A NON-PERTURBATIVE CONTRIBUTION TO THE VACUUM ENERGY IN SUPERSYMMETRIC YANG–MILLS THEORY

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It is shown that the quantum fluctuations around an interacting instanton–anti-instanton field configuration induce negative vacuum energy in supersymmetric Yang–Mills theory.

Supersymmetry [1] as a possible fundamental symmetry of bosons and fermions has been studied extensively in the past few years. It appears to cure the gauge hierarchy [2] problem in the grand unified theories [3,4]. It may also provide an explanation as to why the mass scales are so widely separated in these models. However, at ordinary energy scales, this symmetry is not exact. Whereas perturbative quantum effects respect supersymmetry, it would be desirable if non-perturbative fluctuations were to break it. To that effect it is important to analyse the quantum fluctuations of the instanton [5] type in the vacuum. The role of instantons in supersymmetric gauge theories has been extensively studied recently [6,7].

In Yang–Mills theory there exists an infinity of degenerate classical ground states characterized by an integer topological charge. Instantons provide a description of quantum mechanical tunnelling between ground states of different topological charge, thereby contributing non-trivially to the vacuum energy density [8]. However, when massless fermions are introduced, the tunnelling is completely suppressed due to the zero modes of the relevant Dirac operator in the topologically non-trivial background [9] (with Pontryagin index non-zero). Therefore in supersymmetry, where there are massless fermions, single instantons or anti-instantons (or any other field configuration with non-zero Pontryagin index) do not contribute to the vacuum energy. These cannot break supersymmetry and the vacuum energy stays at zero. However, topologically trivial but non-perturbative configurations, for which there are no exact fermionic zero modes, may contribute to the vacuum energy. An instanton–anti-instanton configuration would be an example of this type.

Indeed, in an earlier publication [10], it was shown that far separated non-interacting instanton–anti-instanton field configurations induce negative vacuum energy. The interaction action was used only to estimate the minimal distance of separation between the instanton and the anti-instanton up to which the approximation is valid. In this work we study the same quantum fluctuations but with interactions included. We find a lower bound on the contribution of these quantum fluctuations to the functional integral. They induce a negative vacuum energy in an agreement with the results found before [10]. The induced vacuum energy is very small but non-zero. If it is not canceled by other non-perturbative effects it suggests an explicit breaking of supersymmetry.

To be more definite, consider a supersymmetric Yang–Mills theory with SU(2) as a gauge group. The euclidean action is given by

$$S_E = \int d^4x \left[\tfrac{1}{4} F^a_{\mu\nu} F^a_{\mu\nu} + \bar{\lambda}^a \left(iD_\mu \bar{\Sigma}_\mu \lambda \right)^a \right]. \tag{1}$$

Here $F^a_{\mu\nu}$ is the field strength,

$$F^a_{\mu\nu} = \partial_\mu W^a_\nu - \partial_\nu W^a_\mu + g\epsilon_{abc} W^b_\mu W^c_\nu ,$$

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$D_\mu^{ac} = \delta^{ac}\,\partial_\mu + g\epsilon^{abc}W_\mu^b$ is the covariant derivative, W_μ^a $(a = 1, 2, 3)$ are the gauge vector potentials and λ^a are the Majorana fermions. They are expressed in euclideanised Weyl basis with the Dirac matrices

$$\gamma_\mu = \begin{pmatrix} 0 & \Sigma_\mu \\ \bar{\Sigma}_\mu & 0 \end{pmatrix}, \quad \Sigma_\mu = (\mathrm{i}\sigma_i, \mathbb{1}), \quad \bar{\Sigma}_\mu = (-\mathrm{i}\sigma_i, \mathbb{1})$$

and

$$\Sigma_\mu\bar{\Sigma}_\nu + \Sigma_\nu\bar{\Sigma}_\mu = 2\delta_{\mu\nu}\mathbb{1}, \quad \bar{\Sigma}_\mu\Sigma_\nu + \bar{\Sigma}_\nu\Sigma_\mu = 2\delta_{\mu\nu}\mathbb{1}.$$

The vacuum in this model is given by $|\theta\rangle = \sum_{n=-\infty}^{\infty} \mathrm{e}^{\mathrm{i}n\theta}|n\rangle$, where $\{|n\rangle\}$ label the degenerate ground states with Pontryagin index n. Tunnelling between vacua differing by one unit of topological charge is provided by the single instanton or single anti-instanton [5]

$$W_\mu^{\mathrm{I}a} = \frac{2}{g}\frac{\eta_{a\mu\nu}(x-x_1)_\nu}{(x-x_1)^2+\rho_1^2}, \quad W_\mu^{\bar{\mathrm{I}}a} = \frac{2}{g}\frac{\bar{\eta}_{a\mu\nu}(x-x_2)_\nu}{(x-x_2)^2+\rho_2^2}. \tag{2}$$

Here $\eta_{a\mu\nu}$, $\bar{\eta}_{a\mu\nu}$ are the 't Hooft symbols [9], x_1, x_2, ρ_1, ρ_2 are the locations and sizes of the instanton and anti-instanton, respectively.

As was mentioned earlier [10] the quantum mechanical tunnelling around these field configurations in a supersymmetric model is completely suppressed due to the existence of the fermionic zero modes of the Dirac operator $(\mathrm{i}\gamma_\mu D_\mu)$ in the background of these configurations. For the instanton we have four left-handed zero modes

$$(\lambda_{0\mathrm{ss}}^{(+)})_\alpha^a = \frac{\sqrt{2}}{\pi}\frac{\rho_1^{5/2}}{\left[(x-x_1)^2+\rho_1^2\right]^2}(\sigma^a)_\alpha{}^\beta u^{(+)}{}_\beta,$$

$$(\lambda_{0\mathrm{sc}}^{(+)})_\alpha^a = \frac{1}{\pi}\frac{\rho_1^{3/2}}{\left[(x-x_1)^2+\rho_1^2\right]^2}(\sigma^a)_\alpha{}^\beta\left(\Sigma_\mu(x-x_1)_\mu\right)_{\beta\dot{\delta}}\bar{v}^{(+)\dot{\delta}}, \tag{3}$$

and for the anti-instanton four right-handed zero modes:

$$\bar{\lambda}_{0\mathrm{ss}}^{(-)a\dot{\alpha}} = \frac{\sqrt{2}}{\pi}\frac{\rho_2^{5/2}}{\left[(x-x_2)^2+\rho_2^2\right]^2}(\sigma^a)^{\dot{\alpha}}{}_{\dot{\beta}}\bar{v}^{(-)\dot{\beta}},$$

$$(\bar{\lambda}_{0sc}^{(-)})^{a\dot{\alpha}} = \frac{1}{\pi}\frac{\rho_2^{3/2}}{\left[(x-x_2)^2+\rho_2^2\right]^2}\left(\Sigma_\mu(x-x_2)_\mu\right)^{\dot{\alpha}\beta}(\sigma^a)_\beta{}^\delta u_\delta^{(-)}. \tag{4}$$

In eq. (3), (4), $u^{(\pm)}$ and $\bar{v}^{(\pm)}$ are unit vectors given by either (1, 0) or (0, 1).

In the absence of tunnelling by configurations with non-zero Pontryagin index, we are led to consider the tunnellings by the non-perturbative configurations with zero topological charge such as an instanton–anti-instanton field configuration. Without loss of generality we may take the distance between the instanton and the anti-instanton in the time-like direction $\Delta_\mu = (x_2 - x_1)_\mu = \Delta\delta_{\mu0}$. Later we will integrate over its direction. Then the instanton–anti-instanton configuration is given by

$$W_\mu^{\mathrm{I}\bar{\mathrm{I}}} = W_\mu^{\mathrm{I}}\theta(R-t) + W_\mu^{\bar{\mathrm{I}}}\theta(t-R), \tag{5}$$

where $R_\mu = \frac{1}{2}(x_1 + x_2)_\mu = R\ \delta_{\mu0}$, and generality is not lost by taking the locations to be such that $x_1 = x_2 = 0$. We later integrate over R_μ as well. Quantum fluctuations around $W_\mu^{\mathrm{I}\bar{\mathrm{I}}}$ consist of the gaussian approximation of $Q_\mu^a = W_\mu^a - W_\mu^{a\,\mathrm{II}}$. When Q_μ^a is integrated over the functional integral, we get the square root of the ratio of the fermionic over the bosonic determinants. The fermionic determinant has eight approximate fermionic zero modes which are associated with the zero modes (3), (4) of the instanton and the anti-instanton. In the limit of infinite separation they become exact zero modes. The bosonic determinant has 16 approximate zero modes associated with the invariance under translations, dilatations and group orientations of the instanton and the anti-instanton. These will be factored out and will be treated by the collective coordinate method. Factoring out also the approximate zero modes of the fermionic determinant we finally get the square root of the ratio of the non-zero modes fermionic determinant over the non-zero modes bosonic determinant in the background of an instanton–anti-instanton. In the approximation of far separation, this determinantal factor can be approximated by the product of the determinantal factors of the instanton and the anti-instanton, and each is equal to one [9] as dictated by supersymmetry [11] [‡1]. As a result we may approximate the functional integral by

$$\langle\theta'|\mathrm{e}^{-HT}|\theta\rangle_{\mathrm{II}} \approx \delta(\theta-\theta')(1/8\pi^4)^2 \int \mathrm{d}^4x_1 \frac{\mathrm{d}\rho_1}{\rho_1^5}\mathrm{d}^4x_2\frac{\mathrm{d}\rho_2}{\rho_2^5}\mathrm{d}\Omega\,\mathrm{d}\Omega_R$$
$$\times\left[8\pi^2/g^2(\rho_1)\right]^4\left[8\pi^2/g^2(\rho_2)\right]^4 K_0(x_1-x_2,\rho_1,\rho_2,\Omega_\mathrm{R})$$
$$\times\exp\left[-8\pi^2/g^2(\rho_1)-8\pi^2/g^2(\rho_2)-S_{\mathrm{int}}\right]. \tag{6}$$

In eq. (6) we have integrated over the locations x_1, x_2 the sizes ρ_1, ρ_2 and the group orientations Ω, Ω_R of the instanton and the anti-instanton respectively. Ω_R is the relative orientation of the anti-instanton compared to the instanton and $K_0(x_1-x_2,\rho_1,\rho_2,\Omega_\mathrm{R})$ is the fermionic determinant evaluated in the subspace of the fermionic zero modes listed in (3) and (4), with σ^a in eq. (4) replaced by $\sigma_\mathrm{R}^a = R_b^a\sigma^b$ and R_b^a represents the anti-instanton relative orientation:

$$K_0 = \det\begin{pmatrix} \lambda_{0\mathrm{ss}}^{(+)}\mathrm{i}\Sigma_\mu D_\mu\bar\lambda_{0\mathrm{ss}}^{(-)} & \lambda_{0\mathrm{ss}}^{(+)}\mathrm{i}\Sigma_\mu D_\mu\bar\lambda_{0\mathrm{sc}}^{(-)} \\ \lambda_{0\mathrm{sc}}^{(+)}\mathrm{i}\Sigma_\mu D_\mu\bar\lambda_{0\mathrm{ss}}^{(-)} & \lambda_{0\mathrm{sc}}^{(+)}\mathrm{i}\Sigma_\mu D_\mu\bar\lambda_{0\mathrm{sc}}^{(-)}\end{pmatrix}. \tag{7}$$

But for the configuration (5)

$$\int \mathrm{d}^4x\lambda_0^{(+)}\mathrm{i}\Sigma_\mu D_\mu\bar\lambda_0^{(-)} = \mathrm{i}\int_{-\infty}^{R}\mathrm{d}t\int\mathrm{d}^3x\;\lambda_0^{(+)}\Sigma_\mu\left(\partial_\mu+gW_\mu^{\mathrm{I}}\right)\bar\lambda_0^{(-)} + \mathrm{i}\int_R^\infty\mathrm{d}t\int\mathrm{d}^3x\;\lambda_0^{(+)}\Sigma_\mu\left(\partial_\mu+gW_\mu^{\mathrm{I}}\right)\bar\lambda_0^{(-)}$$
$$= -\mathrm{i}\int\mathrm{d}^3x\lambda_0^{(+)}(D/2,x)\bar\lambda_0^{(-)}(-D/2,x), \tag{8}$$

and we use the fact that $\lambda_0^{(+)}$, $\bar\lambda_0^{(-)}$ are the zero modes of the Dirac operator in the background of an instanton and anti-instanton respectively. Using (3), (4), (6) and (8) we get for K_0,

$$K_0 = \det\begin{pmatrix} -2\mathrm{i}a\rho_1^{5/2}\rho_2^{5/2}\sigma_a\sigma_\mathrm{R}^a & \mathrm{i}\tfrac{1}{2}\sqrt{2}\,\rho_1^{5/2}\rho_2^{3/2}a\,\Delta\sigma_a\sigma_\mathrm{R}^a \\ -\mathrm{i}\tfrac{1}{2}\sqrt{2}\,\rho_1^{3/2}\rho_2^{5/2}a\,\Delta\sigma_a\sigma_\mathrm{R}^a & \mathrm{i}\left(\tfrac{1}{4}\Delta^2a-b\right)\rho_1^{3/2}\rho_2^{3/2}\sigma_a\sigma_\mathrm{R}^a\end{pmatrix},$$

where

$$a=\frac{1}{\pi}\int_0^\infty\frac{x^2\,\mathrm{d}x}{\left(x^2+\Delta^2/4+\rho_1^2\right)^2\left(x^2+\Delta^2/4+\rho_2^2\right)^2},\quad b=\frac{1}{\pi}\int_0^\infty\frac{x^4\,\mathrm{d}x}{\left(x^2+\Delta^2/4+\rho_1^2\right)^2\left(x^2+\Delta^2/4+\rho_2^2\right)^2}. \tag{9}$$

‡1 This is different from supersymmetric quantum mechanics, where supersymmetry does not dictate the equality of the fermionic and bosonic non-zero mode determinants. This is illustrated in ref. [12].

K_0 can be easily calculated and we get

$$K_0 = 32^2(\rho_1\rho_2)^8(ab)^2 \det(\sigma_a\sigma^a_{\rm R})^2. \tag{10}$$

Denote by u_μ the unimodular four-vector which parametrizes the rotation matrix R_{ab}, then

$$R_{ab} = \delta_{ab} + 2\left[\epsilon_{abc}u_c u_4 + \left(u_a u_b - \delta_{ab}u^2\right)\right], \quad \det(\sigma_a\sigma^a_{\rm R}) = 1 + 8u_4^2.$$

Integrating over the relative orientation we finally find

$$\int {\rm d}\Omega_{\rm R}\, K_0 = 26\pi^2(32)^2(\rho_1\rho_2)^8(ab)^2. \tag{11}$$

For widely separated instanton–anti-instanton we need to keep only the leading term in Δ^{-2} in K_0. This can be seen to be as

$$\int {\rm d}\Omega_{\rm R}\, K_0 = \frac{26\pi^2 \times 64(\rho_1\rho_2)^8}{\left(\Delta^2 + \rho_1^2 + \rho_2^2\right)^8}. \tag{12}$$

We now have to substitute (12) in eq. (6) and we get

$$\langle\theta|{\rm e}^{-HT}|\theta\rangle_{\rm I\bar{I}} \approx VT\frac{64\times 2\pi^2\times 26\pi^2}{(8\pi^4)^2}\int\frac{{\rm d}\rho_1}{\rho_1^5}\frac{{\rm d}\rho^2}{\rho_2^5}{\rm d}^4\Delta$$
$$\times\left(\frac{8\pi^2}{g^2(\rho_1)}\right)^4\left(\frac{8\pi^2}{g^2(\rho_2)}\right)^4\left(\frac{\rho_1\rho_2}{\Delta^2+\rho_1^2+\rho_2^2}\right)^8\exp\left[-\frac{8\pi^2}{g^2(\rho_1)}-\frac{8\pi^2}{g^2(\rho_2)}-S_{\rm int}\right]. \tag{13}$$

Using the renormalization group equation

$$8\pi^2/g^2(\rho) = 8\pi^2/g^2(\mu) - 6\ln(\rho\mu), \tag{14}$$

we may ignore the ρ dependence in the factors $[8\pi^2/g^2(\rho)]^4$ because it is higher order in g, and we can factor out a term $\exp[-16\pi^2/g^2(\mu)]$ from the integral. The interaction action for the instanton–anti-instanton pair was calculated in ref. [13] to yield

$$S_{\rm int} = 4\ln\left(\rho_1\rho_2/\Delta^2\right), \qquad \text{for } \Delta \ll \rho_1,\ \rho_2$$
$$= \frac{32\pi^2}{g^2}\left(\frac{\rho_1\rho_2}{\Delta^2+\rho_1^2+\rho_2^2}\right)^2\left(3-4u_4^2\right), \quad \text{for } \Delta \gg \rho_1,\ \rho_2. \tag{15}$$

Evidently there is a strong suppression factor for small separation and we may consider the far separated configuration in accordance with our approximation. For large separation

$$\exp(-S_{\rm int}) \geqslant \exp\left[-\frac{96\pi^2}{g^2}\left(\frac{\rho_1\rho_2}{\Delta^2+\rho_1^2+\rho_2^2}\right)^2\right] \tag{16}$$

and we get the bound

$$\langle\theta|{\rm e}^{-HT}|\theta\rangle_{\rm I\bar{I}} \geqslant VT(52/\pi^4)\left[8\pi^2/g^2(\mu)\right]^8\exp\left[-16\pi^2/g^2(\mu)\right]$$
$$\times\int\frac{{\rm d}\rho_1}{\rho_1^5}\frac{{\rm d}\rho_2}{\rho_2^5}\frac{{\rm d}^4\Delta(\rho_1\rho_2)^8(\rho_1\mu)^6(\rho_2\mu)^6}{\left(\Delta^2+\rho_1^2+\rho_2^2\right)^8}\exp\left[-\frac{96\pi^2}{g^2(\mu)}\left(\frac{\rho_1\rho_2}{\Delta^2+\rho_1^2+\rho_2^2}\right)^2\right]. \tag{17}$$

Keeping only the leading order in g the integration over Δ yield:

$$\int \frac{d^4\Delta}{\left(\Delta^2+\rho_1^2+\rho_2^2\right)^8} \exp\left[-\frac{96\pi^2}{g^2(\mu)}\left(\frac{\rho_1\rho_2}{\Delta^2+\rho_1^2+\rho_2^2}\right)^2\right] \approx \frac{\pi^2}{2(\rho_1\rho_1)^6}\left(\frac{g^2(\mu)}{96\pi^2}\right)^3. \tag{18}$$

Here we can see the consistency of our approximation; higher orders in $1/(\Delta^2+\rho_1^2+\rho_2^2)$ contribute higher orders in g after integration, and these we consistently ignore. Also we have ignored the contribution from the lower limit of integration over Δ because it is higher order in $\exp(-96\pi^2/g^2)$, which for small g can be ignored.

The ρ_1, ρ_2 integration in eq. (17) is ultraviolet finite but diverges for large instanton and anti-instanton sizes. However we integrate over ρ_1, ρ_2 up to some average instanton and anti-instanton scale ρ_c. To this end we write $\rho_1\mu = v\cos\alpha$, $\rho_2\mu = v\sin\alpha$ and integrate over v and α to obtain

$$\langle\theta|e^{-HT}|\theta\rangle_{I\bar{I}} \geqslant VT\mu^4\tfrac{1}{8}(\rho_c\mu)^8\left[8\pi^2/g^2(\mu)\right]^5(13/12^3\pi^2)\exp\left[-16\pi^2/g^2(\mu)\right] \tag{19}$$

or

$$E(\theta)/V \leqslant -(0.94\times10^{-3}/\pi^2)\left[8\pi^2/g^2(\rho_c)\right]^5\rho_c^{-4}\exp\left[-16\pi^2/g^2(\rho_c)\right]. \tag{20}$$

We would like now to compare this bound with the induced vacuum energy found in ref. [10]. We first note that if instead of using (5) to represent the instanton–anti-instanton field configuration, we replace the covariant derivatives in (7) by ordinary derivatives we get the approximation of ref. [10]. There it was shown that

$$\int d\Omega_R K_0 = \frac{1}{8\left(\Delta^2+\rho_1^2+\rho_2^2\right)^8}\left[45\times121 - 510\ln^2\frac{\rho_2^2}{\rho_1^2} + 13\ln^4\frac{\rho_2^2}{\rho_1^2}\right.$$
$$+\left(248\ln^2\frac{\rho_2^2}{\rho_1^2} - 40\times129\right)\ln\left(\frac{\Delta^2+\rho_1^2+\rho_2^2}{\rho_1\rho_2}\right)^2 + 26\left(67-\ln^2\frac{\rho_2^2}{\rho_1^2}\right)\ln^2\left(\frac{\Delta^2+\rho_1^2+\rho_2^2}{\rho_1\rho_2}\right)^2$$
$$\left.-248\ln^3\left(\frac{\Delta^2+\rho_1^2+\rho_2^2}{\rho_1\rho_2}\right)^2 + 13\ln^4\left(\frac{\Delta^2+\rho_1^2+\rho_2^2}{\rho_1\rho_2}\right)^2\right].$$

Using the bound (16) and integrating over Δ, ρ_1 and ρ_2 as before we get the following bound:

$$E(\theta)/V \leqslant -(10^{-2}/\pi^2)\left[8\pi^2/g^2(\rho_c)\right]^5\rho_c^{-4}\exp\left[-16\pi^2/g^2(\rho_c)\right]F(g), \tag{21}$$

where

$$F(g) = \sum_{n=0}^{4}\alpha_n \ln^n\left[8\pi^2/g^2(\rho_c)\right] \tag{22}$$

and $\alpha_0 = 3.61$, $\alpha_1 = -5.76$, $\alpha_2 = 3.15$, $\alpha_3 = -0.67$, $\alpha_4 = 0.05$. The function $F(g) > 0$ for all g, however we need to consider it only in the region where our approximation is valid, $g^2/4\pi < 1$ ($\ln(8\pi^2/g^2) \gtrsim 2$), because we have consistently ignored higher order perturbative effects. In this region $F(g) \geqslant 0.14$. In fact due to the exponential factor $\exp(-16\pi^2/g^2)$, which is strongly suppressing, it is enough to consider the region $2 \leqslant \ln(8\pi^2/g^2) \leqslant 5$ ($0.14 \leqslant F(g) \leqslant 4.31$), because for smaller values of g the exponential factor is too small to give any meaningful number.

Thus the bounds (20) and (21) are consistent, and they are also consistent with the result of ref. [10] for the contribution to the vacuum energy when interactions are ignored. There it was shown that

$$E(\theta)/V = -(6\times10^{-4}/21\pi^2)\left[8\pi^2/g^2(\rho_c)\right]^8\rho_c^{-4}\exp\left[-16\pi^2/g^2(\rho_c)\right], \tag{23}$$

which agrees with the above bounds in the range of the validity of the approximation. We expect, though, that the final result should be closer to (23) than to (20) or (21) because the average in group space of the interaction action is zero.

We thus have shown that there is an induced vacuum energy by an instanton–anti-instanton field configuration even when interactions are included. It is small and may still be wiped out when other non-perturbative contributions to the path integral are included. If that does not happen, it suggests an explicit breaking of supersymmetry.

The induced vacuum energy, we found, is proportional to the inverse of the fourth power of the scale cut-off ρ_c. Such a cut-off is needed in any scale invariant theory; supersymmetric Yang–Mills theory is only one example. Indeed it was pointed out in ref. [9] that in a spontaneously broken gauge theory this cut-off is not needed, because the contribution of the scalar field to the action (in a background of an instanton or anti-instanton) is $S_H = 4\pi^2 F^2 \rho^2$, which renders the ρ integration finite. Here F is a constant associated with the vacuum expectation value of the scalar field. The same should happen in the supersymmetric extension of this theory. We, however, used the pure supersymmetric Yang–Mills theory for the sake of simplicitly. So in this theory one may take $e\rho_c \sim \Lambda_{QCD}^{-1}$, the strong interaction scale of SU(2) gauge theory.

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