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The Final States $l^{\mp} K^{\pm} K^{\pm} X$ in Jets as Signatures of $B_s^0 - \bar{B}_s^0$ Mixings

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Abstract. Significant mixing is expected between the neutral bottom mesons $B_s^0 - \bar{B}_s^0$ in the standard model of weak interactions. We propose measurements of the processes $\left\{ \begin{matrix} e^+ e^- \\ p\bar{p} \end{matrix} \right\} \to \overset{b\bar{b}}{\to} l^+ K^- K^- X$ +c.c. as a measure of such mixing. Rates are presented for energetic bottom quark jets, produced in $e^+ e^-$ annihilation.

1. Introduction

A non-trivial test of the standard model of electroweak interactions lies in measuring the strength of the flavour changing $|\Delta F| = 2$, $\Delta Q = 0$ transitions among the neutral meson systems $K^0 - \bar{K}^0$, $D^0 - \bar{D}^0$, $B^0 - \bar{B}^0$ and $B_s^0 - \bar{B}_s^0$. Experimentally, such transitions have so far been observed in the $K^0 - \bar{K}^0$ sector only, though there exists some preliminary evidence from the CERN $\bar{p}p$ data that such mixings may also be present among the neutral bottom mesons [1].

The presence of the $|\Delta B| = 2$, $\Delta Q = 0$ transition involving the neutral bottom mesons has a number of interesting phenomenological consequences, two of which have received experimental attention. The first concerns the production of energetic same-sign dileptons in the processes $e^+ e^- \to b\bar{b} \to l^{\pm} l^{\pm} X$ [2, 3] and $\bar{p}p \to b\bar{b} \to l^{\pm} l^{\pm} X$ [1]. The second method involves the measurement of the electroweak charge asymmetry in the process $e^+ e^- \to b\bar{b} \to l^{\pm} X$ [4], since the $|\Delta B| = 2$ interactions lead to the "wrong-sign" lepton and hence tend to decrease the electroweak asymmetry. However, none of these measurements by themselves would distinguish between the mixings in the $B_d^0 - \bar{B}_d^0$ and $B_s^0 - \bar{B}_s^0$ mesons, since either of the two transitions would lead to the $l^{\pm} l^{\pm} X$ final states and the reduction in the rate for charge asymmetry in $e^+ e^- \to b\bar{b}$.

In the standard model one expects significant $B_s^0 - \bar{B}_s^0$ mixing and negligible $B_d^0 - \bar{B}_d^0$ mixing [5]. In a number of non-standard scenarios, however, there exist additional effective interactions which may enhance the $B_d - \bar{B}_d$ mixing amplitude. For example, in the supersymmetric extension of the standard model additional mechanisms which enhance the $B_d - \bar{B}_d$ mixings have been recently emphasized [6]. It is therefore imperative to be able to distinguish experimentally between the $B_d - \bar{B}_d$ and $B_s - \bar{B}_s$ mixings. In principle, some of these tests have already been proposed [7] which consist of measuring the final states $l^+ l^+ \Lambda^0$, $l^- l^- \bar{\Lambda}^0$ and $l^{\pm} l^{\pm} F^{\mp}$ in the fragmentation products of bottom quark jets initiated by the hard collisions $(e^+ e^-, p\bar{p}) \to b\bar{b}X$. Another test, which suggests itself naturally lies in scanning the region at and above the $4S$ resonance in the final state $e^+ e^- \to l^{\pm} l^{\pm} X$. In practice, the former tests would require a very high bottom hadron statistics, which may or may not be available in the near future. Theoretical interpretation of the signal in $e^+ e^- \to l^{\pm} l^{\pm} X$ in the region $\Upsilon(nS)$, $n > 4$, is a potential-model dependent enterprise, since the branching ratios for $\Upsilon(nS) \to B_s \bar{B}_s$, $B_s \bar{B}_s^*$, $B_d \bar{B}_d$ etc. are needed, which can at best be modelled. In any case there is no signal yet from the CESR/DORIS data of either $B_d - \bar{B}_d$ or $B_s - \bar{B}_s$ mixings. Model dependent calculations [8] estimate that a signal is expected at $\Upsilon(5S)$ *but* would require an integrated luminosity of 1,000 pb^{-1}. There is no evidence of $B - \bar{B}$ mixing from the PETRA/PEP data either [3, 4], nor it is expected with the present luminosity [9].

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The aim of this paper is to devise a new test of $B-\bar{B}$ mixing which i) is experimentally measurable, ii) is less dependent on the production cross-section $\sigma(B_s^0 X)$ and the semileptonic branching ratio $\mathrm{BR}(B_s^0 \to l^\pm X)$ of the B_s^0 meson and iii) could distinguish between mixings in the $B_d^0-\bar{B}_d^0$ and $B_s^0-\bar{B}_s^0$ mesons.

2. Proposal

We propose measurements of the final states $l^\pm K^\mp K^\mp X$ in the bottom quark jets produced, for example, in the processes

$$\left\{\begin{array}{c} e^+e^- \\ \bar{p}p \end{array}\right\} \to b\bar{b} \hookrightarrow l^+K^-K^-X + \text{c.c.} \tag{1}$$

as a measure of weak mass mixing in the $B_s-\bar{B}_s^0$ mesons. The test implicitly assumes that the decay product of b and $\bar{b}$ do not contaminate each other, in other words that the b and $\bar{b}$ hadrons are well separated spatially; a criterion well satisfied by jets at PEP and PETRA and in $p\bar{p}$ collisions.

The main idea behind the "decay" of an excited b-quark, $b \to l^+K^-K^-X$, as a measure of $B_s-\bar{B}_s$ mixing is rather simple. The decay of a b-quark produced in a hard collision follows a certain fragmentation weak decay chain, the general features of which are now well known experimentally [10]. Thus, concentrating on the semileptonic decays of the bottom hadrons and the final states containing charged kaons, $K^\pm$, one schematically has

$$\begin{array}{l} b \to b\begin{pmatrix}\bar{u}\\ \bar{d}\end{pmatrix} + \begin{pmatrix}u\\ d\end{pmatrix} \\ \qquad\qquad\quad \hookrightarrow K^+X, \ldots \\ \quad \hookrightarrow (D^0, D^+, D^{*0}, D^{*+}, \ldots)\, l^-\nu_l \\ \qquad\qquad\qquad\quad \hookrightarrow K^-X, \ldots \end{array} \tag{2}$$

$$\begin{array}{l} b \to (b\bar{s}) + s \\ \qquad\quad \hookrightarrow K^-X, \ldots \\ \quad \hookrightarrow (F^+, F^{*+}, \ldots)\, l^-\nu_l \to K^\pm X, \ldots \end{array} \tag{3}$$

where only the Cabibbo-allowed decays of the D^0, D^+ and F^+ mesons are shown. Since $D^0, D^+ \to K^+X$ decays are Cabibbo forbidden but $F^+ \to K^\pm X$ are Cabibbo allowed, a good two-particle inclusive final state to enhance the $B_s^0(\bar{B}_s^0)$ signal should be in the decays $B \to l^-K^+X$, $\bar{B} \to l^+K^-X$. Next, we remark that all experimental measurements of particle production in jets give a suppression in the production of $s\bar{s}$ pair from the vacuum [11]. In fact the world data is in agreement with the probability, $f(s\bar{s}) = 0.1-0.15$ in quark jets. Hence it makes sense to use $f(s\bar{s})$ as an order parameter. To leading order in $f(s\bar{s})$ one would have the fragmentation $b \to (B_d^0, B_u^-)K^\pm X$ and $\bar{b} \to \bar{B}_s^0 K^- X$. Thus, in the approximation of keeping $\sin^2\vartheta_c = 0$ and leading $K^\pm$ mesons from the associated jet accompanying the bottom hadron, the "decay" of the bottom quark jets

$$(b \text{ or } \bar{b}) \to l^\mp K^\pm K^\pm X \tag{4}$$

is forbidden! The situation is quite different in the presence of $B_s^0-\bar{B}_s^0$ mixing. In that case, to leading order in $f(s\bar{s})$ one would have $b \to B_s^0 K^- X$, $\bar{B}_s^0 K^- X$ in the ratio $(1-\chi_s)$ and χ_s where χ_s is the probability of the transition $B_s^0 \to \bar{B}_s^0$. This would lead to the decays

$$\begin{array}{l} b \to (b\bar{s}) \quad + \quad s \\ \quad\updownarrow \qquad\qquad\quad \hookrightarrow K^-X, \ldots \\ (\bar{b}s) \to F^- l^+ \nu_l, \ldots \\ \qquad\quad \hookrightarrow K^\pm X, \ldots \end{array} \tag{5}$$

giving rise to the final states $b \to l^+K^-K^-X$ and $\bar{b} \to l^-K^+K^+X$ which are both Cabibbo allowed and favoured by the fragmentation process $s \to K^-X$. It is easy to see that in the same approximation $B_d^0-\bar{B}_d^0$ mixing would lead to the final states (b or $\bar{b}$)$\to l^\pm K^\pm K^\pm X$. Thus, in the limit stated above observation of (b or $\bar{b}$)$\to l^\pm K^\mp K^\mp X$ in a bottom quark jet would signal $B_s^0-\bar{B}_s^0$ mixings.

To quantify our proposal, we define the following ratio

$$R(l^+K^-K^-/l^-K^-K^-) \equiv \frac{b \to l^+K^-K^-X}{b \to l^-K^-K^-X}. \tag{6}$$

Then, quite generally for collisions leading to $b\bar{b}$ production one can express the ratio R as

$$R(l^+K^-K^-/l^-K^-K^-) = \frac{\sum_q \sigma(B_q)\{\mathrm{BR}(B_q \to l^+K^-X)\, f(q \to K^-X) + \mathrm{BR}(B_q \to l^+X)\, f(q \to K^-K^-X)\}}{\sum_q \sigma(B_q)\{\mathrm{BR}(B_q \to l^-K^-X)\, f(q \to K^-X) + \mathrm{BR}(B_q \to l^-X)\, f(q \to K^-K^-X)\}} \tag{7}$$

with $\sigma(Bq)$ being the inclusive production cross-section for B_q; the sum $\sum_q$ is over all species of the bottom hadrons Bq and $f(q \to K^+X)$ etc. represents the probability of a quark giving a K^+. In principle, one could additionally have the decays $B_q \to l^+K^-K^-X$, due to, for example,

$\bar{B}_s \to l^+ F^- X$; $F^- \to K^- K^+ K^-$, but being suppressed by phase space and $\sin^2\vartheta_c$, they must have negligible branching ratios ($\lesssim 10^{-3}$).

Now, in the limit $\sin^2\vartheta_c = 0$ and leading order in $f(s\bar{s})$ (keeping only the valence $K^{\pm}$ hadron from the fragmentation process $q \to K^{\pm} X$), we have

$$R(l^+K^-K^-/l^-K^-K^-)$$

$$\approx \frac{\sigma(B_s^0)\,\mathrm{BR}(B_s^0 \to l^+K^-X)\,f(s\to K^-X)}{\sigma(B_s^0)\,\mathrm{BR}(B_s^0 \to l^-K^-X)\,f(s\to K^-X)} \tag{8}$$

$$= \frac{\mathrm{BR}(B_s^0 \to l^+K^-X)}{\mathrm{BR}(B_s^0 \to l^-K^-X)} = \frac{\chi_s}{1-\chi_s}\,\frac{\mathrm{BR}(F^- \to K^-X)}{\mathrm{BR}(F^+ \to K^-X)} \tag{9}$$

where $\chi_s \equiv \Gamma(B_s^0 \to l^+ \nu_l X)/\Gamma(B_s^0 \to l^{\pm} \nu_l X)$ is the measure of $B_s^0 - \bar{B}_s^0$ mixing. Thus, in the limit that (8) holds, the quantity $R(l^+K^-K^-/l^-K^-K^-)$ is independent of the B_s^0-production cross-section $\sigma(B_s^0 X)$, the probability $f(s \to K^- X)$ and the semileptonic branching ratio for the B_s^0 meson. It depends, however, on the inclusive branching ratio of the $F^{\pm}$ mesons, namely $F^- \to K^{\pm} X$. The quantity $\mathrm{BR}(F^- \to K^- X)/\mathrm{BR}(F^+ \to K^- X)$ in (9) is expected to be around 1 in the quark-parton model description of the $F^{\pm}$ decays, and hopefully should be available soon experimentally.

In the real world, Cabibbo suppressed transitions in the charmed hadron decays $D^0, D^+ \to K^+ X$ and the suppressed fragmentation processes $u, d \to K^- X$ and $s \to K^+ X$ would contribute to the final state $b \to l^- K^+ K^+ X$, thus giving non-zero contribution to the ratio $R(l^-K^+K^+/l^-K^-K^-)$. In fact, as the virtuality of the b-quark jet increases, there would be an increase in the $K^{\pm}$ multiplicity in the jet, and the background to $b \to l^+ K^- K^- X$ would become rather formidable. This is certainly not the case at PETRA and PEP.

In the rest of this paper, we present a calculation to estimate the signal and background for the final state $b \to l^+ K^- K^- X$ for realistic experimental conditions in e^+e^- annihilation at PETRA/PEP and LEP/SLC energies.

3. Rate Estimates

It is obvious from (7) that we need three quantities to estimate $R(l^+K^-K^-/l^-K^-K^-)$ i) the bottom hadron cross-section $\sigma(B_q X)$, ii) the semileptonic branching ratios of the bottom hadrons $\mathrm{BR}(B_q \to l^{\mp} K^{\pm} X)$ and iii) the fragmentation probabilities $f(q \to K^{\pm} X)$. We use the QCD improved quark-parton model to estimate $\sigma(e^+e^- \to b\bar{b})$ and adapt the fragmentation model for the quark jets, originally due to Field and Feynman [12] and adjusted to describe the general profile of heavy quark jets measured in e^+e^- experiments at PETRA and PEP. The most important feature of this modification for our purpose is in the longitudinal momentum distribution of the bottom hadrons, which have distinctly harder momenta as compared to the leading particles in the light quark or gluon jets [10, 11]. We use the parametrization of Petersen et al. [13] for the bottom hadron

$$f(Z_B) = \frac{Z_B(1-Z_B)^2}{[Z_B(1-Z_B)-(1-Z_B)-\varepsilon_B Z_B]^2} \tag{9}$$

where $Z_B = \frac{E_B}{E_b} = \frac{2E_B}{\sqrt{s}}$ and $\varepsilon_B = 0.02$, compatible with the measurements in e^+e^- experiments. The momentum distribution of the K mesons in the process

$$e^+e^- \to b\bar{b},\quad b \to (b\bar{q}) + q,\quad q \to K^{\pm} X \tag{10}$$

is then considerably soft, as can be seen in Figs. 1a, b, where we have plotted the fragmentation functions $D(Z = 2E_K/\sqrt{s})$ for $q \to K^{\pm} X$, $q = u, d, s$ with the normalization $\int_0^1 D(Z)\,dz = f(q \to K^{\pm} X)$. Figure 1a shows the $D(Z)$ distributions for a typical PETRA energy $\sqrt{s} = 43$ GeV and should be representative of the high energy data from PETRA and PEP collected at $\sqrt{s} = 29-46$ GeV. Figure 1b shows the same distributions at $\sqrt{s} = m_Z \simeq 93$ GeV, relevant for the LEP and SLC experiments. Since the general features of the quark jets observed at PETRA and PEP are remarkably well described by the fragmentation models [14] of the type we are discussing, we expect that the distribution functions $D(Z)$ shown in Figs. 1a, b should be rather close to the experimental distributions, though we are aware that an analysis of the bottom quark jets as depicted in Figs. 1a, b has not yet been undertaken.

The flavour distributions $D(Z)$ shown in Figs. 1a, b are quite instructive. They give a concrete meaning to the concept of allowed (valence) and suppressed (sea) $q \to K^{\pm} X$ fragmentation transitions. For example, for an s-quark accompanying a B_s^0 meson in a jet, the ratio $D^{s\to K^-/K^+}(Z) = \frac{D^{s\to K^-X}(Z)}{D^{s\to K^+X}(Z)}$ has the value: $D^{s\to K^-/K^+}(Z) \geq 8$ for $Z \geq 0.1$ and this ratio becomes >20 for $Z > 0.3$. Thus, the ability to detect energetic $K^{\pm}$ mesons is at a premium! This is all that one needs to show that $\sigma(B_s^0 K^+ X)/\sigma(B_s^0 K^- X) \ll 1$ for realistic fragmentation functions in the absence of $B_s - \bar{B}_s^0$ mixing.

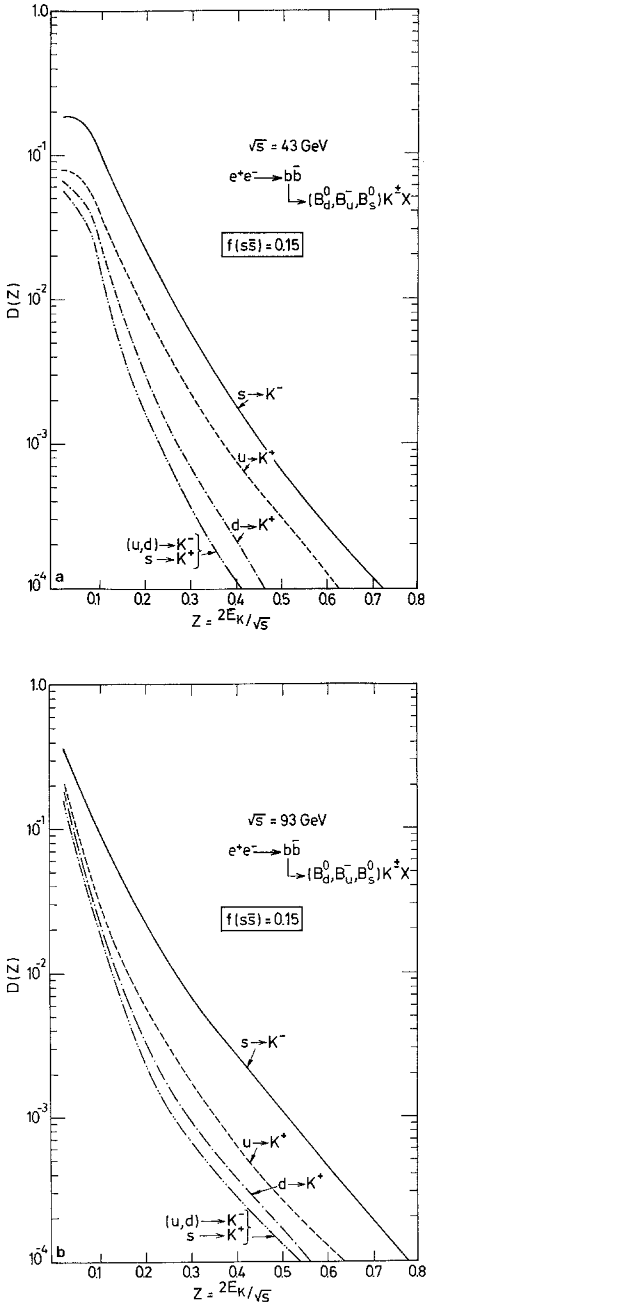

Fig. 1. **a** Fragmentation functions $D(Z)$ for $q \to K^{\pm} X$ produced in association with a bottom meson in the process $e^+ e^- \to b\bar{b}$ at $\sqrt{s} = 43$ GeV, with $f(s\bar{s}) = 0.15$. **b** Same as **a** but at $\sqrt{s} = m_Z \simeq 93$ GeV

Our next task is to calculate the inclusive bottom hadron cross-section $\sigma(B_q X)$ in continuum $e^+ e^-$ and $\bar{p}p$ collisions. We argue that it is very plausible to assume that given the inclusive bottom quark cross-section, $\sigma(bX)$, the inclusive cross-section for producing a bottom hadron $\sigma(B_q X)$ is determined essentially by the probability of producing a quark-antiquark or diquark pair from the vacuum $f(q\bar{q}), f(qq)$. These probabilities have been well measured in the light-quark fragmentation processes [11]. We argue that the functions $f(q\bar{q}), f(qq)$ are approximately independent of the flavour of the decaying quark. This assumption is implicit in most fragmentation models in vogue [14] and appears rather plausible. However, it has not yet been directly tested. We remark that measurements of the cross-section ratio $\sigma(F^{\pm} X)/\sigma(D^{0, \pm} X)$ would be very welcome to determine $f(s\bar{s})$ for the charm quark jets. With this assumption we have

$$\begin{aligned} \sigma(B_q X) &= \sigma(bX) f(q\bar{q}) \\ \sigma(\Lambda_B X) &= \sigma(bX) f(qq) \end{aligned} \tag{11}$$

where $\sigma(bX)$ is obtained from perturbative QCD. For example,

$$\frac{\sigma(e^+ e^- \to bX)}{\sigma(e^+ e^- \to \mu^+ \mu^-)} = \frac{1}{3}\left(1 + \frac{\alpha_s(Q^2)}{\pi}\right).$$

It is well known that exact $\vartheta(\alpha_s^2)$ contributions [15], as well as all order contributions in leading log [16] to $\sigma(e^+ e^- \to bX)$ are entirely negligible upto and including LEP energies. This means that the multiplicity of the bottom hadron in a jet is at most 1 [16].

In Fig. 2a we show the differential cross-section $\frac{1}{\sigma(b\bar{b})} \frac{d\sigma}{dZ}(e^+ e^- \to BKX)$ for the final states $e^+ e^- \to B_u K^{\pm} X$, $B_d^0 K^{\pm} X$ and $B_s^0 K^{\pm} X$ for $\sqrt{s} = 43$ GeV, assuming $f(s\bar{s}) = 0.15$. Similar cross-sections are shown for $\sqrt{s} = m_Z = 93$ GeV in Fig. 2b. The hierarchy in the cross-sections is very striking, with $\sigma(B_s^0 K^- X) \simeq \sigma(B_u^- K^+ X)$ being the largest and $\sigma(B_s^0 K^+ X)$ the smallest cross-section. For the particular choice of $f(s\bar{s})$, we note that $\sigma(B_s^0 K^- X)/\sigma(B_s^0 K^+ X) \simeq 5$ for $Z = 2E_k/\sqrt{s} \geqq 0.1$ and this ratio becomes $\gtrsim 10$ for $Z \geqq 0.2$. In the presence of $B_s^0 - \bar{B}_s^0$ mixing, no change is really expected in the cross-sections for $e^+ e^- \to B_u K^{\pm} X$, $B_d^0 K^{\pm} X$ but the cross-sections $\sigma(B_s^0 K^- X)$ and $\sigma(B_s^0 K^+ X)$ are expected to change. They would tend to each other and become identical for complete $B_s^0 - \bar{B}_s^0$ mixing, in which case

$$\frac{d\sigma}{dZ}(e^+ e^- \to B_s^0 K^- X) = \frac{d\sigma}{dZ}(e^+ e^- \to B_s^0 K^+ X)$$

for all values of Z. This is an effect hard to miss for detectors having a good B_s^0 and $K^{\pm}$ identification!

How does one tag on the B_s meson in a bottom quark jet produced for example at PETRA/PEP and LEP/SLC energies? There might be more favourable decay modes discovered in the course of time, but as of now we recall from the discussion above that the decays $B \to l^- K^+ X$ are expected to provide a B_s^0-

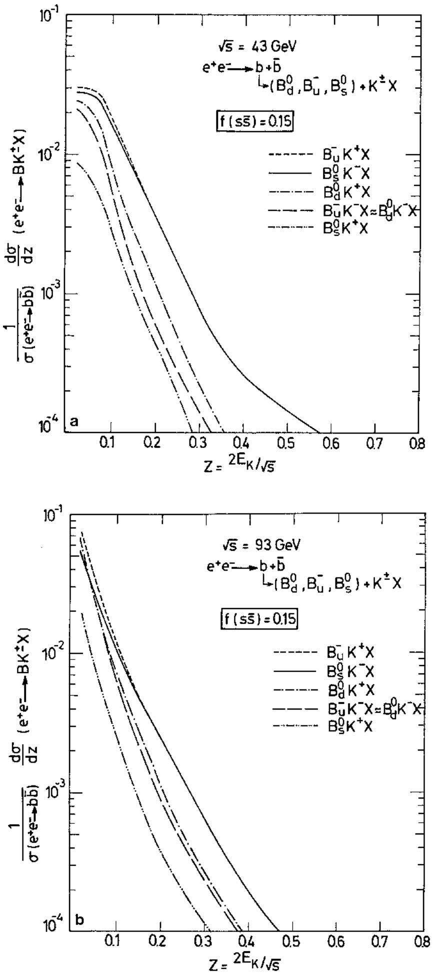

Fig. 2. **a** The differential cross-section $\frac{1}{\sigma(b\bar{b})}\frac{d\sigma}{dZ}$ $\cdot(e^+e^-\to BK^{\pm}X)$ for the various bottom mesons at $\sqrt{s}=43$ GeV, with $f(s\bar{s})=0.15$. The B and $K^{\pm}$ mesons belong to the same jet (hemisphere). **b** Same as **a** but at $\sqrt{s}=m_Z\simeq 93$ GeV

enriched sample. To that end we need the branching ratios for the decays $B_u^-\to l^-K^{\pm}X$, $B_d^0\to l^-K^{\pm}X$ and $B_s^0\to l^-K^{\pm}X$, none of which are presently available. We make the assumption based on the quark-parton model that the semileptonic branching ratios for all three bottom mesons are close to each other,

$$\mathrm{BR}(B_u^-\to l^-X)\simeq\mathrm{BR}(B_d^0\to l^-X)\simeq\mathrm{BR}(B_s^0\to l^-X). \quad (12)$$

Then, since the transition $b\to c$ completely dominates over the transition $b\to u$, [17] and the two-body leptonic decays of the bottom mesons are also negligible, the X in (12) essentially consists of $(D^0, D^+)+\dots$ and $F^+ +\dots$ for the (B_u^-, B_d^0) and B_s^0 semileptonic decays, respectively [18]. We, therefore need to know the inclusive branching ratios for $D^0\to K^{\pm}X$, $D^+\to K^{\pm}X$ and $F^+\to K^{\pm}X$.

The experimental situation about D^0 and D^+ decays is quite satisfactory [17], with about 80 % of the exclusive D^0 decays and 85 % of the D^+ decays now well measured. Only one of the measured exclusive D^0 decays namely $D^0\to K^-K^+$ involves a K^+ with a branching ratio $\mathrm{BR}(D^0\to K^-K^+)=(0.6\pm 0.09)$ %. There are several exclusive modes known for the D^+ decays involving K^+. Summing over all of them, one gets $\mathrm{BR}(D^+\to K^+X)=(2.8\pm 0.5)$ %. Thus, it seems that the Cabibbo suppression is really working very efficiently in the inclusive $D^0, D^{\pm}$ decays. The experimental situation about $F^{\pm}$ decays is rather unsatisfactory. However, since the dominant decays of the F^+ would involve the decays $F^+\to(s\bar{s})l^+\nu_l$, $(s\bar{s})(u\bar{d})$, and $s\bar{s}$ would give $\eta, \eta', \varphi, K\bar{K}$, we expect substantial branching ratios for the decays $F^+\to K^+X$ and $F^+\to K^-X$. Certainly in this case, no Cabibbo suppression is operative for $F^+\to K^+X$. In Table 1 we list the inclusive branching ratios for the D^0, D^+ and F^+ decays that we have used in our calculations. The two set of values for the $F^{\pm}$ decays bracket the expectations in a number of models [19] and probably represent a fair indication of the uncertainty in the F^- decays. We have used the Cabibbo suppressed $D^0\to K^+X$, $D^+\to K^+X$ inclusive rates from the 1984 Particle Data Table (PDT) [20] and have incorporated all the known and updated exclusive decays of D^0 and $D^{\pm}$ [17]. In our opinion the inclusive branching ratios for $D^{0,\pm}\to K^+X$ are rather high in PDT and so our background estimates are somewhat exaggerated. Before we leave the discussion of the $F^{\pm}$ and D^0 decays, we note that since the decays $D^+, D^0\to\eta X, \eta' X$ are very much suppressed but the decays $F^+\to\eta X$ have been measured, another potentially interesting final state to detect $B_s^0-\bar{B}_s^0$ mixing would be in the final states $b\to l^{\pm}K^{\mp}\eta X$, $l^{\pm}K^{\mp}\eta' X$.

Table 1. Inclusive branching ratios for the charmed meson decays used in the calculations

Modes	Branching Ratios (%)	
$D^0\to K^-X$	55.0	
$D^0\to K^+X$	9.5	
$D^+\to K^-X$	30.0	
$D^+\to K^+X$	6.5	
	case (i)	case (ii)
$F^+\to K^-X$	20.0	30.0
$F^+\to K^+X$	30.0	45.0

4. Results

We now discuss the results for the final states most sensitive to the issue of $B_s^0-\bar{B}_s^0$ mixing, namely

$$e^+e^-\to b\bar{b}\to l^{\pm}l^{\pm}X,\ l^{\pm}l^{\mp}X$$

Table 2. Estimated rates based on $10^5 b\bar{b}$ events for the production of final states $e^+e^-\to b\bar{b}\to l\bar{l}X$, lXX $(l=e+\mu)$ at $\sqrt{s}=43$ GeV with cuts (i) described in the text

χ_s	$l^\pm l^\pm X$	l^+l^-X	$l^\pm l^\pm X/l^+l^-X$	$l^\mp K^\pm K^\pm X$	$l^\mp K^\mp K^\mp X$	$l^\mp K^+K^-X$	$\Delta(lKK)$
0.0	845	4065	0.21	590	1275	6010	0.37
0.20	1100	3800	0.29	685	1220	5970	0.28
0.30	1200	3710	0.32	750	1180	5945	0.22
0.40	1270	3640	0.35	800	1140	5935	0.17
0.50	1370	3540	0.38	845	1110	5920	0.14

Table 3. Estimated rate based on $10^5 b\bar{b}$ events for the production of final states $e^+e^-\to b\bar{b}\to l\bar{l}X$, lXX $(l=e+\mu)$ at $\sqrt{s}=m_Z=93$ GeV with cuts (ii) described in the text

χ_s	$l^\pm l^\pm X$	l^+l^-X	$l^\pm l^\pm X/l^+l^-X$	$l^\mp K^\pm K^\pm X$	$l^\mp K^\mp K^\mp X$	$l^\mp K^+K^-X$	$\Delta(lKK)$
0.0	35	735	0.045	420	1000	3640	0.41
0.20	55	715	0.075	475	965	3620	0.34
0.30	90	680	0.130	535	935	3600	0.28
0.40	105	665	0.160	540	925	3595	0.25
0.50	115	655	0.175	570	900	3590	0.23

and

$$e^+e^-\to b\bar{b}\to l^\pm K^\mp K^\mp X,\, l^\pm K^\pm K^\pm X,\, l^\pm K^- K^+ X.$$

To remove the background from the processes $e^+e^-\to c\bar{c}\to l^\pm X$ and the cascade decays

$$e^+e^-\to b\underset{\hookrightarrow \bar{c}\to l^- X}{\bar{b}}$$

leading also to the final states $e^+e^-\to l^\pm l^\pm X$, etc., without any $B-\bar{B}$ mixings, we have out a cut-off on the transverse momentum of the leptons measured with respect to the jet-axis. Similarly, since the signal/background ratio is expected to improve with the increase in the momentum of the $K^\pm$, we have put a lower cut-off on the momentum of the $K^\pm$. However, in view of the limited range available in most $\frac{dE}{dX}$ devices, one cannot arbitrarily increase the cut-off on E_K. We have chosen the cut-offs (i) $p_T^l>1.0$ GeV, $p_K>0.5$ GeV at $\sqrt{s}=43$ GeV and (ii) $p_T^l>1.5$ GeV, $p_K>0.5$ GeV at $\sqrt{s}=93$ GeV.

In Table 2 we present our results for the final states $e^+e^-\to llX$, $lKKX$ for $\sqrt{s}=43$ GeV with all possible charge combinations for $l^\pm$ and $K^\pm$. The corresponding results for $\sqrt{s}=93$ GeV + cuts (ii) are presented in Table 3. The choice of $10^5 b\bar{b}$ events in e^+e^- continuum represents a reasonable sample for the kind of effects we are discussing, though for maximum mixing $\chi_s=0.5$, one may already be able to see a definite trend with $10^4 b\bar{b}$ events also.

Concentrating on the results of Table 2, which are representative for the PETRA/PEP energy, we note that the dilepton ratio $l^\pm l^\pm X/l^+l^-X$ rises from 0.21 to 0.38 for $\chi_s=0$ to $\chi_s=0.5$. The ratio

$$\Delta(lKK)=\frac{\sigma(l^-K^-K^-)-\sigma(l^-K^+K^+)}{\sigma(l^-K^-K^-)+\sigma(l^-K^+K^+)}$$

decreases from 0.37 to 0.14 in the same interval. With the cut-offs (i), the rates for $\mu^\pm K^\mp K^\mp X$ are higher as compared to $\mu^\pm\mu^\pm X$ events. At $\sqrt{s}=93$ GeV, relevant for LEP/SLC energies and the cut-off (ii) the dilepton ratio $l^\pm l^\pm X/l^+l^-X$ rises from 0.045 $(\chi_s=0)$ to 0.175 $(\chi_s=1)$. In contrast, the ratio $\Delta(lKK)$ decreases from 0.41 to 0.23. The rates for the $\mu^\pm K^\mp K^\mp X$ states are now substantially higher than those for the $\mu^\pm\mu^\pm X$ states. The decrease in the ratio $\Delta(lKK)$ as a function of χ_s is a special case of the results shown in Figs. 2a, b, where it was shown that the cross-section $B_s^0K^+X$ and $B_s^0K^-X$ tend to each other with increasing χ_s. As is obvious, for equal number of $b\bar{b}$ events, the change in $\Delta(lKK)$ due to $B-\bar{B}$ mixing is more marked at lower energies than at higher energies, since the $K^\pm$ multiplicity in the bottom quark jet is lower at lower $\sqrt{s}$, resulting in smaller background.

The actual contribution to $\Delta(lKK)$ from the $B_s^0-\bar{B}_s^0$ mixing, namely

$$\frac{\Delta(lKK,\chi_s)-\Delta(lKK,\chi_s=0)}{\Delta(lKK,\chi_s=0)},$$

is independent of $f(s\bar{s})$. In contrast, the dilepton ratio $l^\pm l^\pm X/l^\pm l^\mp X$ depends on $f(s\bar{s})$, since the numerator receives contribution only when a B_s^0 is

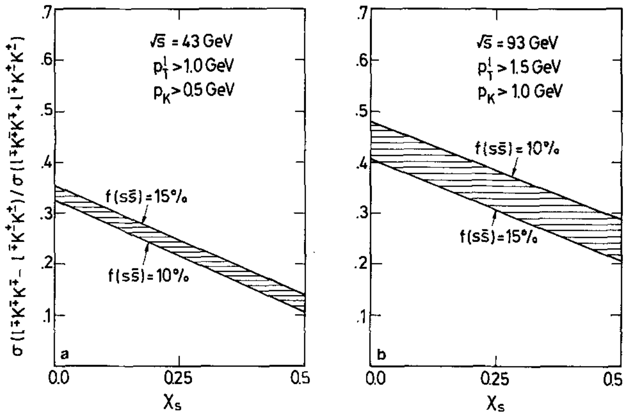

Fig. 3. a. The quantity

$$\Delta(lKK) = \frac{[\sigma(e^+e^- \to l^{\mp} K^{\mp} K^{\mp} X) - \sigma(e^+e^- \to l^{\mp} K^{\pm} K^{\pm} X)]}{[\sigma(e^+e^- \to l^{\mp} K^{\mp} K^{\mp} X) + \sigma(e^+e^- \to l^{\mp} K^{\pm} K^{\pm} X)]}$$

as a function of the mixing measure χ_s at $\sqrt{s} = 43$ GeV with the indicated values of $f(s\bar{s})$ and cuts. Note that lKK belong to the same jet (hemisphere). **b** Same as **a** but at $\sqrt{s} = m_Z \simeq 93$ GeV

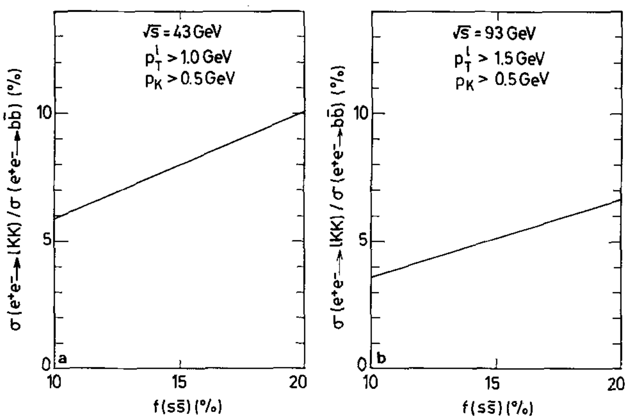

Fig. 4. **a** The branching ratio $\frac{\sigma(e^+e^- \to lKKX)}{\sigma(e^+e^- \to b\bar{b})}$ summed over all charged combinations of $l^{\pm}$ and $K^{\pm}$ as a function of $f(s\bar{s})$ with the indicated cuts at $\sqrt{s} = 43$ GeV. Note that (lKK) belong to the same jet (hemisphere). **b** Same as **a** but at $\sqrt{s} = m_Z = 93$ GeV

present. The actual measurement of $\Delta(lKK)$ i.e. signal background) however, would depend on $f(s\bar{s})$ as shown in Fig. 3, where we have shown $\Delta(lKK)$ as a function of χ_s for $\sqrt{s} = 43$ and 93 GeV.

In Fig. 4, we show how to determine $f(s\bar{s})$ in bottom quark jets, where we plot the branching ratio for the inclusive final states

$$\sigma(e^+e^- \to (l^{\pm} K^+ K^- + l^{\pm} K^{\pm} K^{\pm} + l^{\pm} K^{\mp} K^{\mp}) X$$

involving $l^{\pm}$ and $K^{\pm}$, as a function of $f(s\bar{s})$ for the range $0.1 < f(s\bar{s}) < 0.2$. As expected, the cross-section

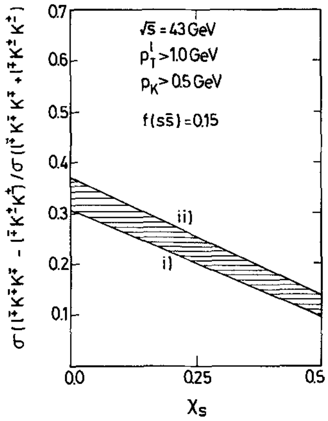

Fig. 5. The dependence of the quantity $\Delta(lKK)$ on the assumed branching ratios for the inclusive decays $F^+ \to K^{\pm} X$ (sets (i) and (ii) from Table 1 at $\sqrt{s} = 43$ GeV, $f(s\bar{s}) = 0.15$ and indicated cuts

$\sigma(e^+e^- \to lKKX)$ depends linearly on $f(s\bar{s})$ and a measurement of this quantity should already be possible with the existing data at PETRA/PEP energies. This would then provide an independent measurement of $f(s\bar{s})$ for bottom jets in the same experiment where effects of $B_s^0 - \bar{B}_s^0$ mixing are being looked at and would fix $f(s\bar{s})$ for both the $\Delta(lKK)$ and $l^{\pm} l^{\pm}/l^+ l^-$ measurements.

Finally, in Fig. 5 we show the dependence of $\Delta(lKK)$ on the assumed branching ratios for the inclusive decays $F^+ \to K^{\pm} X$. This serves as a fair indication of the attendant uncertainty. However, this uncertainty is not due to some artifact of our model but due to lack of experimental data on $F^{\pm}$ decays and we hope would be soon removed.

5. Conclusions

In conclusion, we have discussed a new measure, $\Delta(lKK)$, of $B_s^0 - \bar{B}_s^0$ mixing, which we hope is useful for experiments studying bottom hadron production in the continuum e^+e^- and $\bar{p}p$ collisions. The measure is meaningful only for those experiments having good $l^{\pm}$ and $K^{\pm}$ detection ability. It goes beyond the standard technique of measuring mixings namely through the ratio $l^{\pm} l^{\pm}/l^+ l^-$ in $(e^+e^-, \bar{p}p) \to b\bar{b} \to l\bar{l}X$ and the reduction in the electroweak charge asymmetry in the process $e^+e^- \to b\bar{b} \to l^{\pm} X$, in so far as it is sensitive only to the $B_s^0 - \bar{B}_s^0$ mixing. It depends on the semileptonic branching ratio $\mathrm{BR}(B_s^0 \to l^- X)$ and the production cross-section $\sigma(B_s^0 X)$ like its other two aforementioned counterparts. We propose a method to determine $f(s\bar{s})$ in the bottom quark jets; very probably this will determine $\sigma(B_s^0 X)$. However, we do expect the dependence of $\Delta(lKK)$ on both $\sigma(B_s^0 X)$ and $\mathrm{BR}(B_s^0 \to l^- X)$ to be mild, since in the limiting case of $\sin^2 \vartheta_c = 0$ and keeping leading $K^{\pm}$ mesons this dependence drops out.

Though we have discussed in detail only the production of bottom quark jets in e^+e^- annihilation, but the calculations, in principle, are also applicable for $p\bar{p}$ collisions, or for bottom jets produced in some other hard collision, like for example in the process $ep \to bx$ at HERA. The calculations of the background from the so-called minimum bias events will have to be performed. Detailed studies of the jet profile at the CERN $\bar{p}p$ collider suggest that additional cuts on the transverse energy and pseudorapidity of the kaons will drastically reduce the contamination of the jet fragments due to the beam fragments.

We hope that the lepton-kaon correlations, suggested in this paper will bring us closer to an understanding of the heavy quark jet flavour profile.

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