ON THE QCD 2 → 3 CONTRIBUTIONS TO THE HADROPRODUCTION OF HEAVY QUARKS

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The use of tree level QCD 2 → 3 cross sections for heavy quark production are examined in detail; they are known to give significant contributions to $c\bar{c}$ and $b\bar{b}$ production at hadron colliders. Results are presented for the 2 → 3 contributions which can be reliably calculated from QCD, namely (A) the production of three isolated high p_T jets, (B) heavy flavour excitation where only one of the heavy quarks has large p_T and, (C) collinear heavy quark pair production where the pair form a single high p_T jet. There are no appreciable 2 → 3 contributions for t-quark production.

Heavy quark production at hadron colliders has become a central issue not only because of the evidence for the top-quark [1], but also because of the richness of the dimuon signal [2], the possibility of detecting $B^0-\bar{B}^0$ mixing [3,4], and the observation of an unexpectedly large charged meson content in high transverse momentum (p_T) jets [5]. It has become clear that in addition to the well-known 2 → 2 QCD fusion processes [6], $q\bar{q}$, gg → $Q\bar{Q}$, the higher order 2 → 3 contributions [7] to charm- and bottom-quark production must be included to study the above phenomena. Various approaches have been taken to calculate the higher order contributions; the massless 2 → 3 matrix elements [8] have often been used as a guide [9,10,3], the g → $Q\bar{Q}$ splitting approximation has been used for studying the charmed quark content of gluon jets [11–13], and also the exact 2 → 3 matrix elements [7] have been used [13–15,4]. However the 2 → 3 contributions presented in the literature have widely different magnitudes due to the sensitivity to the particular cut-off which is used to regulate the soft and collinear singularities. This cut-off dependence is unphysical, and arises because of our ignorance of the complete order α_s^3 corrections, namely the one-loop corrections to the 2 → 2 processes. Here we attempt to clarify the situation by introducing appropriate cut-offs and by emphasizing those final state configurations which can be readily calculated by the tree-level 2 → 3 amplitudes alone and by demonstrating their insensitivity to the particular cut-off values used. We also examine the accuracy of using massless matrix elements (that is setting $m_Q = 0$ in the numerators of the 2 → 3 amplitudes) and of making use of the g → $Q\bar{Q}$ splitting approximation.

To simplify the discussion we introduce in fig. 1 different possible 2 → 3 configurations using the subprocess gg → g$Q\bar{Q}$ as an example. The diagrams correspond to (A) three-jet production, (B) heavy quark excitation, (C) collinear $Q\bar{Q}$ production, (D) collinear gluon emission from a heavy quark, (E) initial state collinear gluon emission, and (F) soft gluon emission. Upon integration over the relevant phase space region, configuration A is free from singularities, whereas large logarithms $\ln(\hat{s}/m_Q^2)$ appear from configurations B, C, and D when the colliding parton CM energy $\sqrt{\hat{s}}$ is much larger than the heavy quark mass, whilst configurations E and F suffer from collinear and soft singularities. Configurations D, E and F become degenerate in energy with the 2 → 2 configuration in the collinear and/or soft limit and hence should be considered only in conjunction with the virtual emission (loop) corrections. However the remaining three configurations A, B, and C give the "finite piece" of the 2 → 3

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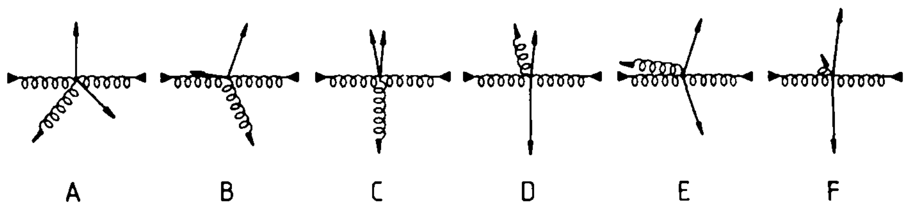

Fig. 1. Typical three-momentum configurations for the 2 → 3 heavy quark pair production process $gg \to gQ\bar{Q}$ are: three-jet production (A), heavy quark excitation (B), collinear $Q\bar{Q}$ production (C), collinear gluon emission from heavy quarks (D), from initial gluons (E) and soft gluon emission (F).

cross sections when suitably cut-off from the 2 → 2 region. The cut-offs we use are

$$p_T(Q\bar{Q})/m(Q\bar{Q}) > \epsilon\,, \quad \theta_{Qc}, \theta_{\bar{Q}c} > \delta\,, \qquad (1a,b)$$

for $ab \to cQ\bar{Q}$ which collectively denotes the subprocess $gg \to gQ\bar{Q}$, $qg \to qQ\bar{Q}$ and $q\bar{q} \to gQ\bar{Q}$. Condition (1a) ensures that the final light parton, c, is hard and acollinear to the initial parton momenta so that regions E and F are avoided, while condition (1b) on the angles in the parton CM frame excludes region D by requiring acollinearity with the heavy quark momenta.

First we examine the relative contribution of the 2 → 3 processes, as defined by the cuts of eq. (1), to the total heavy quark production cross section at fixed $Q\bar{Q}$ invariant mass:

$$R(\epsilon, \delta) \equiv \frac{d\sigma(2 \to 3; \epsilon, \delta)/dm(Q\bar{Q})}{d\sigma_0(2 \to 2)/dm(Q\bar{Q})}\,, \qquad (2)$$

where $d\sigma_0/dm$ denotes the lowest order 2 → 2 cross section summed over the $gg \to Q\bar{Q}$ and $q\bar{q} \to Q\bar{Q}$ subprocesses. The total heavy quark production cross section can be expressed in the next-to-leading order as

$$\frac{d\sigma(p\bar{p} \to Q\bar{Q}X)}{dm(Q\bar{Q})} = \frac{d\sigma_0(2 \to 2)}{dm(Q\bar{Q})}\,[1 + (\alpha_s/\pi)A(\epsilon,\delta)] + \frac{d\sigma(2 \to 3; \epsilon, \delta)}{dm(Q\bar{Q})} = K\,\frac{d\sigma_0(2 \to 2)}{dm(Q\bar{Q})}\,, \qquad (3)$$

where the cut-off dependence of the term $A(\epsilon, \delta)$, which comes from the regions D, E, F and the loop corrections, exactly cancels that of the 2 → 3 cross section to give a cut-off independent correction to the lowest order result, the K-factor. In the absence of one-loop calculation, the best estimate of the total cross section is still the lowest order result. On the other hand we do get reliable leading-order estimates for heavy quark production in configurations A, B, and C. A compromise exclusive distribution, which makes the best use of our limited knowledge without giving a cut-off dependent total cross section, is obtained as follows:

$$d\sigma(p\bar{p} \to QQX) = d\sigma_0(2 \to 2)\,[1 - R(\epsilon, \delta)] + d\sigma(2 \to 3; \epsilon, \delta)\,, \qquad (4)$$

where R is the $m(Q\bar{Q})$ dependent ratio given by eq. (2). Upon integration over the phase space, the distribution (4) trivially reproduces the lowest order cross section.

If the ratio R is sufficiently less than one and if the cut-offs ϵ and δ are sufficiently small that the distinction between the configurations D, E, F in fig. 1 and the 2 → 2 configuration is not too large, then we can use the distribution (4) to generate heavy quark events and study their cut-off independent signals. This is the approach taken in this paper.

Fig. 2 shows the ratio $R(\epsilon, \delta)$ as a function of $m(Q\bar{Q})$ for charm and bottom quark production in $p\bar{p}$ collisions at $\sqrt{s}$ = 630 GeV for two choices of cut-off, $\epsilon = 0.2$ and $\epsilon = 0.3$. A fixed $p_T(Q\bar{Q})$ cut-off is frequently used in the literature and so we also compare the ϵ cut-offs with a $p_T(Q\bar{Q}) > 5$ GeV cut. We take $\delta = 20°$ in the colliding parton CM frame; the dependence on the cut-off parameter δ is weak for bottom and moderate for charm at the energy scale explored in this paper. The maximum sensitivity is for charm in the region $m(c\bar{c}) \gtrsim 20$ GeV where increases of about 5% may occur for $\delta = 0°$. From now on we set $\delta = 20°$. Also we use throughout set I of the Duke–Owens

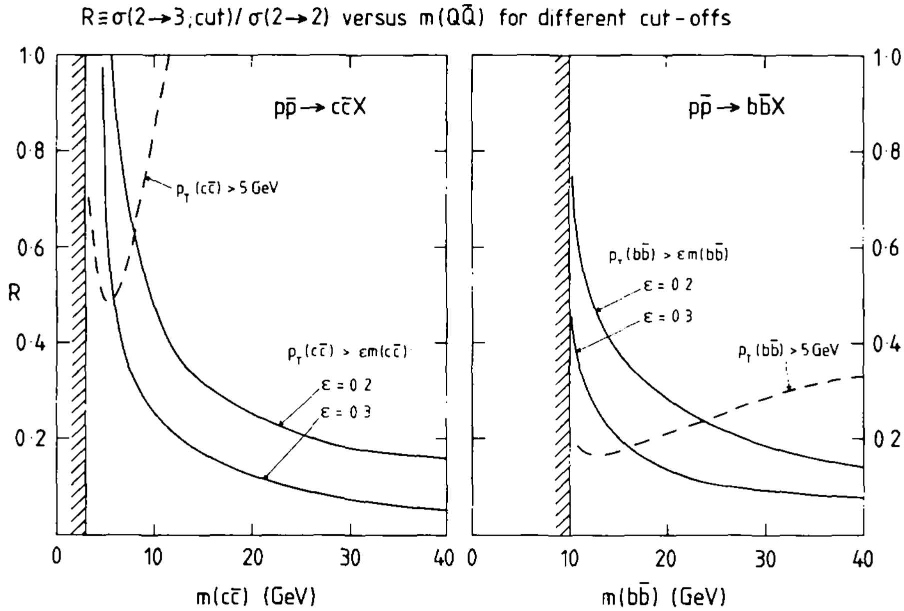

Fig. 2. Ratio of $2 \to 3$ and $2 \to 2$ contributions to heavy quark production in $p\bar{p}$ collisions at $\sqrt{s} = 630$ GeV shown as a function of the heavy quark pair invariant mass $m(Q\bar{Q})$ for different $p_T(Q\bar{Q})$ cut-offs, all with $\theta_{Qc}, \theta_{\bar{Q}c} > 20°$ in the colliding parton CM frame. The heavy quark masses are chosen as $m_c = 1.5$ GeV and $m_b = 5$ GeV.

parton distributions [16] and the running coupling constant $\alpha_s(Q^2)$, both evaluated at $Q^2 = \hat{s}$, where

$$\frac{\pi}{\alpha_s(Q^2)} = \frac{25}{12} \ln \frac{Q^2}{\Lambda_4^2} - \frac{2}{3} \sum_{q=b,t} \theta(Q^2 - 4m_q^2) \ln \frac{Q^2}{4m_q^2}, \tag{5}$$

with $\Lambda_4 = 0.2$ GeV and $(m_c, m_b, m_t) = (1.5, 5, 40)$ GeV. It is worth noting that the ratio R for $t\bar{t}$ production is very small (typically 0.03 for $\epsilon = 0.2$–0.3) at CERN collider energies and so in this case the $2 \to 2$ fusion predictions should be reliable.

The commonly used fixed p_T cut, $p_T(Q\bar{Q}) > 5$ GeV, leads to large values of R exceeding unity for charm and hence is not suited for our purpose. Here, we argue it is better to use a scaled cut-off ($\epsilon = 0.2$ or $\epsilon = 0.3$). We find that this gives reasonable ratios except at very small $m(c\bar{c})$ values; small-p_T small-mass $c\bar{c}$ production is not our immediate concern. The use of a scaled cut-off can be motivated by the results of the complete next-to-leading order studies [17] in the Drell–Yan processes where the lowest-order calculation was found to be reliable for $p_T(\ell\bar{\ell})/m(\ell\bar{\ell}) \gtrsim 0.3$ almost independent of the $m(\ell\bar{\ell})/\sqrt{s}$ values. Indeed the recent investigation by Collins et al. [18] showed that the factorization of $Q\bar{Q}$ production can be proven in much the same way as that of the Drell–Yan process. This suggests that the all-order summation of the leading singularities can be done in a similar manner leading to the regularized distribution over the full $p_T(Q\bar{Q})$ region. A first attempt in this direction has been made very recently by Barger and Phillips [19]. We make one comment on this promising approach: in the absence of the complete next-to-leading order calculation, it is necessary to test the insensitivity of the final results to the shape of the regularized distribution in the region $p_T(Q\bar{Q}) < \epsilon m(Q\bar{Q})$.

We are now ready to examine the distributions arising from the $2 \to 3$ configurations of physical interest, namely those insensitive to the cut-off. First the three-jet region, A, which may be defined by the jet-defining algorithm similar to the one used by the UA1 collaboration [20];

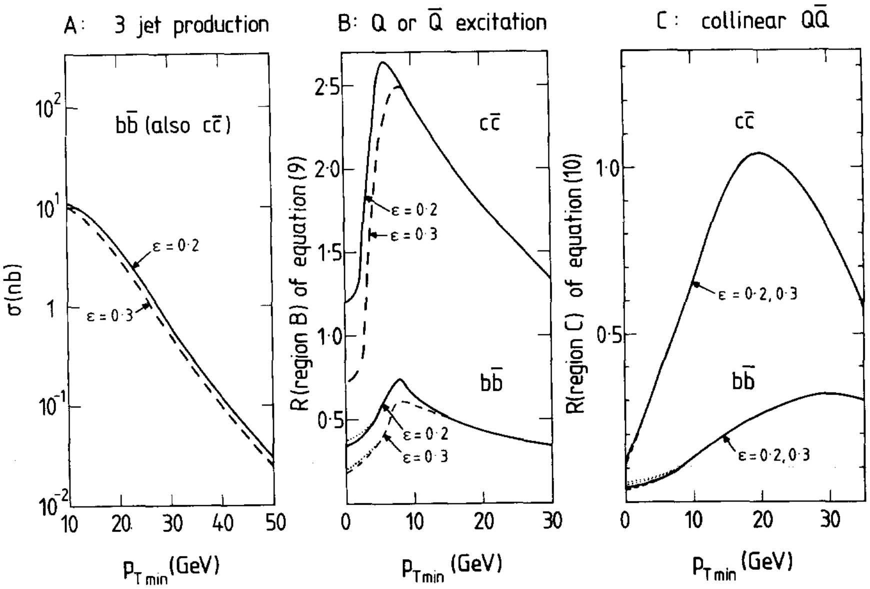

Fig. 3. The $2 \to 3$ cross sections for the production of heavy quarks in $p\bar{p}$ collisions at $\sqrt{s} = 630$ GeV in regions A, B, C respectively (the latter two normalised to the $2 \to 2$ contribution) as functions of the minimal trigger p_T of a heavy quark defined by Max$\{p_{QT}, p_{\bar{Q}T}\} > p_{T\,min}$. Solid and dashed curves are obtained by the cut-offs $(\epsilon, \delta) = (0.2, 20°)$ and $(0.3, 20°)$, respectively. The dotted lines are obtained by using the massless matrix element approximation of ref. [9], and are only distinguishable from the exact results for $b\bar{b}$ at small $p_{T\,min}$.

$$\text{A}: d(i,j) > 1 \quad \text{for } (i,j) = (Q,\bar{Q}), (c,Q), (c,\bar{Q}), \qquad (6a)$$

$$: p_T(i) > 10 \text{ GeV} \quad \text{for } i = Q, \bar{Q}, c, \qquad (6b)$$

where d is the separation in the pseudorapidity-azimuthal angle plane

$$d(i,j) = [(\Delta\eta_{ij})^2 + (\Delta\phi_{ij})^2]^{1/2}. \qquad (7)$$

The cross section in this region is shown in fig. 3a, for $p\bar{p}$ collisions at $\sqrt{s} = 630$ GeV, as a function of the minimal trigger ‡1 p_T of a heavy quark. We find no observable difference between the charm- and bottom-quark contributions and, moreover, the massless matrix element approximation [9,10,3] gives identical results. We conclude that the perturbative estimate of the three-jet contribution, as defined by eq. (6), is reliable because almost the entire contribution comes from the "safe" region, $p_T(Q\bar{Q})/m(Q\bar{Q}) > 0.3$. The three-jet configuration of $b\bar{b}$ production has a special physical significance as it forms a background to the μ + dijet signal of $W \to b\bar{t}$ events [9]. Since the high-p_T muon can only come from higher p_T b-quarks, we can estimate from fig. 3a the cross section of having μ + dijet events from this source and hence see the number of events which have to be eliminated by muon isolation techniques [9].

The excitation region, B, may be most clearly recognized by the absence of a heavy quark jet, and hence of a muon, in the back-to-back configuration of the triggered high-p_T heavy quark jet. We therefore require

$$\text{B: Min}\{p_{QT}, p_{\bar{Q}T}\} < 5 \text{ GeV}, \qquad (8)$$

so that one of the pair can only give rise to a small-p_T muon. We show in fig. 3b the cross-section satisfying the condition (8) as a function of the minimal trigger p_T of the other heavy quark normalized to the $2 \to 2$ cross section, in the form

‡1 One heavy quark must have sufficient p_T to give rise to a trigger muon (or an identifiable vertex).

$$R(\text{region B}) = [\sigma(2\to 3; \text{Min}\{p_{QT}, p_{\bar{Q}T}\} < 5 \text{ GeV},$$
$$\text{Max}\{p_{QT}, p_{\bar{Q}T}\} > p_{T\,min})/$$
$$\sigma(2\to 2; p_{QT} = p_{\bar{Q}T} > p_{T\,min}) \,. \quad (9)$$

Here, unlike region A, there is striking flavour dependence; recall that the $2 \to 2$ normalization factor in eq. (9) is essentially flavour independent in the high-p_T region. It is remarkable that the very large $2 \to 3$ contributions at higher trigger p_T ($p_{T\,min} > 10$ GeV for charm, $P_{T\,min} > 15$ GeV for bottom) come entirely from the region of phase space where the $2 \to 3$ cross sections are reliable (i.e. cut-off independent). We therefore conclude that the production of a high-p_T charm- or bottom-quark jet balanced essentially by a light parton jet (with the remaining heavy quark jet at low p_T) is very significant and is a reliable prediction of QCD perturbation theory. This phenomenon gives an excess of single-muon events as compared to the back-to-back dimuon events resulting from the $2 \to 2$ processes alone.

The region C, in which Q and $\bar{Q}$ are collinear, may simply be defined by requiring that the Q and $\bar{Q}$ lie within the same jet, that is $d(Q,\bar{Q}) < 1$. Fig. 3c shows the appropriate ratio

$$R(\text{region C}) = \frac{\sigma(2\to 3; d(Q,\bar{Q})<1, \text{Max}\{p_{QT}, p_{\bar{Q}T}\} > p_{T\,min})}{\sigma(2\to 2; p_{QT} = p_{\bar{Q}T} > p_{T\,min})}\,, \quad (10)$$

as a function of the minimal trigger p_T of a heavy quark. We again find strong flavour dependence. The rather large value of these ratios are coming from the $2 \to 3$ contributions far away from our cut-off region. For each high-p_T charm- or bottom-quark jet, there is an appreciable chance of the accompanying heavy quark being within the same jet.

To gain insight into the m_Q dependence displayed

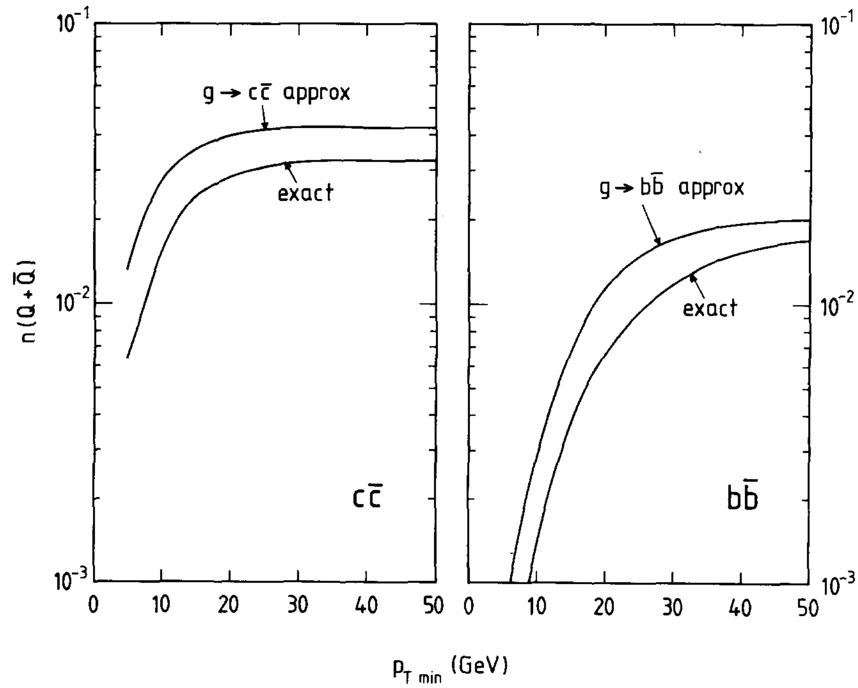

Fig. 4. Charm and bottom quark multiplicity in a jet in $p\bar{p}$ collisions at $\sqrt{s} = 630$ GeV, shown as a function of the minimal trigger p_T of a jet. No cut-off dependence appears in these distributions. The curves are obtained by using respectively, the exact $2 \to 3$ matrix elements, and the $g \to Q$ splitting approximation of ref. [12].

in fig. 3 consider, first, cofigurations B and C. B contains a region of phase space where the heavy quark propagator can become as large as order of m_Q^{-2}. Similarly C contains a region where the gluon propagator, $(m_{Q\bar{Q}})^{-2}$, can be as large as order m_Q^{-2}. Contributions B and C therefore depend on the value of m_Q. (Indeed, the success of the massless matrix element approximation in these regions is a non-trivial result). However in region A neither of the propagators can become as large as m_Q^{-2}, because the collinear singularities are cut-off by fractions of the large energy scale $\sqrt{s}$ and not by m_Q. Thus it is safe to neglect m_Q in region A.

For region C we may compute the heavy quark multiplicity in a high-p_T jet

$$n(Q+\bar{Q}) = 2\,\frac{\sigma(2\to 3;\, d(Q,\bar{Q})<1,\, p_T(Q\bar{Q})>p_{T\min})}{\sigma(\text{all } 2\to 2;\, p_T(j)>p_{T\min})}, \quad (11)$$

where here the denominator denotes the sum of all QCD 2 → 2 contributions to high-p_T jet production. We show in fig. 4 the resulting charm and bottom quark multiplicity in a jet as a function of the minimal jet p_T, using (i) the exact 2 → 3 matrix elements [7], and (ii) the gluon-to-massive quark splitting approximation [12]. It is interesting to observe that the g → $Q\bar{Q}$ splitting approximation [12] consistently overestimates the multiplicity by about 20% all the way up to the highest p_T jets and indeed gives a reasonable description of the charmed quark multiplicity in high-p_T jets. The calculated rate of about 0.03 is far below the reported observation [5], and is consistent with previous estimates [10–13].

In summary, we confirm the importance of the $O(\alpha_s^3)$ processes, ab → $cQ\bar{Q}$, to heavy quark production. Ignorance of the loop corrections means the result is cut-off dependent and this had led to an uncertainty in the values of the $Q\bar{Q}$ cross sections available in the literature. By working at the parton level we have attempted to clarify the position. In particular, we have emphasized and presented results for those 2 → 3 configurations which can be calculated in a reliable, cut-off independent, way. We find cross sections comparable to 2 → 2 $Q\bar{Q}$ production.

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References

[1] UA1 Collab., G. Arnison et al., Phys. Lett. 147B (1984) 493.

[2] UA1 Collab., G. Arnison et al., CERN report EP/85-19.

[3] V. Barger and R.J.N. Phillips, Phys. Lett. 143B (1984) 259; Phys. Rev. D32 (1985) 1128.

[4] A. Ali and C. Jarlskog, Phys. Lett. 144B (1984) 266; A. Ali, Proc. 5th Topical Workshop on Proton–antiproton collider physics (Aosta, February 1985) p. 272.

[5] UA1 Collab., G. Arnison et al., Phys. Lett. 147B (1984) 222.

[6] L.M. Jones and H. Wyld, Phys. Rev. D17 (1978) 782; J. Babcock, D. Sivers and S. Wolfram, Phys. Rev. D18 (1978) 162; K. Hagiwara and T. Yoshino, Phys. Lett. 80B (1979) 282; B.L. Combridge, Nucl. Phys. B151 (1979) 429.

[7] Z. Kunszt, E. Pietarinen and E. Reya, Phys. Rev. D21 (1980) 733; Z. Kunszt, private communication.

[8] T. Gottschalk and D. Sivers, Phys. Rev. D21 (1980) 102; Z. Kunszt and E. Pietarinen, Nucl. Phys. B 164 (1980) 45; F.A. Berends, R. Kleiss, P. De Causmaecker, R. Gastmans and T.T. Wu, Phys. Lett. 103B (1983) 124.

[9] V. Barger, H. Baer, K. Hagiwara, A.D. Martin and R.J.N. Phillips, Phys. Rev. D29 (1984) 1923.

[10] V. Barger and R.J.N. Phillips, Phys. Rev. D31 (1985) 215; F. Halzen and P. Hoyer, Phys. Lett. 154B (1985) 324.

[11] K. Hagiwara and S. Jacobs, Z. Phys. C28 (1985) 95.

[12] K. Hagiwara, DESY report 85-137; K. Hagiwara and S. Jacobs, in preparation.

[13] G. Köpp, J.H. Kühn and P.M. Zerwas, Phys. Lett. 153B (1985) 315; A. Ali and G. Ingelman, Phys. Lett. 156B (1985) 111.

[14] L. Schmitt, L.M. Sehgal, H.D. Throll and P.M. Zerwas, Phys. Lett. 139B (1984) 99.

[15] B. van Eijk, Proc. 5th Topical Workshop on Proton–antiproton collider physics (Aosta, February, 1985) p. 165.

[16] D.W. Duke and J.W. Owens, Phys. Rev. D30 (1984) 49.

[17] G. Altarelli, R.K. Ellis, M. Greco and G. Martinelli, Nucl. Phys. B246 (1984) 12; G. Altarelli, R.K. Ellis and G. Martinelli, Z. Phys. C27 (1985) 617; Phys. Lett. 151B (1985) 457.

[18] J.C. Collins, D.E. Soper and G. Sterman, University of Oregon preprint OITS-292 (1985).

[19] V. Barger and R.J.N. Phillips, Phys. Rev. Lett. 55 (1985) 2752.

[20] UA1 Collab., G. Arnison et al., Phys. Lett. 123B (1983) 115; 136B (1984) 294.