L \times \infty^2 \times \infty$ system.

[1] Supported in part by Deutsche Forschungsgemeinschaft, contract BE 915/2-1.

[‡1] For a review see ref. [5].

Table 1
MC statistics (number of sweeps, 1 k = 1000) and used lattices.

$\beta = 2.25$		2.40		2.55		2.70		2.85	
120 k	$2^3 \times 32$	120 k	$2^3 \times 32$	130 k	$2^3 \times 32$	120 k	$2^3 \times 60$		
130 k	$4^3 \times 16$	210 k	$4^3 \times 24$	120 k	$4^3 \times 32$	120 k	$4^3 \times 64$	65 k	$4^3 \times 64$
60 k	$6^3 \times 16$	120 k	$6^3 \times 24$	120 k	$6^3 \times 32$	140 k	$6^3 \times 64$	55 k	$6^3 \times 64$

In case of the 2D O(3) σ-model the gap between MC calculations and the finite volume weak coupling expansion has been bridged recently [8]. In this letter we make a similar attempt for the 4D SU(2) lattice gauge theory. In the σ-model the MC calculation of ref. [8] is possible, because the spin field seems to have a reasonably good projection on the mass gap wave function at small *and* at large β-values (spacelike lattice length fixed). By dimensional reasons no local operator with this property exists for 4D lattice gauge theories. We therefore consider correlations between the simplest non-local operator which couples to the glueball wave function. This is the Polyakov loop in the adjoint SU(2) representation. An advantage of this operator, relevant at intermediate β-values, is that it allows multi-hit improved measurements [9]. This has extensively been used for investigating the Polyakov loop in the fundamental SU(2) representation [10], where the Polyakov loop does not couple to the glueball wave function, but is related to the string tension.

Let us denote the three spacelike directions of our lattice by x, y and z. By means of the periodic boundary conditions we close the Polyakov loop in z-direction. Summing over the x, y-positions we project out momentum $\boldsymbol{p} = 0$, i.e. we have constructed a translation invariant operator $P^a(t)$ ("a" stands for adjoint). One may further project on appropriate irreducible representations of the cubic group [1,11]. For this first study, however, we discard this option and measure directly the correlations

$$c(t) = \langle 0|P^a(0)P^a(t)|0\rangle_{\text{connected}} . \tag{3}$$

Previous results on the string tension are also improved by analysing the correlations between Polyakov loops in the fundamental representation.

Our MC calculations for various lattices and β-values are summarized in table 1, where the final statistics is given for each case. To reach equilibrium we have carried out between 1000 and 2000 sweeps without measurements. The SU(2) gauge group was approximated by using the 120 element icosaeder subgroup and multi-hit improved measurements were done every 10 sweeps.

The multi-hit improvement is efficient when the dominant fluctuations are short range. This is true when L is large as compared to the correlation length. At the high β-values $\beta = 2.70$ (except $N = 6$) and $\beta = 2.85$ CPU time was saved by doing only normal measurements. In any case normal measurements were done for the sake of comparison.

Our mass gap estimates are collected in table 2. We define the effective mass at distance t, $m(t)$, by the implicit formula

Table 2
Final mass gap estimates in units of Λ_L and in lattice units. The number in parenthesis gives the distance from which the final estimate was taken.

Lattice	$\beta = 2.25$		2.40		2.55		2.70		2.85	
2^3N_t	186 ± 5	(3)	230 ± 13	(4)	228 ± 19	(5)	376 ± 28	(5)		
4^3N_t	194 ± 5	(3)	176 ± 19	(4)	180 ± 10	(5)	260 ± 10	(6)	339 ± 7	(5)
6^3N_t	noise		180 ± 16	(4)	159 ± 10	(5)	204 ± 15	(4)	251 ± 14	(5)
2^3N_t	1.25 ± 0.03	(3)	1.06 ± 0.06	(4)	0.72 ± 0.06	(5)	0.81 ± 0.06	(5)		
4^3N_t	1.30 ± 0.03	(3)	0.81 ± 0.04	(4)	0.57 ± 0.03	(5)	0.56 ± 0.02	(6)	0.50 ± 0.01	(5)
6^3N_t	noise		0.83 ± 0.07	(4)	0.50 ± 0.03	(5)	0.44 ± 0.04	(4)	0.37 ± 0.02	(5)

$$\rho(t) = c(t)/c(t-1)$$
$$= \frac{e^{-m(t)t} + e^{-m(t)[N_t - t]}}{e^{-m(t)[t-1]} + e^{-m(t)[N_t - t + 1]}} . \quad (4)$$

Neglecting the "cosh effect" this reduces to the usual definition

$m(t) = \ln \rho(t)$.

In table 2 the number in parenthesis gives the distance t from which the final estimate was taken. In case of stable correlations over several distances the error bars can be corrected towards lower values. For two example points ($\beta = 2.55, 4^3 \times 32$ and $\beta = 2.70, 4^3 \times 64$) the thus obtained t-dependence of mass gap estimates, $m(0^+)(t)$, is illustrated in table 3. From the viewpoint of $t \to \infty$ stability the correlations at $\beta = 2.70$ ($4^3 \times 64$ lattice) are among our nicest. Altogether the results are very encouraging: The signal can be followed to much larger distances than in previous MCV calculations and we are able to obtain also results at a much larger correlation length than before. At $\beta = 2.85$ ($6^3 \times 64$ lattice) the correlation length is close to $\xi = 3$,

Table 3
t dependence of mass gap estimates $m(0^+)(t)$ (given in lattice units).

t	$\beta = 2.55, 4^3 \times 32$	$\beta = 2.70, 4^3 \times 64$
2	0.736 ± 0.006	0.618 ± 0.004
3	0.661 ± 0.010	0.568 ± 0.006
4	0.63 ± 0.02	0.557 ± 0.009
5	0.57 ± 0.03	0.556 ± 0.011
6	0.51 ± 0.05	0.56 ± 0.02
7	noise	0.56 ± 0.04
final estimate	0.57 ± 0.03	0.56 ± 0.02

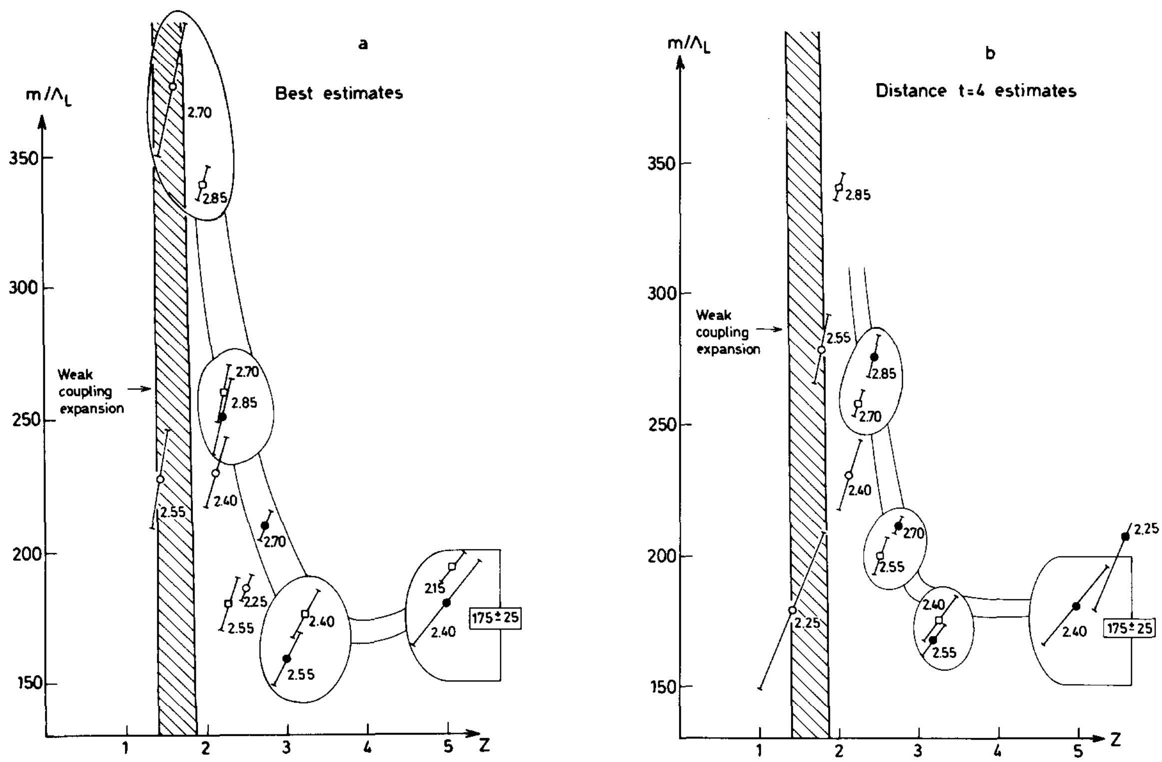

Fig. 1. Mass gap m as a function of z in units of Λ_L. Lattice sizes are indicated as follows: $2\ {}^3N_t$, $4\ {}^3N_t$ and $6\ {}^3N_t$. The attached numbers give the β-values corresponding to the data points. The two full lines are from the small z-expansion of ref. [7], if $\Lambda_{\overline{MS}} = 19.82\Lambda_L$ is used. (a) relies on the estimates of table 2, whereas (b) depicts for comparison $m(t = 4)$. Data points supporting universal behaviour are encircled.

Table 4
Final $\sqrt{\text{string tension}}$ estimates in units of Λ_L and in lattice units. The first number in parenthesis gives the distance t from which the final estimate was taken, the second number indicates the distance up to which consistency is achieved.

Lattice	$\beta = 2.25$		2.40		2.55		2.70		285	
$2^3 N_t$	52.0 ± 0.8	(5–12)	64.8 ± 1.6	(7–14)	82.5 ± 1.0	(6–14)	109.2 ± 2.8	(8–18)		
$4^3 N_t$	49.4 ± 0.6	(5–8)	49.6 ± 1.1	(6–12)	60.0 ± 1.0	(5–14)	74.7 ± 1.0	(5–18)	99.1 ± 3.4	(6–18)
$6^3 N_t$	55.8 ± 2.0	(4–6)	45.6 ± 0.7	(5–10)	47.0 ± 1.0	(8–15)	59.4 ± 1.0	(5–18)	77.4 ± 1.4	(3–10)
$2^3 N_t$	0.349 ± 0.005	(5–12)	0.298 ± 0.007	(7–14)	0.260 ± 0.003	(6–14)	0.235 ± 0.006	(8–18)		
$4^3 N_t$	0.331 ± 0.004	(5–8)	0.228 ± 0.005	(6–12)	0.189 ± 0.003	(5–14)	0.161 ± 0.002	(5–18)	0.146 ± 0.005	(6–18)
$6^3 N_t$	0.374 ± 0.013	(4–6)	0.210 ± 0.003	(5–10)	0.148 ± 0.003	(8–15)	0.128 ± 0.002	(5–18)	0.114 ± 0.002	(3–10)

whereas previously the largest correlation length at which reliable results could be obtained was only slightly above $\xi = 1$.

The z-dependence of our mass gap data is summarized in fig. 1a. For comparison we depict in fig. 1b the mass gap results obtained at distance $t = 4$. (At small β-values $t = 4$ gives of course rather large error bars.) In both figures the crossover from small z-behaviour to large z-behaviour is extremely sharp. Around $z \lesssim 2$ the mass gap rises rapidly for decreasing z and approaches the weak coupling expansion [7] from the right. In converting the results of ref. [7] to the Λ_L-scale we have used the one-loop perturbative result $\Lambda_{\overline{MS}} = 19.82\ \Lambda_L$ $(\beta = \infty)$ and neglected $1/\beta$-corrections. Because of the extremely rapid increase of m for decreasing small z the picture would, however, remain unaffected including $1/\beta$-corrections, and a more detailed analysis like the one carried out in ref. [8] seems to be impossible for 4D lattice gauge theory.

We have encircled MC data from different β-values supporting a universal curve in the m–z-plane. Up to an instability of the $\beta = 2.55$ $(4^3 \times 32)$ result both figures are consistent and we obtain nearly identical shapes indicating the z-dependence of the mass gap. For large z the MC data approach

$$m(0^+) = (175 \pm 25)\Lambda_L , \qquad (5)$$

in good agreement with previous MC estimates [4]. Fig. 1 clearly reveals that Lüscher's weak coupling expansion does not yield information about the $z \to \infty$ limit. In case of 2D σ-models the situation is more subtle, see ref. [8].

Following the lines of refs. [9,10] we obtain the string tension K from correlations between Polyakov loops in the fundamental representation. Table 4 summarizes our final estimates for $\sqrt{K}$ [in analogy to table 2 for $m(0^+)$], and table 5 illustrates for two example points (again $\beta = 2.55$, $4^3 \times 32$ and $\beta = 2.70$, $4^3 \times 64$ lattice) the t-dependence of the string tension estimates $K(t)$ (in analogy to table 3). The stability over many distances is quite impressive. Fig. 2 plots $\sqrt{K}$ (in units of Λ_L) versus the variable

$$z' = 3.5\sqrt{K}\,L . \qquad (6)$$

The factor 3.5 is introduced to achieve

$$z' \sim z , \qquad (7)$$

where z is defined by eq. (2). For completeness we

Table 5
t dependence of string tension estimates $K(t)$.

t	$\beta = 2.55, 4^3 \times 32$	$\beta = 2.70, 4^3 \times 64$
2	0.0374 ± 0.0006	0.0274 ± 0.0003
3	0.0360 ± 0.0007	0.0262 ± 0.0003
4	0.0359 ± 0.0009	0.0260 ± 0.0004
5	0.0358 ± 0.0010	0.0259 ± 0.0005
6	0.0359 ± 0.0012	0.0260 ± 0.0006
7	0.0360 ± 0.0013	0.0260 ± 0.0007
8	0.0360 ± 0.0014	0.0261 ± 0.0009
9	0.0357 ± 0.0016	0.0262 ± 0.0011
10	0.0356 ± 0.0019	0.0264 ± 0.0013
11	0.0357 ± 0.0021	0.0267 ± 0.0015
12	0.0357 ± 0.0024	0.0269 ± 0.0017
13	0.0356 ± 0.0027	0.0270 ± 0.0020
14	0.0356 ± 0.0030	0.0271 ± 0.0023
15	0.0359 ± 0.0030	0.0272 ± 0.0028
16	0.0355 ± 0.0040	0.0274 ± 0.0033
17		0.0278 ± 0.0041
18		0.0280 ± 0.0051
final estimate	0.0358 ± 0.0010	0.0259 ± 0.0005

have also included MC data of ref. [10] in fig. 2. The results are now as follows: For decreasing $z' \lesssim 3$ the string tension rises sharply, but the crossover seems to be smoother than in case of the mass gap. For $4 < z' < 5$ our MC data indicate universal behaviour and a value $45\Lambda_L < \sqrt{K} < 50\Lambda_L$. For larger z' (up to $z' \sim 9.5$) $\sqrt{K}$ rises smoothly by about 10–20%. It is, however, not completely clear whether this behaviour is indeed universal or has to be attributed to using too small lattices. For the sake of definiteness, we have plotted the z behaviour of the finite size string tension implied by the Coulomb correction [13]

$$\sqrt{K_\infty} = \sqrt{K}\{1 - \pi[3(z'/3.5)^2]^{-1}\}^{1/2}. \tag{8}$$

Using our data for $z > 4$, a least squares fit to eq. (8) gives $\sqrt{K} = 61\Lambda_L$. The estimate

$$\sqrt{K_\infty} = (61 \pm 5)\Lambda_L \tag{9a}$$

encloses all the data used. Eq. (8) relies on unproven relations between nonabelian gauge theories and string theory. Assuming instead of eq. (8) an expo-

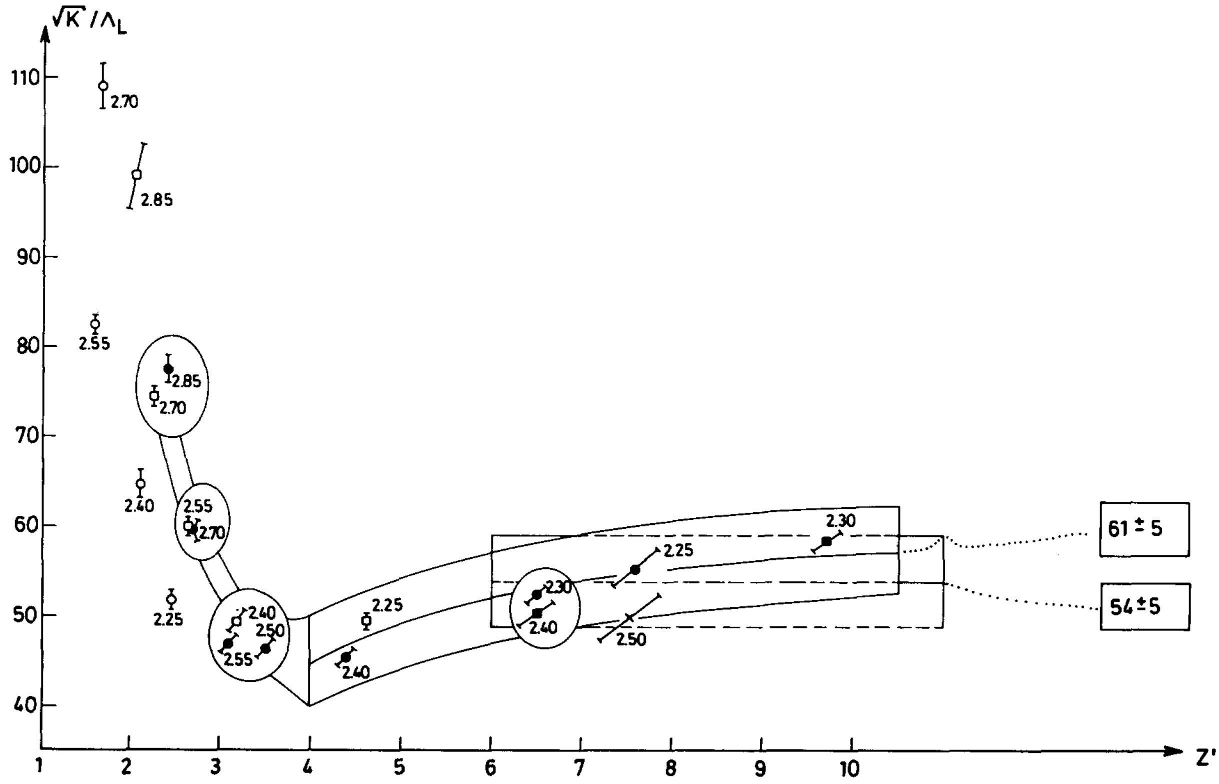

Fig. 2. $\sqrt{\text{String tension}}$ as a function of $z' = 3.5\sqrt{K}\,L$ in units of Λ_L. Lattice sizes are indicated as in fig. 1. For completeness the following data points from ref. [10] are included: $\beta = 2.3$ ($6^3 \times 24$, $8^3 \times 24$), $\beta = 2.4$ ($8^3 \times 16$) and $\beta = 2.5$ ($6^3 \times 24$, $12^3 \times 24$). The $8^3 N_t$ lattices are indicated by ⧳ and the $12^3 \times 24$ lattice by ⧲.

nential approach to the asymptotic value $\sqrt{K}$ would lead to the estimate

$$\sqrt{K_\infty} = (54 \pm 5)\Lambda_L \tag{9b}$$

and is in good agreement with previous results of ref. [10]; see also refs. [4,13].

Tables 2 and 4 show that one may very well push for results at even larger β-values and lattices. We plan to do this in a similar investigation for the SU(3) gauge group, hoping that the first-order deconfinement phase transition which occurs on an $L \times \infty \times \infty$ lattice will not be an obstacle.

Computer calculations were carried out at the Supercomputer Institute, Tallahassee. We would like to thank Alberto Devoto and Dennis Duke for their help. Further we want to thank Enzo Marinari without whom this work would have been impossble. This paper was completed at DESY and one of the authors (A.B.) thanks Roberto Peccei and the DESY theory group for their kind hospitality. Finally we appreciated help of Bernd Baumann.

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