

## HALF-INTEGER CHARGED HADRONS FROM HIGHER DIMENSIONS?

C. WETTERICH

*Deutsches Elektronen-Synchrotron DESY, Notkestrasse 85, D-2000 Hamburg 52, Fed. Rep. Germany*

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Certain embeddings of the low energy  $SU(3)_C \times SU(2)_L \times U(1)_Y$  gauge group within the higher dimensional symmetry lead to exotic chiral generations in addition to a number of standard generations. An example is studied, leading to an anomaly free chiral exotic generation consisting of quarks with electric charge  $+\frac{1}{6}$  and  $-\frac{2}{3}$ , the corresponding antiquarks and additional integer charged leptons. The most striking prediction from such a compactification would be the existence of stable half-integer charged hadrons with a mass of the order of the Fermi scale. Detection of such particles would be an impressive hint for the existence of more than four dimensions.

Higher dimensions are an attractive theoretical idea [1] – but possible experimental tests are disappointingly missing. This is due to the high degree of consistency of the standard low energy  $SU(3)_C \times SU(2)_L \times U(1)_Y$  theory plus the wide gap between the Fermi scale and the Planck scale. If we assume that the only particles with mass considerably smaller than the compactification scale  $M_c$  (typically of order  $10^{17}$ – $10^{18}$  GeV) are the  $SU(3)_C \times SU(2)_L \times U(1)_Y$  gauge bosons, a certain number of standard chiral fermion generations and one light Higgs doublet, the only implications of higher dimensions for these particles are predictions of the parameters of the standard model plus nonrenormalizable interactions suppressed by inverse powers of  $M_c$ . Although it is true that higher dimensions modify [2] the standard  $SU(5)$  predictions on proton decay and the weak mixing angle and generally lead to nonvanishing neutrino masses through gravitational interactions [3], these phenomena also occur in four-dimensional unified theories and can hardly be used as a characteristic test of higher dimensions. Higher dimensions lead to rare decays like  $\mu \rightarrow 3e$ , but the rate is much too small to be observable. There are in principle very interesting gravitational effects – the four-dimensional principle of equivalence is perturbed [4], different particles fall with different speed and do in general not move on geodesics. Unfortunately these effects are very likely much too small to be detected in precision measurements like the equality of gravitational and inertial mass etc. (Such effects may, however, play an important role in early cosmology [4,5].) In addition, there may be superheavy stable particles (pyrgons [6]) or topological configurations left over from the big bang. Again, monopoles or strings are not a very characteristic signature for higher dimensions. Also today's number of density for such remnants may well be below experimentally detectable levels.

We have depicted a scenario very boring for experimentalists. Unfortunately, such a scenario is consistent and it is not unlikely that it is realized. In this case the idea of higher dimensions could only be tested by possible predictions on the fermion mass matrices [7] (including neutrino mass matrices), the Higgs mass and the gauge couplings or by early cosmology. Even if we release our strong assumptions and allow for intermediate scales or low energy supersymmetry, it is not easy to find effects characteristic for higher dimensions.

In this letter we explore another possible scenario, namely that the low energy theory derived from a higher dimensional model leads to additional light fermions with exotic quantum numbers. Large masses for these exotics are forbidden by chirality with respect to  $SU(3)_C \times SU(2)_L \times U(1)_Y$ . The restrictions from anomaly cancellation for chiral fermions are automatically fulfilled if we start with anomaly free higher dimensional models. We find that certain embeddings of  $SU(3)_C \times SU(2)_L \times U(1)_Y$  within the higher dimensional symmetry group lead to one or several exotic generations in addition to a certain number of standard generations. In our example, an exotic

generation consists of a weak doublet of antiquarks (G, H) with electric charges  $-\frac{1}{6}$  and  $+\frac{5}{6}$  and two quarks  $G^c$  and  $H^c$  in  $SU(2)_L$  singlets with charge  $+\frac{1}{6}$  and  $-\frac{5}{6}$ . As usual, quarks and antiquarks are colour triplets and antitriplets, respectively. In addition, an exotic generation comprises two standard leptonic doublets and two particles with the quantum numbers of the positron. (We only count here the left-handed particles.) There are also examples where the exotic generation is replaced by an exotic mirror generation with quarks in doublets and antiquarks in singlets of  $SU(2)_L$ . In this case one needs at least five standard charge  $\frac{2}{3}$  and  $-\frac{1}{3}$  quarks since two charged leptons from the standard generations can form  $SU(3)_C \times SU(2)_L \times U(1)_Y$  invariant mass terms with the mirror leptons of the exotic mirror generation and disappear from the low energy spectrum. The embedding of our example is possible in a six-dimensional  $SO(12)$  model [8] or in a ten-dimensional  $E_8 \times E_8$  model [9] favoured by superstrings [10].

Before discussing some details of our embedding and dimensional reduction, let us first focus on the most striking predictions of such a scenario. Since the quarks have charge  $+\frac{1}{6}$  and  $-\frac{5}{6}$  they cannot decay into any number of particles with charges  $\frac{1}{3}n_i$  ( $n_i$  integer). The lightest one of these quarks is stable. Because of confinement, we will of course rather observe a stable hadron. All baryons containing one or three exotic quarks are half-integer charged, whereas baryons with two exotic and one standard quark are integer charged. Similarly, mesons with one exotic quark and one standard quark have half-integer charge and mesons with two exotics are integer charged. The appearance of only half-integer charged or integer charged hadrons is a generalization of triality. The lowest mass half-integer charged baryon or meson is absolutely stable. Its mass is typically of the order of the W-boson mass. Pairs of exotic half-integer charged hadrons would be copiously produced through strong or electromagnetic interactions at accelerators with sufficient energy. Stability and half-integer charge would give a very clear signal.

One may ask if the stable exotic hadrons could be detected in ordinary matter. This largely depends on their density in terrestrial material. The low energy renormalizable interactions imply a separate conserved exotic baryon number. There are, however, nonrenormalizable interactions violating exotic baryon number conservation like  $u + d \leftrightarrow G^c + G^c$ . They are mediated by superheavy particles with mass of order  $M_c$  and become, therefore, quickly suppressed once the temperature of the universe falls sufficiently below  $M_c$ . Decay of those superheavy particles in the very early universe can create an exotic baryon number asymmetry which ensures that a certain amount of exotic hadrons survives the big bang. The total hadronic matter density in the universe puts upper bounds on the exotic asymmetry, but it is well conceivable that the exotic baryon asymmetry is much smaller than the asymmetry for standard baryons. (For example, there is no  $CP$  violating mixing matrix for only one exotic generation.) To derive bounds for terrestrial material requires an investigation if exotic hadrons are more or less distributed like ordinary baryons or if they tend to concentrate in the center of galaxies, for example. It may also be worthwhile to think about the possible presence of exotic hadrons in cosmic rays.

Another "signature" for an exotic generation is the existence of two additional left-handed neutrinos, heavy electrons and positrons which is required by anomaly cancellation. For three standard generations this would lead to five neutrinos. If all five neutrinos are light – this is the case if all antineutrinos of the model can acquire superheavy masses – the upper bound from nucleosynthesis would be violated. An independent determination of the number of neutrinos by laboratory experiments is certainly needed. If the cosmological bound persists, the case of an exotic mirror generation would be preferred compared to the exotic generation discussed above. Our arguments apply to this case as well with only minor modifications.

Let us now describe in more detail our assumptions and the embedding leading to exotic generations. We take a conservative approach and assume that the low energy gauge group is  $SU(3)_C \times SU(2)_L \times U(1)_Y$ . All low mass fermions must be chiral with respect to this group and anomalies should cancel. There is only one Higgs doublet with small mass. With these assumptions the standard chain of unifications in four dimensions  $E_6 \rightarrow SO(10) \rightarrow SU(5) \rightarrow SU(3)_C \times SU(2)_L \times U(1)_Y$  cannot lead to chiral exotics.

The situation changes in more than four dimensions. Groups with only real or pseudoreal representations like  $E_8$  or  $SO(12)$  become viable for  $d = 2 \pmod 4$  dimensions [11]. Consider the case that the higher dimensional gauge group contains a subgroup  $SO(8) \times SU(2)_L \times SU(2)_R$  with some spinors in the representations  $(8_1, 2, 1) + (8_2, 1, 2)$  or  $(8_1, 1, 2) + (8_2, 2, 1)$  where  $8_1$  and  $8_2$  are the inequivalent spinor representations of  $SO(8)$ . Within  $SO(12)$  this

is the decomposition of the two spinor representations  $32_1$  and  $32_2$ . It is a simple exercise to check that these representations appear for  $E_8$  using the chain  $E_8 \rightarrow SO(16) \rightarrow SO(12) \times SO(4)$ . In addition there may be other interesting embeddings for a large group like  $E_8$ . There are two different ways to embed the Pati–Salam group [12]  $SU(4)_C \times SU(2)_L \times SU(2)_R$  into  $SO(8) \times SO(4)$ : For the standard embedding  $SO(8) \rightarrow SO(6) \times SO(2)$  the vector decomposes as follows:

$$8_v \rightarrow 6 + 1 + 1, \tag{1}$$

and the spinors transform under  $SU(4)_C \times SU(2)_L \times SU(2)_R$  as

$$(8_1, 2, 1) \rightarrow (4, 2, 1) + (\bar{4}, 2, 1), \quad (8_2, 1, 2) \rightarrow (4, 1, 2) + (\bar{4}, 1, 2). \tag{2}$$

All particles contained in these representations have quantum numbers of standard quarks and leptons or mirror particles. An inequivalent embedding of  $SU(4)_C \times U(1)$  in  $SO(8)$  (parallel to the  $SU(5)$  embedding in  $SO(10)$ ) gives

$$8_v \rightarrow 4_{-1} + \bar{4}_1, \quad 8_1 \rightarrow 4_1 + \bar{4}_{-1}, \quad 8_2 \rightarrow 6_0 + 1_2 + 1_{-2} \tag{3}$$

(we have also indicated the  $U(1)$  charge and we note that the embeddings (1), (2) and (3) are related by  $SO(8)$  triality). Spinors in the  $8_2$  of  $SO(8)$  will lead to exotic quarks. The 6plet of  $SU(4)$  gives the colour representations

$$6 \rightarrow 3 + \bar{3}, \tag{4}$$

but the hypercharge of  $SU(2)_L$  doublets is twice the hypercharge of standard quark doublets. The representation  $8_1$  leads to standard quarks so that exotic quarks and standard quarks can coexist.

The fact that a real representation as the 6 of  $SU(4)$  can induce *chiral* exotic quarks in four dimensions is an intrinsic higher dimensional effect. In the process of dimensional reduction, the chirality index [13] may be different for the 3 and  $\bar{3}$  colour states in the 6. Unwanted representations can simply be absent due to a vanishing index and the possibilities of unification with a complex spectrum are much wider than in four dimensions. We emphasize that the systematic search for possible embeddings of  $SU(3)_C \times SU(2)_L \times U(1)_Y$  into the higher dimensional symmetry group has to be redone, since the criteria for viable embeddings differ completely from four-dimensional unification. We will make the following requirements for candidates of viable (anomaly free) chiral theories:

- (1) All known fermions have to be reproduced.
- (2) All charged chiral fermions must get a mass from  $SU(2)_L \times U(1)_Y$  breaking by an  $SU(2)_L$  doublet scalar.
- (2) The weak mixing angle at the unification scale should be

$$\sin^2 \vartheta_w^0 = \frac{3}{8}. \tag{5}$$

For the purpose of this letter we will in addition assume that corrections to gauge couplings from nonrenormalizable terms of the type discussed in ref. [2] are small, so that  $\sin^2 \vartheta_w^0$  can be calculated from the embedding of  $SU(3)_C \times SU(2)_L \times U(1)_Y$  into the unification group.

As an example for such a systematic analysis we discuss monopole solutions [14] in the six-dimensional  $SO(12)$  model [8]. The embedding problem into groups like  $E_8$  will be of similar nature, but more involved. The monopole solutions are simple enough to serve as a nice theoretical laboratory to generate different four-dimensional chiral models. Our analysis proceeds in nine steps:

- (1) One determines the symmetries left unbroken by the monopole solutions (compare ref. [8]).
- (2) We look for all possible embeddings of  $SU(3)_C$  and  $SU(2)_L$  into these symmetries. We find from our first requirement that this embedding must be the same for all cases. We write the monopole field as



Table 1  
Quantum numbers of fermions.

$SU(3)_C \times SU(2)_L$ representation	$H_1 + H_2$	$H_3 + H_4 + H_5$	$H_6$	$Y_{B-L}$	$q$	$Q$	name	$SU(2)_G$ representation
$32_1 (3, 2)$	0	1/2	1/2	1/3	1	2/3, -1/3	u, d	$(n + p)/2$
$(1, 2)$	0	-3/2	1/2	1	-1	0, 1	$\bar{u}, \bar{e}$	$(n - 3p)/2$
$(\bar{3}, 1)$	1	-1/2	1/2	2/3	0	-1/6	$\bar{G}^c$	$(n - p + 2m)/2$
$(3, 1)$	-1	-1/2	1/2	2/3	0	5/6	$\bar{H}^c$	$(n - p - 2m)/2$
$(1, 1)$	1	3/2	1/2	0	2	-1/2	$J^c$	$(n + 3p + 2m)/2$
$(1, 1)$	-1	3/2	1/2	0	2	1/2	$K^c$	$(n + 3p - 2m)/2$
$(\bar{3}, 2)$	0	-1/2	-1/2	-1/3	-1	-2/3, 1/3	$\bar{u}, \bar{d}$	$-(n + p)/2$
$(1, 2)$	0	3/2	-1/2	-1	1	0, -1	$\nu, e$	$-(n - 3p)/2$
$(3, 1)$	-1	1/2	-1/2	-2/3	0	1/6	$G^c$	$-(n - p + 2m)/2$
$(3, 1)$	1	1/2	-1/2	-2/3	0	-5/6	$H^c$	$-(n - p - 2m)/2$
$(1, 1)$	-1	-3/2	-1/2	0	-2	1/2	$\bar{J}^c$	$-(n + 3p + 2m)/2$
$(1, 1)$	1	-3/2	-1/2	0	-2	-1/2	$\bar{K}^c$	$-(n + 3p - 2m)/2$
$32_2 (3, 2)$	0	1/2	-1/2	-2/3	0	1/6, -5/6	$\bar{G}, \bar{H}$	$(n - p)/2$
$(1, 2)$	0	-3/2	-1/2	0	-2	1/2, -1/2	J, K	$(n + 3p)/2$
$(\bar{3}, 1)$	1	-1/2	-1/2	-1/3	-1	-2/3	$u^c$	$(n + p - 2m)/2$
$(\bar{3}, 1)$	-1	-1/2	-1/2	-1/3	-1	1/3	$d^c$	$(n + p + 2m)/2$
$(1, 1)$	1	3/2	-1/2	-1	1	-1	$\bar{e}_c$	$(n - 3p - 2m)/2$
$(1, 1)$	-1	3/2	-1/2	-1	1	0	$\bar{\nu}^c$	$(n - 3p + 2m)/2$
$(\bar{3}, 2)$	0	-1/2	1/2	2/3	0	-1/6, 5/6	G, H	$(n - p)/2$
$(1, 2)$	0	3/2	1/2	0	2	-1/2, 1/2	$\bar{J}, \bar{K}$	$-(n + 3p)/2$
$(3, 1)$	-1	1/2	1/2	1/3	1	2/3	$\bar{u}^c$	$-(n + p - 2m)/2$
$(3, 1)$	1	1/2	1/2	1/3	1	-1/3	$\bar{d}^c$	$-(n + p + 2m)/2$
$(1, 1)$	-1	-3/2	1/2	1	-1	1	$e^c$	$-(n - 3p - 2m)/2$
$(1, 1)$	1	-3/2	1/2	1	-1	0	$\nu^c$	$-(n - 3p + 2m)/2$

for the corresponding representations. If negative integers or zeros appear, the corresponding representation is not present in the massless four-dimensional spectrum. In table 2 we list the number of chiral particles for three examples.

(6) After spontaneous symmetry breaking  $SU(2)_L \times U(1)_Y \rightarrow U(1)_{em}$  all particles must be in vectorlike representations of  $SU(3)_C \times U(1)_{em}$ . Otherwise they must remain massless forever [7,8]. Looking at example (c) one finds four charge  $\frac{1}{3}$  antiquarks and only three charge  $-\frac{1}{3}$  quarks. In fact, for all  $m \neq 0$  the fermion representation is chiral with respect to  $SU(3)_C \times U(1)_{em}$  and we have to eliminate this case. This embedding requires  $m = 0$  so that the monopole solutions are  $SU(2)_L \times SU(2)_R$  symmetric.

(7) The symmetry group of the monopole solutions is larger than  $SU(3)_C \times SU(2)_L \times U(1)_Y$ . Further spontaneous symmetry breaking to the low energy gauge group will induce  $SU(3)_C \times SU(2)_L \times U(1)_Y$  invariant mass terms like  $(J^c K^c)$  and  $(JK)$ . We see in example (b) that only three standard generations remain at low energies.

Table 2  
Examples for exotic particles.

Example	$n$	$p$	$m$	u, d	$\nu, e$	$u^c$	$d^c$	$\nu^c$	$e^c$	G, H	$G^c$	$H^c$	$\bar{G}^c$	J, K	$J^c$	$K^c$
(a)	2	4	0	3	5	3	3	5	5	1	1	1	-	7	7	7
(b)	3	3	0	3	3	3	3	3	3	-	-	-	-	6	6	6
(c)	3	3	1	3	3	2	4	2	4	-	-	1	1	6	7	5

The alternative embedding does not necessarily lead to exotics. In this case all half-integer charged particles are superheavy. The lightest one of those is stable – an example for pyrgons [6]. Due to  $SU(2)_L$ -symmetry, the mass term (JK) is antisymmetric in generation indices and can only give masses to an even number of pairs (J, K). In example (a) one pair J, K can only acquire mass from  $SU(2)_L$  breaking. This is an example of a modulo-two chirality index [13]. Such a modulo-two chiral, half-integer charged leptonic doublet may cause a problem: If there is no light  $J^c K^c$ , it can only acquire mass from coupling to an  $SU(2)_L$  triplet operator. Its mass would be of order of typical neutrino masses [15]. We will not consider this possibility further.

(8) The most striking feature of the embedding (B) is the appearance of an exotic chiral generation in example (b). Such a generation consists of a doublet of antiquarks (G, H), two quarks  $G^c$  and  $H^c$ , two doublets ( $\nu$ , e) and two  $e^c$ . The appearance of whole generations with exotic charges besides standard generations is a generic feature of this embedding for  $n \neq p$ . (The number of standard generations is  $(n+p)/2$  and for exotic generations it is  $(p-n)/2$ . For  $n > p$  we have instead the corresponding number of mirror exotic generations.) It is easily checked that an exotic generation is free of all  $SU(3)_C \times SU(2)_L \times U(1)_Y$  anomalies and mixed gauge and gravitational anomalies. An exotic generation has one weak doublet more than a standard generation and the ratio  $\Sigma I_{3L}^2 / \Sigma Q^2$  is  $\frac{3}{10}$  instead of  $\frac{3}{8}$ . Exotic generations induce therefore small deviations from the standard renormalization group equations even at the one-loop level, a feature not common in four-dimensional unification.

(9) Finally we have to check that all chiral exotic quarks can acquire mass from  $SU(2)_L \times U(1)_Y$  breaking through Yukawa couplings to Higgs doublets. As for the standard embedding, this is indeed the case if scalar fields are added to the six-dimensional action [7,8].

In conclusion we find that certain embeddings of the low energy group  $SU(3)_C \times SU(2)_L \times U(1)_Y$  within the higher dimensional symmetry group lead to exotic chiral generations predicting half-integer charged stable hadrons with mass at the Fermi scale. These embeddings are not possible within the framework of four-dimensional unification. They involve real representations at some step, so that chirality requires  $(4n+2)$  dimensions. Detection of low mass half-integer charged particles would be a strong hint for higher dimensional unification. Nevertheless, the existence of exotic generations is consistent within the low energy  $SU(3)_C \times SU(2)_L \times U(1)_Y$  framework alone <sup>#1</sup>. A systematic search for such particles should be done.

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<sup>#1</sup> For a discussion of exotic particles with integer charged hadrons see ref. [16].

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