TWO-STATE SIGNAL AT THE CONFINEMENT–HIGGS PHASE TRANSITION IN THE STANDARD SU(2) HIGGS MODEL

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In a Monte Carlo calculation, numerical evidence is found for metastable coexisting phases in the variable length ($\lambda = 1.0$) and fixed length ($\lambda = \infty$) SU(2) Higgs model with doublet scalar field. This supports earlier conjectures about the first order nature of the phase transition for finite gauge coupling (β) and an arbitrary scalar self-coupling (λ).

Introduction. The SU(2) Higgs model with a scalar doublet Higgs field ("standard Higgs model") is an important part of the standard SU(3) ⊗ SU(2) ⊗ U(1) model of strong and electroweak interactions. The vacuum expectation value of the Higgs field renders the W- and Z-bosons a mass and is responsible also for the fermion masses via the fermion Yukawa couplings. From the point of view of possible high energy extensions of the standard model, it is important to know, what happens if one tries to decrease the lattice spacing in order to reach a continuum limit in the lattice formulation of the standard Higgs model. In addition, as a prototype Higgs model with non-abelian gauge symmetry, SU(2) Higgs models can serve as a working laboratory for the understanding of more complicated (and more complete) Higgs models, like the Higgs sector in an SU(5)-like grand unified theory.

In a previous paper of one of us [1] some numerical evidence was obtained, from the study of the correlations and static energies, for the existence of asymptotically free critical points at vanishing bare gauge coupling ($g = 0$ or $\beta \equiv 4/g^2 = \infty$) and arbitrarily fixed scalar self-coupling (λ = const.). (For a discussion and summary see also ref. [2].) This means that it is possible to define a non-trivial continuum limit of the standard Higgs model by keeping λ constant. The resulting continuum theory is possibly λ-independent. The λ-independence implies that the continuum theory has one free parameter less, therefore, if the continuum limit is assumed to be relevant for phenomenology, then the physical value of the Higgs boson mass (m_H) can be predicted from the value of the W-boson mass (m_W) and of the renormalized gauge coupling. The first Monte Carlo measurement gave $m_H \simeq 6\, m_W$ [2]. Assuming that the classical Higgs potential is a good approximation also in this situation, the high Higgs boson mass corresponds to a low energy effective theory with strong physical Higgs self-coupling at the W-boson mass scale: $\lambda_{phys}(\mu = m_W) \gg 1$. (Note, however, that the triviality of the pure ϕ^4 theory and the asymptotic freedom of the gauge coupling suggest that $\lambda_{phys}(\mu)$ vanishes at asymptotically high energies: $\lim_{\mu\to\infty} \lambda_{phys}(\mu) = 0$.)

In the standard Higgs model, besides the conventionally assumed "Higgs-phase" there is also a "confining phase" corresponding to a QCD-like theory with a scalar matter field. In the lattice regularization scheme there is a phase transition between these two phases. The position and order of this phase transition has already been studied in the first numerical Monte Carlo investigations [3–6]. The phase transition was first considered second order for finite β and large enough λ-values, but finite size scaling studies [5] and the abrupt change of the correlation lengths near the phase transition [1] prefer a first-order transition for all values of λ. In a strong gauge-coupling approximation, the extension of the calculations in ref. [7] to the case of the fundamental Higgs field also indicates a first-or-

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der transition for every λ [7]. A recent, more detailed Monte Carlo investigation by the Aachen–Graz group [8] established the first-order nature for small λ-values ($\lambda \leqslant 0.03$), but still left open the question of the order at larger λ (and in particular at $\lambda = \infty$). In general, it is clear that the transition weakens for increasing λ and therefore it becomes increasingly more difficult to tell the order at large λ.

The order of the confinement–Higgs phase transition is relevant also for the existence and properties of the critical points at $\beta = \infty$. In the case of a second-order phase transition line in the λ = const. planes the correlation lengths are infinite along this line. This allows for the expected exponential rise of the correlation lengths exp(const. β) along the renormalization group trajectories (RGT's) going to the critical point

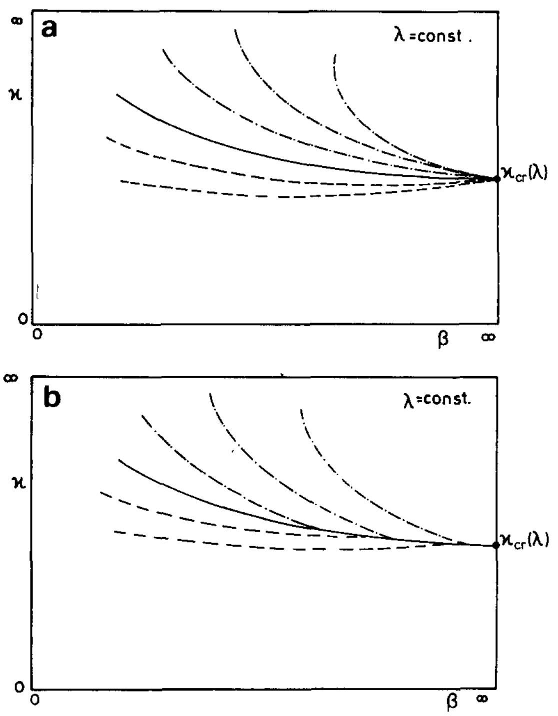


Fig. 1. (a) The schematic picture of RGT's in a λ = const. plane in the case of a second-order phase transition line (full line). The dashed–dotted lines are the RGT's in the Higgs phase, the dashed ones the RGT's in the confinement phase. The correlation lengths diverge for $\beta \to \infty$. (b) The same as (a) in the case of a first-order phase transition line (full line) ending in a second-order point $\beta = \infty$, $\kappa_{cr}(\lambda)$, provided that the correlation length along the phase transition line is not increasing fast enough for $\beta \to \infty$.

at $\beta = \infty$ near the phase transition line. The picture of the RGT's in the (β, κ)-plane is then qualitatively given, as suggested in refs. [1,2], by fig. 1a. If, however, the phase transition line is first order everywhere except for the endpoint at $\beta = \infty$ (where it is second order), then there is no reason for the correlation lengths to diverge for finite β. If the maximum of the correlation lengths does not increase sufficiently fast for $\beta \to \infty$, then the (approximate) RGT's do not reach the critical point at $\beta = \infty$: they are going to the discontinuity at the first-order line for some finite correlation length, like it is shown by fig. 1b. In this case the critical point at $\beta = \infty$ is likely to be trivial (i.e. equivalent to the pure gauge theory in the confinement phase and to a free theory of massive bosons in the Higgs phase). Therefore, in the case of a first-order phase transition line, the sufficiently fast increase of the maximum correlation length along the line is a non-trivial requirement for the existence of a non-trivial continuum limit in the $\beta = \infty$ critical point. A theoretically appealing situation would be, if the confinement–Higgs phase transition would everywhere be first order for finite β and λ but second order for infinite β or λ. In this case the RGT's could safely reach $\beta = \infty$ at $\lambda = \infty$.

In this letter, as a first result of a high-statistics Monte Carlo investigation of the standard Higgs model, we show strong evidence for coexisting phases at $\lambda = 1.0$ and a somewhat weaker evidence at $\lambda = \infty$. This indicates, that the phase transition is probably first order everywhere at the chosen β value ($\beta = 2.3$). In addition, we also performed precise correlation length measurements close to the phase transition. More details and a comparison between high-statistics data on 12^4 and 8^4 lattices will be published in a forthcoming paper [9].

Monte Carlo calculation. The numerical calculation was performed on a 12^4 lattice for the full SU(2) group. The lattice action in the gauge invariant variables on sites ($\rho_x > 0$) and on links ($V(x,\mu) \in$ SU(2)) is [1,2]

$$S = \beta \sum_{\mathrm{P}} (1 - \tfrac{1}{2}\mathrm{Tr}\, V_{\mathrm{P}}) + \sum_x \Big(\rho_x^2 - 3 \log \rho_x + \lambda(\rho_x^2 - 1)^2 - \kappa \sum_{\mu=1}^{4} \rho_{x+\mu}\rho_x \,\mathrm{Tr}\, V(x,\mu)\Big) . \quad (1)$$

The first term (a sum over plaquettes P) is the pure

SU(2) gauge action. The third coupling parameter, besides λ and β, is the "hopping parameter" κ. In the $\lambda \to \infty$ limit the Higgs field length ρ_x is frozen to $\rho_x = 1$, and we are left with

$$S_{\lambda=\infty} = \beta \sum_{\mathrm{P}} (1 - \tfrac{1}{2}\mathrm{Tr}\, V_{\mathrm{P}}) - \kappa \sum_{x,\mu} \mathrm{Tr}\, V(x,\mu). \qquad (2)$$

The value of the gauge coupling was fixed to $\beta = 2.3$, where a large part of the numerical calculation in ref. [1] was done. λ was either 1 or ∞. The points were chosen in the vicinity of the phase transition at the hopping parameter $\kappa = \kappa_{\mathrm{pt}}(\lambda, \beta)$. The value of κ was tuned in the third or fourth digit. So close to the phase transition there are very long time correlations in the updating (we used the Metropolis updating procedure with 6 hits per link or site). In order to overcome this, 80–120 000 sweeps per point were performed. The first 10–20 000 sweeps were left out from the statistics for equilibration. This turned out to be essential, because after changing the κ-value from some neighbouring point, we observed a typical equilibration process characterized by oscillatory collective changes of the lattice configuration. In the latter stages of the updating such collective changes do occur, too, but usually with smaller amplitudes and less frequently. The effect of the equilibration process is influencing mainly the correlation length in the Higgs-boson (scalar, isoscalar) channels: including some part of the first 10–20 000 sweeps into the final statistics reduces the Higgs mass (am_{H}) considerably. This effect is most substantial in the points where am_{H} is near its minimum. In view of this, the number of sweeps we performed is by no means too large. On the contrary, still higher statistics would be desirable, especially if one would try to go to larger lattices.

Results. Some average quantities at $\lambda = \infty$ are shown in table 1. The statistical errors given in the table are determined from estimates of the standard deviation by binning the data in bins of 2^k ($k = 0, 1, 2, ...$). The correlations in the Higgs-boson (scalar, isoscalar) channel were measured by the quantities ρ_x, $\mathrm{Tr}\, V(x,\mu)$ and $\phi_{x+\mu}\rho_x\, \mathrm{Tr}\, V(x,\mu)$. Those in the W-boson (vector, isovector) channel by $\mathrm{Tr}\{\tau_r V(x,\mu)\}$ and $\rho_{x+\mu}\rho_x \times \mathrm{Tr}\{\tau_r V(x,\mu)\}$.

The correlations in the three Higgs-boson, respectively, two W-boson channels are strongly correlated, that is the correlation lengths determined in the same channel by different quantities deviate from each other much less than the individual statistical errors. (The statistical errors were obtained also here by 2^k binning.) The errors are somewhat smaller in the case of $\rho_{x+\mu}\rho_x\, \mathrm{Tr}\, V(x,\mu)$, respectively, $\rho_{x+\mu}\rho_x\, \mathrm{Tr}\{\tau_r V(x,\mu)\}$, therefore these are slightly better to use if one wants to rely on a single quantity.

Table 1
Some average quantities calculated on a 12^4 lattice at $\lambda = \infty$ and $\beta = 2.3$. The link expectation value $L \equiv \frac{1}{2}\langle \mathrm{Tr}\, V(x,\mu)\rangle$, plaquette expectation value $P \equiv \langle 1 - \frac{1}{2}\mathrm{Tr}\, V_{\mathrm{P}}\rangle$ and average action per site $s \equiv 6\beta P + 1 + 8\kappa(1 - L)$ is given. The errors in the last numerals are in parentheses.

κ	L	P	s
0.390	0.2485(2)	0.39126(8)	8.744(2)
0.392	0.2535(3)	0.39047(9)	8.729(2)
0.394	0.2599(4)	0.38937(15)	8.706(4)
0.395	0.2677(8)	0.3876(3)	8.663(7)
0.396	0.2635(6)	0.3864(4)	8.634(9)
0.397	0.2783(6)	0.3854(3)	8.610(8)
0.398	0.2856(4)	0.38377(14)	8.570(4)
0.400	0.2931(3)	0.38253(11)	8.541(4)
0.410	0.3214(2)	0.37860(5)	8.450(2)

The obtained masses (inverse correlation lengths) are shown in fig. 2 as a function of $\langle \frac{1}{2}\, \mathrm{Tr}\, V(x,\mu)\rangle$. In most cases the correlation between timeslices can be fitted well for time distances 3–6 by a single cosine-hyperbolicus (corresponding to a single mass). In the points, however, where the masses are around 0.2–0.3, there is still some appreciable contribution from higher states. In these cases one has to use a single mass fit to the distances 4–6 and/or a two-mass fit to the distances 2–6. It is interesting, that near the phase transition the effect of higher states is usually stronger in the W-channel than in the Higgs-boson channel.

As one can see from fig. 2, the approximate universality of the masses between $\lambda = \infty$ and $\lambda = 1.0$ is good within the present errors, therefore it is much better than it was shown by the lower-statistics 8^4 data in ref. [1]. This means that a low-order strong self-coupling expansion [2] works presumably well for the correlation lengths at $\lambda = 1.0$.

It can also be seen, that below the minimum of the masses there is a region in κ, where both am_{H} and am_{W} are below or around 1. This is a hint for the existence of a non-trivial confinement-like phase below the phase transition surface. (In ref. [1] no such points

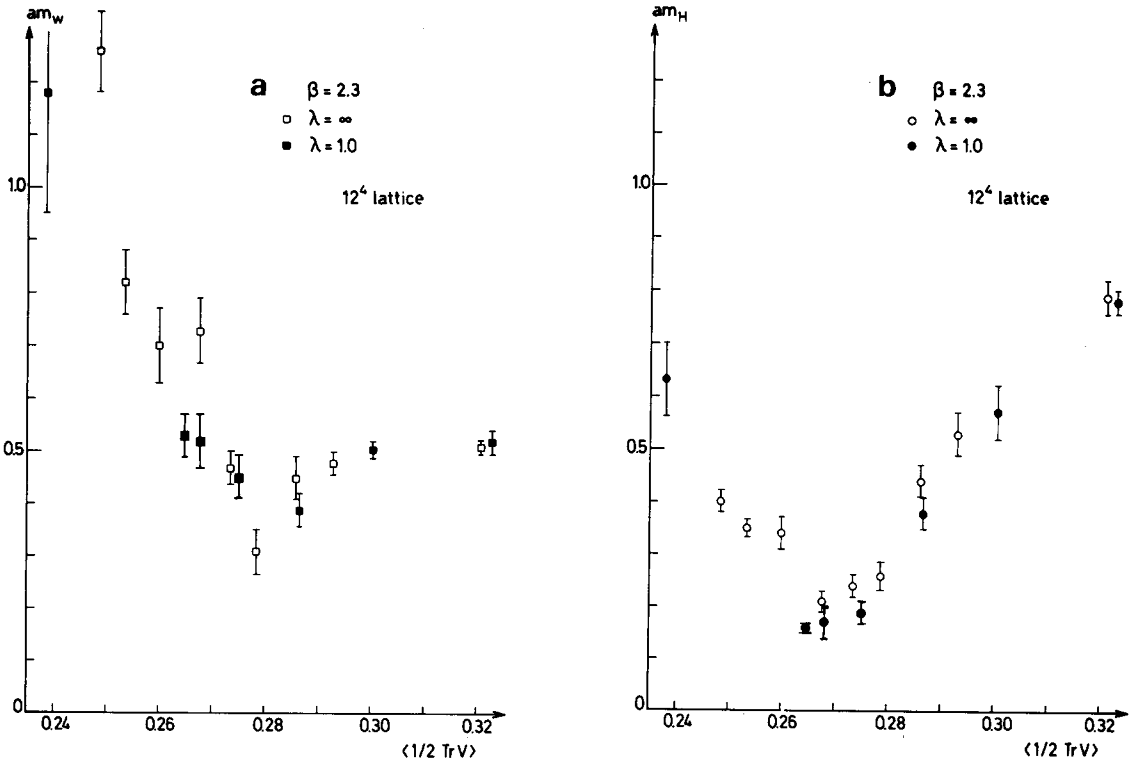


Fig. 2. (a) The mass in lattice units in the W-boson channel as a function of the link expectation value $L \equiv \frac{1}{2}\langle \mathrm{Tr}\, V(x,\mu)\rangle$. (For the corresponding κ values see table 1.) (b) The same as (a) for the Higgs-boson mass.

were found, because the steps in κ were too large.)

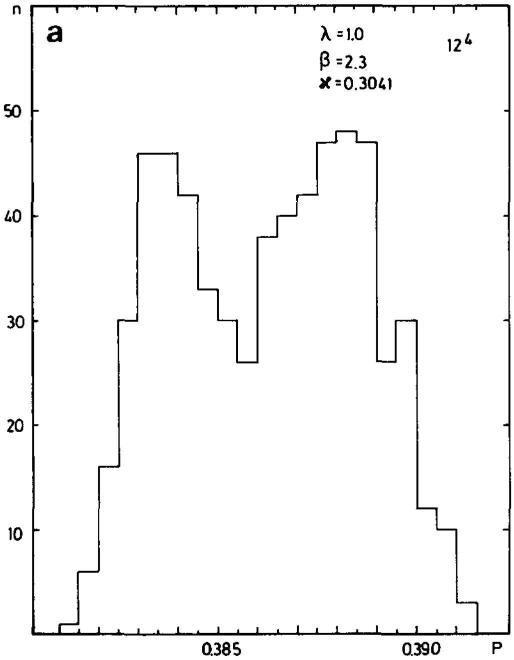
Defining the position of the phase transition κ_{pt} by the maximum of the correlation lengths, we obtain on the 12^4 lattice

$$\kappa_{\mathrm{pt}}(\beta = 2.3, \lambda = \infty) = 0.395 \rightarrow 0.397,$$

$$\kappa_{\mathrm{pt}}(\beta = 2.3, \lambda = 1.0) = 0.3041 \rightarrow 0.3045. \qquad (3)$$

Above this region (i.e. in the Higgs phase) the mass ratio $m_{\mathrm{H}}/m_{\mathrm{W}}$ is greater than (1.0 ± 0.2), below it (in the confinement phase) we have $m_{\mathrm{H}}/m_{\mathrm{W}} \leqslant 0.5$. There is a rapid change of $m_{\mathrm{H}}/m_{\mathrm{W}}$ in the phase transition region itself, from a smaller value 0.3–0.5 at the lower edge to a value of about 1 near the upper edge.

In order to get more information about the phase transition, we extensively studied the fluctuations of average quantities (like link expectation value, plaquette expectation value or average action per site etc.) during the updating. To reduce intrinsic fluctuations, the quantities were first averaged in a number of sweeps, typically 50–200. The distribution of the obtained average plaquette values is shown in the point $\lambda = 1, \kappa = 0.3041$ in fig. 3a, and for $\lambda = \infty, \kappa = 0.395$ in fig. 3b. In the first point there is a clearly separated two-peak structure, which shows that the configuration is oscillating between two metastable states. This can, of course, also be seen as a function of time (for more details, see ref. [9]). The same can be observed also in the other average quantities [9]. At a slightly larger κ-value, $\kappa = 0.3042$, the two peaks are at the same place, but there the left peak (at the smaller P-values) is stronger than the right one. The two-state signal becomes weak at $\kappa = 0.3045$, and disappears for still higher κ. In the case of fixed length $(\lambda = \infty)$ at $\kappa = 0.395$ there is a two-peak structure, too, but the distance of the peaks is smaller, and the system does not stay very long in one state. (Changes between the two metastable states occur on the 12^4 lattice typically after 1000–5000 sweeps.)



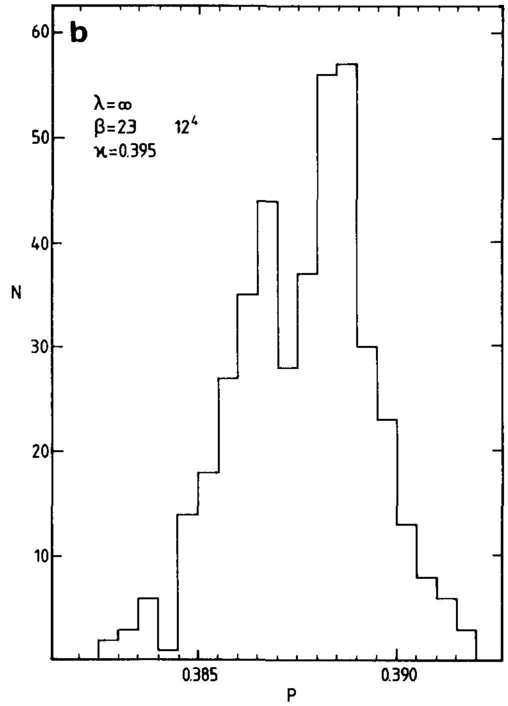


Fig. 3. (a) The distribution of the mean plaquette expectation value $P \equiv \langle 1 - \frac{1}{2}\mathrm{Tr}\, V_P \rangle$ averaged over 200 consecutive sweeps during the updating in the point $\lambda = 1.0, \beta = 2.3, \kappa = 0.3041$. (b) The same as (a) for $\lambda = \infty, \beta = 2.3, \kappa = 0.395$.

Discussion. The most probable interpretation of fig. 3 is that the confinement–Higgs phase transition is weakly first order at $\lambda = 1.0$ and $\lambda = \infty$, and therefore it is first order for any λ at the given gauge coupling ($\beta = 2.3$). The two-state signal is rather convincing at $\lambda = 1$. An indirect additional evidence for the first-order phase transition at $\lambda = \infty$ is provided by the first-order nature at $\lambda = 1.0$ and by the good universality between $\lambda = 1.0$ and $\lambda = \infty$ shown, for instance, by fig. 2. It is, nevertheless, not fully excluded, that a second-order phase transition produces a fake two-state signal on our 12^4 lattice. Therefore, a first-order phase transition surface with an edge of second order at $\lambda = \infty$ is still possible, although not probable.

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